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Reclaiming our African pride through mathematics teaching

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Foreword

The language of learning and teaching (LoLT), diversity, context, resources and culture are important factors to consider when teaching mathematics. The plenaries, presentations and workshops of the AMESA 2016 Congress highlight and explore these factors in addition to various approaches to assist teachers to reclaim our African pride in the mathematics classroom. The National and International studies that are presented emphasise the sharing of ideas, resources and good practice.

We encourage delegates to reflect on these ideas, practices and resources as a way of rethinking how mathematics is taught in South Africa. It is only through dedication, commitment, recognising ones' role and responsibility when teaching as well as actively debating our challenges that we can reclaim our African pride through mathematics teaching. It is through this reflection, sharing and rethinking that we can be proud of teaching and learning mathematics in South Africa.

Busisiwe Goba & Jayaluxmi Naidoo

June 2016

Review process

The papers accepted for publication in this volume of the Proceedings (*Plenary Papers* and *Long Papers*) were subject to double-blind peer review by two experienced mathematics educators. The academic committee considered the reviews and made a final decision on the acceptance or rejection of each submission, as well as changing the status of submissions. Authors of accepted submissions were given the option of submitting an extended abstract rather than their full submission for publication in the Proceedings if they wished to submit their reviewed submissions for possible publication elsewhere.

Number of submissions:	105
Number of plenary paper submissions:	5
Number of long paper submissions:	35
Number of short paper submissions:	16
Number of workshop submissions:	25
Number of 'How I Teach' paper submissions:	20
Number of poster submissions:	4
Number of submissions accepted:	95
Numbers of submissions rejected:	9
Number of submissions withdrawn by authors:	1

We thank the reviewers for giving their time and expertise to reviewing the submissions.

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SUCCESS IN MATHEMATICS TEACHING AND LEARNING: A CASE OF REMOTE EDUCATION PROVISION IN AUSTRALIA

Robyn Jorgensen (Zevenbergen)

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This plenary draws on the outcomes of a national study that has been exploring the practices in successful remote (rural) schools that provide mathematics education for Australia's First People – Aboriginal and Torres Strait Islanders. To date, more than 30 case studies have been undertaken across Australia in schools that have been experiencing success in mathematics. This plenary discusses the major trends across the study, but also draws on the many nuanced practices, consistencies, contradictions, and possibilities that have emerged from the case studies. What is clear from the study is that the practices are shaped by the contexts and that there is no one way to teach mathematics. However, there are clearly some salient practices that shape access and success in remote education. These include having high expectations of teachers, students and community; middle leadership in supporting teachers and leaders to develop high quality mathematics programs; the role of language in learning mathematics and how practices need to build bridges and mediate between home languages and mathematics; and importance of data to inform teaching and the need for targeted teaching and differentiated learning. The framework that is emerging from the study, along with practical examples will be shared in this session.

Keywords: Learning, mathematics, success, teaching

INTRODUCTION

Educational research into issues associated with the scholastic success of the most vulnerable groups of learners often assumes a deficit approach and, as a consequence, highlights the issues (mostly negative) that contribute to the lack of success for marginalised learners. There are serious issues in Australia, and internationally, for First People and their success in mainstream education. In the Australian context, under successive Federal governments, there has been a strong push to 'close the gap' in education, health and housing between the First people and mainstream Australians. Despite billions of dollars being spent on intervention policies, the gap has remained at best, and indeed, widened in many cases. While there are questions about what is seen to constitute success, particularly in education, for remote peoples (Guenther, 2013), there is an undeniable difference between remote and very remote outcomes than for those students living in urban areas. This significantly differentiated outcome raises the complexity of issues impacting on the provision of education, and the capacity for success. While research (and policy) has highlighted the many issues impacting on education provision and outcomes, little is understood (or researched) as to what makes for success in remote education provision. The research outcomes shared in this plenary arise from a national (Australian) study that has explored 'successful' practice in remote and very remote contexts where there are large numbers of First Australians attending school.

ISSUES IN REMOTE EDUCATION PROVISION

There is a considerable literature that outlines the issues confronting education provision in remote Australia. In considering this expansive and long standing literature, there are a number of key themes that emerge, most notably the vast difference between the achievement of remote Aboriginal learners and other cohorts of learners (Ford, 2013). This has been the cause for on-going concerns for policy as well as education. A long standing policy for two successive governments has been to "Close the Gap" for Indigenous people in housing health and education. Many factors have been identified as potential areas of concern including the quality of teachers and teacher preparation, including both teachers and leaders within the schools and the capacity to provide professional learning for the staff in remote areas; the provision of quality cultural experiences for teachers (Luke, Shield, Theroux, Tones, & Villegas, 2012); the early career status of most of the staff in schools – both teachers (A. Sullivan & Johnson, 2012) and leaders (Niesche & Jorgensen, 2010); the motives for teaching in remote contexts (Schulz, 2015). Issues around the student cohorts have included the background of the learners including culture and language (Rahman, 2013; Rennie, 2006) as well as attendance (Ladwig & Luke, 2013; Prout, Quicke & Biddle, 2016) and the capacity of early career teachers to cater for the cultural diversity of learners (Luke et al., 2012). At the same time, there are challenges posed by the contexts themselves including living conditions which collectively impact on the transience of teachers; the cultural incongruence between teachers, students and communities and the impact on education provision (Guenther, Osborne, Arnott, & McRae-Williams, 2015); and, interconnected with many of these issues, is the involvement of First People in the education process (Thorpe, Bell-Booth, Staton, & Thompson, 2013). Furthermore, issues around identity, low expectations (Bell-Booth, Staton, & Thorpe, 2014; Sullivan, Jorgensen, Boaler, & Lerman, 2013)and independence of learners (Nielsen, Mushin, Tomaselli, & Whiten, 2014) have been seen to also impact on the ways of teaching and outcomes in remote education provision. At the same time, the importance of ensuring local people are well supported, trained and valued in the education process (Maher, 2013) has been seen to be critical in developing quality mathematics education. These are significant areas of concern in remote education provision and have a considerable research basis. They are recognised as issues that impact on the capacity to provide quality education. It is beyond the scope of this paper to provide a comprehensive review of these literatures but rather for the purposes here, it is important to note this expansive literature, for the issues identified within the broader literature were evident across the schools in this study and were the catalyst for many of the initiatives undertaken within the schools. For example, there are considerable issues around attendance, so much so, that the government has mandated that schools must report against a benchmark of 80% attendance.

Targeted funding has been deployed to schools to develop strategies to build attendance to this level (Department of Prime Minister and Cabinet, 2014). This is in recognition that the Federal government sees that 80% would seem to be a reasonable target if students are to attend and learn commensurate with age-appropriate standards (or benchmarks) and that schools need to adopt strategies to assure that this attendance benchmark is met. While there is considerable controversy over the intent and practices associated with this policy, there is an acknowledgement that students do need to attend school in order to learn. What is more at issue is the quality, relevance, cultural relevance, etc. of school education for First People. This tension would appear to be the core of the complex issue rather than 'forcing' students to attend school. Until the issues around (non) attendance are better understood, compelling students (and their families) to attend school when they are uninterested and/or disruptive can be counter-productive. Of importance to this study, was that issues around attendance were evident in the schools but significant to success was how such issues were framed and addressed. How teachers, schools, administrators, strategies and school policies around attendance were framed seemed to have a greater impact than the issue per se.

OVERVIEW OF THE STUDY

The study has been funded by the Australian Research Council through its Discovery Grant Scheme[1]. The intent of the project is to develop 32 case studies of successful remote and very remote schools across Australia which are experiencing success in the teaching of numeracy/mathematics. Due to budgeting success, the study will undertake more than the initial 32 case studies. The study is continuing into 2016. As can be seen in Table One below, the study has included

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[1] The views expressed in this paper are those of the author and not of the funding agency. The Discovery Grant is DP130103585.

the main states and territories in Australia that assume responsibility for remote education, and includes schools from state, Catholic and Independent sectors of education; along with schools that cover primary, secondary, combined sectors of schooling, along with day schools and boarding schools (both in country and out of country). This distribution represents the diversity of remote education provision across the nation.

Table 1: Distribution of schools to the end of April 2016.

	Government	**Catholic**	**Independent**	**Confirmed (Term 2 2016)**	**To be sought**	**Total**
WA	8	3	8		-	19
QLD	4				6	10
SA	2			2		4
NSW	3			3	1	7
NT			1		2	3
Total	**17**	3	9	5	9	**43**

It is noted that there are no government schools in the Northern Territory in this study due to the Department of Education not giving permission to access their schools. This is the only Department to have refused entry into schools. This does not mean that there are no schools in the Northern Territory that would qualify for inclusion in the study, only that the project could not access them. More schools will be included in 2016 from Queensland, New South Wales, Northern Territory (Independent schools) and South Australia so that a more balanced representation can be achieved.

DEFINING SUCCESS

To be included in the study, schools had to be listed at 'remote' or 'very remote' on the MySchools website. This is a national website that reports on the outcomes of the National Assessment Plan for Literacy and Numeracy (NAPLAN). This site was used as a source of information for the inclusion of schools as it is the only national comparative database for Australian schools. Data from the MySchools site were also used to identify remote and very remote schools that served high percentages of First Australians. To be included, schools needed to have a school population of around or over 80% Indigenous students. While the vast majority of schools are over 80%, often around 100%, a small number of schools have had enrolments in the high 70%.

Two sources were used for the selection of schools having 'success in numeracy' component of the study. Initially, data from the MySchool site were used to identify successful schools. It is recognised that this process is somewhat flawed due to the inherent issues of national testing processes and that the results are only a snapshot of a particular performance. However, it was seen to be the only reliable and comparable tool available for assessing success in numeracy across the cohorts of schools. Schools were selected that performed better or significantly better against comparable schools (rather than the national achievements) as this was seen to be a fairer process. Schools that also showed growth were included. Success had to be on-going (over 3 years) rather than once-off successes. The second approach was that schools could also be recommended to the research team as it was recognised that NAPLAN may not pick up success in other measures. All schools were asked to provide data to show their successes in the teaching of mathematics. These data included school-based assessments which often reflected better outcomes than those on NAPLAN. In two cases, data was collected from schools but their stories were not included in the final case studies for a range of reasons.

METHOD

The approach used in the study is two-levels. In the first instance, ethnographic case studies are undertaken at each school. The case study involves qualitative open-ended questions to the leadership team, teachers and other staff at the schools in which they describe and discuss the practices (and rationale) used at the school. Lessons are observed and a profiling of lessons is undertaken (using the Productive Pedagogies framework) so that there is a common (and robust) tool for lesson observations. School documents are also collected. At the end of each site visit, a case study is developed and approved by the principal (or nominee). These case studies are published on a project website hosted at University of Canberra (Jorgensen (Zevenbergen), 2016). The case studies are highly visual so that they can be used by schools for promotional purposes, particularly among the community.

A secondary meta level analysis is now being undertaken and will continue to evolve as schools are finalised. All interviews have been coded in NVivo and analysis of key themes is emerging. All interviews also have all been entered into Leximancer and analyses undertaken for each school and across the study. Cross referencing between the two programs will help to ensure the validity of the categories developed from NVivo and for a robust analysis to be ensured. A statistical analysis is being undertaken of the Productive Pedagogies and for the complete data set of the schools acquired from ACARA in relation to NAPLAN results. Collectively these analyses begin to illustrate the emergence of key themes – both qualitatively and quantitatively. As more school case studies will be completed in 2016, the data emerging from these analyses is only preliminary at the time of writing this paper.

FRAMEWORK

While the initial intent of the project had been to develop thick descriptions of classroom practice, the study has shown that there are multiple levels of practice that need to be considered when looking at effective practice in teaching mathematics. What is now emerging is a holistic framework for developing successful practice that encompasses multiple levels of practice.

Table 2: Levels of Practice.

Level of practice	**Description and examples**
Envisioned	Schools need to have a coherent and agreed set of values that underpin their practices. Often this is provided through the leadership team at the school who work with staff to develop and implement the vision for the school. This level of practice goes beyond the rhetorical 'vision' for the school and ensures that the vision is grounded and sustained.
Enabling	An intermediary level of practice enables the vision of the school to be translated into practice. The intermediary level includes targeted people who are able to work with both leaders of the school and the teachers in the classrooms to build the practices and repertoire of skills of teachers in line with the vision. This level engaged numeracy leaders/middle leaders and/or local people employed to support and engage students, community and teachers in culturally responsive teaching practices.
Enacted	This level provides practical strategies used by teachers in the classrooms such as group work, language resources, grouping students by attendance and behaviour, lesson planning, using data to inform teaching, explicit goals and intents being made transparent to students, pacing of lessons, the use of humour, revision and consolidation of concepts, use of digital resources, visual resources in the classrooms, creating classroom contexts for learning, differentiating and targeting teaching to the needs of learners,

In summarising the model that is emerging from the analysis being undertaken, Figure One below, captures this multi-level model of practice. It is beyond the scope of this paper to elucidate the various practices within each level but suffice to say, it is clear that the levels of practice represent the various practices that have emerged from the study.

Figure 1: Levels of Practice.

What is clear from the analysis and the case studies that have been created is that the practices at the schools have been informed by the needs of the learners. By focusing on the needs and issues around remote numeracy/mathematics education, the participants in the study have illustrated both in words and actions how they go about bringing about change for success.

ISSUES AS THE CATALYST FOR CHANGE: ATTENDANCE AS A CASE ISSUE

What is clear from the data is that the success of the schools in this study has been founded in the recognition of issues and then taking positive, proactive changes to address these issues. The issues identified at the start of this paper were evident in all the schools in this study but what has emerged from the study is the ways in which the schools have addressed the issues. For the remainder of the paper, I will focus on the issue of attendance as this is a key issue across remote education, and indeed education for most marginalised learners. There are many complexities around attendance, or more specifically, non-attendance. Attendance can refer to the physical absence of students who fail to appear at school, but equally there are students who attend school physically but do not engage in learning so effectively they are cognitively absent. Creating practices that engage learners to want to attend school both physically and cognitively has been evident across the study.

IDENTIFYING REASONS FOR NON-ATTENDANCE

There is a plethora of reasons, often quite legitimate, for non-attendance. For remote students, the culture of schooling is not aligned with the home culture making for a strong cultural dissonance between the home and school. The language of instruction – Standard Australian English – is often a second, third, or even foreign language for remote/very remote students. The only time English is spoken is at school or with government agencies. Where there have been long periods of white contact, such as the Kimberley region, a creole has developed so that the original languages of the region have been lost or are dying. There is a considerable tension in this aspect of education and raises the very serious question as to the purpose of schooling, and mathematics education, in remote lifestyle. Where the home cultures and languages are very strong and hence resonate very little with schooling, there is a question as to the relevance of schooling.

The purpose of schooling, and mathematics education, is a preparation for life which includes work and financial independence. In remote communities, there are often very few work opportunities in community. Most employment is with government agencies operating within the community. So, the rationale of mathematics education providing a pathway to work is very limited. Similarly, money handling is a life-skill that impacts significantly on people's lives but with the current policies requiring government assistance being quarantined for food, First People families have a cash card and use this to purchase food from the local stores. So, there are considerable and vexed questions as to the relevance and purpose of school and mathematics.

While this is the case, there is a very strong tension here. In the first instance, compelling students and communities to participate in schooling and mathematics education is to indoctrinate First Australians into the culture of the dominant group which then denies and ameliorates the home culture – often at the expense of the death of the home culture and language. However, and herein lies the tension, to deny access to the dominant, and hence powerful, forms of knowledge is a denial of human rights and access to power. Many families and communities want their children to have access to the dominant forms of knowledge knowing that this is how they will succeed in mainstream society and culture, but also recognise that this comes with some cost – the loss or deterioration of their home cultures and languages.

ENVISIONED PRACTICE: CREATING A HAPPY SPACE

At school A, the leadership team encountered a considerable issue with non-attendance where students would walk to the school gate and taunt teachers that they were not coming to school. The leadership team employed a consultant to come to the school and work with community and students to identify the issues around the non-attendance of the students. Feedback suggested that the community and students had not had good experiences at the school and felt they did not belong to the school and hence non-attendance resulted. The levels of discontent were quite high and this had resulted in the students taunting the staff about non-attendance. It was recognised that attendance was necessary for the staff to have employment so if they did not attend school, then the staff would not have jobs, thus giving some sense of power to the local people. To this end, the school focused on bringing about a culture within the school that was welcoming and supportive, where students felt happy and safe, and hence wanted to come to school. The development of such a culture took about three years to embed and was a whole-of-community strategy. Involving local people in the changes to the school and embedding this across the wider community was key to the strategy, so that the community felt involved and had ownership of the approaches being adopted. The pillars to the reform became central to the initiative, so much so that upon entering the community, large colourful banners were placed on power poles that identified and celebrated the pillars.

The staff was proactive in encouraging and noting positive behaviours (as opposed to punishing negative behaviours). Teachers had to identify five positive behaviours if they were concerned with one negative behaviour. Home visits were conducted in which a teacher and one on the local people employed at the school would visit homes to talk to parents about the positive work of their children. School work, including mathematics, was shared with the families so that they could see what their children were achieving at school. The leadership team would stand at the school gate in the mornings welcoming all students, by name, to school and wishing them a successful and happy day. At the end of each day, the same would happen. Where a student may have been in trouble, care was taken to say to that student that they should go home and have a good evening and that the staff was keen to see them tomorrow as it was a new day and a new start.

The strategy met with large successes. Attendance had reached over 90% which was well above the targeted 80% set by the government but more importantly, students were happy at school and had engaged in learning. While the school had not focused on implementing any targeted numeracy (or literacy) programs, the engagement and attendance of students had reflected positively on learning and improvement in learning outcomes, in mathematics/numeracy (and measures of literacy).

ENABLING PRACTICES: LANGUAGE, CULTURE AND AEW[2]S

Across many schools in the study, there was a strong recognition of the impact of home languages on mathematics. The degree of incongruence between school English and home languages was commensurate with the degrees of difference between the home language and SAE. Many of the schools were in very isolated communities where home languages were still very strong, whereas other schools were in areas where a local creole had developed that were along a continuum in their proximity to SAE. In most sites, the recognition that SAE was different from the languages spoken outside school meant that schools developed a range of strategies to support a bridging between the home and school languages. Creating spaces for learners where their languages and cultures are a recognised part of the school curriculum creates a place where students are more likely to feel they belong and their backgrounds are seen as legitimate.

In all schools, the staff was predominantly early career teachers – often in their first year/s of teaching, often in their first move away from home, and with little (or no) previous contact with First People or living in remote/isolated areas. Local people were a powerful resource for teachers and often had many years of experience working in the schools. Depending on the school, the local people had very different experiences in terms of professional learning opportunities in relation to teaching and supporting teachers/education. One of their strengths was their knowledge of culture, language and community. Across the study, there were powerful stories of how the local people were the backbone to the school in terms of developing practices around culture and language.

Many of the students commence school strong in their home language but with little experience with SAE, so some schools have adopted transition programs where the early years of schooling offer a "both ways" or "two ways" approach where instruction is in both the home language and school language. The AEWs have a pivotal role in these approaches as frequently (in nearly all cases), the teacher has little to no knowledge of the students' home languages. The AEW becomes a co-teacher who works as a teacher partner and would translate to home language the instructions of the teacher. Both teachers, for example, may sit at the front of the class with the 'teacher' providing the initial instructions to the students and then the AEW explaining the instructions in the home language. In those

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[2] Across the study, various terms were used to refer to the local people employed at the schools. For consistency across the study, the term Aboriginal Education Workers (AEWs) has been adopted. This term is used in the Kimberley region of Western Australia.

schools who had adopted this approach, there was a seamlessness to the process. It enabled the students to hear spoken SAE and then hear the translation.

The language of mathematics is very nuanced and specific. There are many unique terms and concepts so coming to learn mathematics is as much about the learning of concepts as it is about language. For students whose home language may not have the same terms as SAE and mathematics, instruction in the meta-language of mathematics becomes a salient teaching process. Terms such as prepositions, or nuanced terms, need to be explicitly taught and or scaffolded for learners. The AEWs are well placed to recognise the stumbling blocks for learners. In one school, the AEWs were provided with training on how to make reading resources for students so they would take many photographs of students engaging in local contexts (such as the playground or events in school or community) and then use these as focal points in the books. In one case (Jorgensen, 2015), the teaching of locational language (use of terms such as under, near, beside) was a feature of the book. AEWs constructed pages with photographs of the students in these positions (e.g. Margaret is UNDER the slide) with the positional term highlighted (in coloured text). The story on each page corresponding to the relevant photo was written in both home language and SAE so that students could see and hear the terms being used.

The AEWs served a critical role in these transition programs since their knowledge of the language, culture, students and community were far greater than those of the teachers. They often remained in the schools well past the time served by the transient teachers who often stay for 2-3 years. Many schools and systems were investing in the local people in terms of their professional learning so that they could be valuable resources in the classrooms and schools.

ENACTED PRACTICES: GROUP WORK AND TARGETED TEACHING

Attendance, or more specifically, non-attendance impacts on the possibilities for learning. High levels of absenteeism, and transience between schools, is common across remote communities making it a challenge to cater for the learning needs of students. One of the more profound statements made by a teacher in the study was that non-attendance cannot be used as an excuse for offering an impoverished curriculum. She went on to argue that it was the professional responsibility of a teacher to ensure that when a student is at school that this is an achievement and that teachers must use that opportunity to make the best possibilities for learning while that child is at school. Rather than assuming that students cannot undertake particular mathematics, there was a significant shift across all schools in the study away from deficit models of learning to proactively seeking to know what students could do, and then to build appropriate teaching opportunities to match the learning needs of the students.

Attendance impacts on students' capacity to learn and retain mathematics. Students who attended regularly were more likely to be attaining benchmark levels whereas students whose attendance was irregular, sporadic and/or very poor, were more at risk of being well below benchmarks. Most school systems have some type of report to show families that irregular attendance impacts on the potential to learn. For example in one system, charts are placed around the schools and classrooms to show the long term impact of missing half days and days each week on the longer term learning. For example missing one day per week effectively means that after 5 years, a student has missed a whole year of schooling.

Every school in the study had moved away from 'whole class teaching' where there was a common approach being used to teach all the students the same content. All schools and teachers collected data (regularly) on their students and used these data to identify the needs of the students. Most teachers had some record of student levels of learning and would map student progress. It was not uncommon across the study for teachers to have some visible form of student data within the classroom. Mostly these data displays were used to show students their levels of understanding but more importantly to show students their progression, that is, to celebrate their successes. In some schools, this involved whole school data walls and regular sessions (2 per term) where all teachers would move their students along learning continua to celebrate publicly the success of the students. The data were displayed in an area where staff and community members working in the school could have access. It was not available to the general public. Having these displays of students' progress also meant that community members associated with the school could see how students were progressing and hence be involved and informed about success within the school. The highly visual nature of the displays also meant that the progress (or not) was easy to see. Community members were also able to share their knowledge of students and families to help teachers better understand the wider contexts of students' lives and the impact on their learning which was invaluable in trying to better understand the lack of progress from some students.

Differentiated learning and targeted teaching were evident in all schools in the study. Group work was common across nearly all schools in the study. Mostly groups were heterogeneous groupings where teachers believed that students would learn more when working with peers who could support each other. Often when particular concepts may be explicitly taught, then homogenous groups would be used so that targeted teaching could be achieved for the needs of learners.

Working within groups or in whole class teaching, students were provided with the same resources but with learning activities appropriate to their needs. For example, all students would work with the same resource, for example dominoes, but the activity might be different for each child – in the same group, one student might be counting the dots and recording the number; another might be adding the numbers while another might be multiplying. The importance of having all students feeling part of the same group yet having targeted activities relevant to their needs was an important strategy across the schools. Mathematically it meant that students were having their cognitive needs being met, but importantly, culturally the students were able to 'save face' and not be 'shamed' by being different from their same aged peers. Culturally, *shame* is a very embedded and important concept in First People culture. Ensuring students did not feel shame helped facilitate the student to be engaged with activities.

SUMMARY

Throughout this paper, my aim has been to provide insights into the outcomes of a large study that has been exploring what works in remote numeracy education. It is difficult to capture the breadth and diversity of the outcomes of the study. What is clear from the work is that the schools have taken very positive steps around identified issues and worked from a strengths-based philosophy and not bought into deficit models of education. The strategies employed by the schools and teachers are very diverse but underpinned by some common factors which are now the emphasis in the on-going analysis of the data set.

In this paper, the focus has been on the levels of practice that have emerged from the study, and the issues-based approaches being adopted by the schools. Taking one key issue – attendance – it becomes clear that schools have addressed this complex issue from a range of levels. This multi-level approach provides insights into not only the complexity of the issue but also the practical approaches adopted within the study on how to create spaces to engage learners, both physically and cognitively. While some of the strategies are not specifically targeting mathematics/numeracy per se, they ultimately impact on the learning of mathematics. For example, the school that adopted a whole of school (and community) approach to ensure that students were happy when they came to school also saw the flow on effect to mathematics learning. Others clearly were related to mathematics, such as group work and targeted learning, and the effect on mathematics learning was tangible.

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RECLAIMING OUR AFRICAN PRIDE THROUGH THE INTEGRATION OF ETHNOMATHEMATICAL STUDIES IN MATHEMATICS TEACHING AND LEARNING

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The focus of this Plenary Address is on ethnomathematical studies in South Africa and their potential to assist the mathematics education community in the country to reclaim the African pride in mathematics education. The Plenary Address will also refer to ethnomathematical studies in the rest of the African Continent (especially in the Southern African region) and other parts of the world.

Keywords: Ethnomathematics, learning, mathematics, teaching.

INTRODUCTION

The 13th International Congress on Mathematical Education is scheduled to take place in Hamburg, Germany from 24 to 31 July 2016. This international conference brings together many mathematics education researchers from all over the world, including South Africa. There are 54 Topic Study Groups on a variety of research areas in mathematics education, ranging from Early Childhood Mathematics Education, through Reasoning, Proof and Problem Solving in Mathematics Education to the Philosophy of Mathematics Education. Topic Study Group 35 focuses on 'The Role of Ethnomathematics in Mathematics Education'. The summary of the Study Group and the invitation to Ethnomathematics researchers to submit research papers states the following.

The aim of TSG 35 at ICME-13 is to gather mathematics educators interested in the issues connected with the role of Ethnomathematics in Mathematics Education. The application of ethnomathematical approaches allows us the opportunity to examine local knowledge systems and give insight into forms of mathematics used in diverse contexts and cultural groups. The pedagogical approach that connects this diversity of mathematics is best represented by a process of translation and elaboration of the problems and questions taken from daily phenomena. It is necessary to broaden the discussion of possibilities for the inclusion of an ethnomathematics perspective that respects the social diversity of distinct cultural groups with guarantees for the development of the understanding of different ways of doing mathematics through dialogue and respect.

The summary further identifies the following aspects and questions to be discussed and addressed by ethnomathematical researchers and those who take an interest in this area of mathematics education at the Topic Study Group meetings during the conference:

1. What is the mathematical thinking developed by people in non-traditional and traditional academic-western contexts?
2. How can mathematics education use information regarding this mathematical thinking that has developed outside schools to improve our understanding of the mathematics and the mathematics teaching and learning in schools?
3. How can a wider socio-critical/cultural view of mathematics expand the possibilities for peace, prosperity and elimination of discrimination?
4. What has been done in terms of the research on the role of ethnomathematics in mathematics education? What are current lines and tendencies for new and relevant research?
5. Ethnomathematics can be defined both broadly and narrowly. How do these many definitions influence/impact the ways in which is incorporated into formal educational settings?
6. What impact does an appreciation of non-western cultural contexts and the mathematics relate to these diverse contexts have on mathematics education
7. What evidences are there, and how do we get more, that school programs incorporating ethnomathematical ideas succeed in achieving their aims for the mathematical education of the learners and of their ethnomathematical aims?

The summary closes by indicating that discussions on these issues identified for the Topic Study Group imply that ethnomathematics is an instrument to improve mathematical education where it has a role in helping us to clarify the nature of mathematical knowledge.

The issues raised for discussion in the Topic Study Group made me to think about ethnomathematical studies and the extent to which they are reflected and integrated in the school mathematics curriculum in South Africa. Following this reflection I have decided to focus the Plenary Address on ethnomathematical studies in South Africa and their potential to assist the mathematics education community in the country to reclaim the African pride in mathematics education. The Plenary Address will also refer to ethnomathematical studies in the rest of the African Continent (especially in the Southern African region) and other parts of the world.

ETHNOMATHEMATICAL RESEARCH

Ethnomathematical research and focus in mathematics education can be traced back to the seminal work of Ubiratan D'Ambrosio (Brazil), followed closely by the work of Paulus Gerdes (Mozambique). Both have contributed extensively to the definitions of ethnomathematics and to the conceptual development of this area in mathematics education. Their definitions and ideas have subsequently been embraced, extended, and critiqued by other mathematics educators working in this area. One of the earlier definitions of ethnomathematics by D'Ambrosio states:

> [Societies] have, as a result of the interaction of their individuals, developed practices, knowledge and in particular, jargons...and codes, which clearly encompass the way they mathematise, that is the way they count, measure, relate, and classify and the way they infer. This is different from the way all these things are done by other cultural groups. [We are] interested in the relationship... between ethno-mathematics and society, where 'ethnos' comes into the picture as the modern and very global concept of ethno both as race and/or culture, which implies language, codes, symbols, values, attitudes, and so on, and which naturally implies science and mathematics practices. (D'Ambrosio, 1984).

Here D'Ambrosio looks at the cultural elements such as language, codes, symbols, values, and attitudes which characterise a particular practice. Such elements are largely unique from one cultural group to another.

In a subsequent definition, he defines the cultural groups as national tribal societies, labour groups, children of a certain age bracket, and so on.

> ...we will call ethnomathematics the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labour groups, children of a certain age bracket, professional classes and so on. (D'Ambrosio, 1985, p. 45).

Then he (D'Ambrosio) emphasizes the importance of the cultural group to be well-identified:

> ...we use the term ethnomathematics (ethno + mathema + tics) for the art or technique of understanding, explaining, learning about, coping with, and managing the natural, social, and political environment through processes like counting, measuring, sorting, ordering and inferring - processes that result from well-identified cultural groups (D'Ambrosio, 1989).The processes of counting, measuring, sorting, ordering, and inferring in D'Ambrosio's definition relate very closely to Bishop's (1988, p. 22-23) six fundamental activities that he contends are characteristic of every culture. The six activities are counting, locating, measuring, designing, playing and explaining. Indeed these six fundamental activities by Bishop are identifiable in various cultures.

Gerdes (1994, p.20) who sadly passed away in November 2014, defines ethnomathematics as 'the field of research that tries to study mathematics (or mathematical ideas) in its (their) relationship to the whole of cultural and social life'. Gerdes (1996, p. 915; 1997, p. 343) goes on to indicate that as a research field, ethnomathematics may be defined as the 'cultural anthropology of mathematics and mathematical education.' Although Gerdes provides this definition, he also stresses the importance of seeing ethnomathematics as a movement and he provides a framework for understanding this notion of an ethnomathematical movement and ethnomathematicians – researchers involved in the movement (Gerdes, 1996, p. 917) as follows:

i. Ethnomathematicians adopt a broad concept of mathematics, including, in particular, counting, locating, measuring, designing, playing, and explaining (Bishop, 1988);
ii. Ethnomathematicians emphasize and analyse the influences of socio-cultural factors on the teaching, learning and development of mathematics;
iii. Ethnomathematicians argue that the techniques and truths of mathematics are a cultural product, and stress that all people – every culture and every subculture – develop their own particular forms of mathematics;
iv. Ethnomathematicians emphasise that the school mathematics of the transplanted, imported 'curriculum' is apparently alien to the cultural traditions of Africa, Asia and South America;
v. Ethnomathematicians try to contribute to and affirm the knowledge of the mathematical realisation of the formerly colonised peoples. They look for cultural elements which have survived colonialism and which reveal mathematical and other scientific thinking;
vi. Ethnomathematicians in 'Third World' countries look for mathematical traditions, which survived colonisation, especially for mathematical activities in people's daily lives. They try to develop ways of incorporating these traditions and activities into the curriculum;
vii. Ethnomathematicians also look for other cultural elements and activities that may serve as a starting point for doing and elaborating mathematics in the classroom;
viii. In the educational context, ethnomathematicians generally favour a socio-critical view and interpretation of mathematics education that enables students to reflect on the realities in which they live, and empowers them to develop and use mathematics in an emancipatory way.

Mathematics educators working in the area of ethnomathematics have either explored one specific aspect given above or a component thereof, for instance research projects that have investigated how games may be used in the mathematics classroom, may be classified as dealing with a few of the aspects above.

MATHEMATICAL ANALYSIS IN ETHNOMATHEMATICAL STUDIES

Many ethnomathematical studies have focused upon analysis of various indigenous activities to reveal mathematical concepts and principles that are associated with such activities. Gerdes (1995, p. 8) has identified that ethnomathematical studies revolve around two forms of analysis: (i) mathematical traditions that survived colonization and mathematical activities in people's daily life and ways to incorporate them into the curriculum (ii) culture elements that may serve as a starting point for doing and elaborating mathematics in and outside school. In both forms of analysis, researchers use their mathematical understanding to interpret an indigenous activity and reveal a variety of mathematical concepts associated with the activity.

Two examples have been selected in this plenary address for illustration purposes with respect to analysing indigenous activities. The examples were selected from some of the ethnomathematical research I have been involved in over the past few years. The first relates to an activity that takes place outside the classroom and is brought to bear upon mathematical activities in the classroom. The second was conducted in a mathematics classroom to determine the level of knowledge of indigenous activities and the kinds of analysis that may done in relation to the activity.

Example One: Analysis in Beadwork

The following excerpt reports on the interview conducted with two ladies at the Lesedi Cultural Village. The Lesedi Cultural Village is located in the Gauteng Province, South Africa. In the excerpt, the questions focus on how the ladies engage in beadwork activities and explore related mathematical concepts. It focuses specifically on how they use beads to engrave names of people on beadwork activities, but also refers to other beadwork activities. In the excerpt, R refers to the researcher and SS to the first lady and LM to the second lady. The ladies speak Isindebele, however, they also understand the Sesotho languages like Setswana and Sesotho to a large extent, as a result the interview used both Isindebele and Sesotho as the latter is mostly spoken and understood by the researcher in comparison to Isindebele.

R: Le rutilwe ke mang ho sebetsa ka dibeads? [Who taught you to work with beads?]

SS: Si fundiswe uGogo. [We were taught by our grandmothers].

R: Ni fundiswe nini? Le rutilwe leng ho etsa dibeads? [When were you taught to work with beads?].

LM: Si fundiswe sise bancane. Si ne minyaka e 10. [We were taught when we were very young. We were 10 years old].

R: Nkgono yo a le rutileng, ene o ne a rutwa ke mang? [Who taught the grandmother who taught you to work with beads?].

LM: U fundiswe ngo mama wakhe. [She was taught by her mother].

R: So ho raya hore hangata Gogo o ruta ngwana, ngwana yo ha a setse a hodile o ruta bana ba hae. Jwalo jwalo. [So it means that many a times grandmothers teach their children, and when these children have grown up they also teach their children, and it continues like that].

LM: Njalo njalo [It continues like that].

R: Ho raya hore ha ho hlokahale hore le ye sekolong ho ithuta ho sebetsa ka dibeads? [It means you do not need to go to school to learn to work with beads.

SS: A siyanga a skoleni [We have never attended school].

R: Jwale le entse tsohle tse di leng mona [So have you done all beadwork items that are here?](Researcher asks pointing to all the items displayed around the ladies).

SS: Si enze konke, na nga se stolo. [We have done all the beadwork things here, including all the things in the store](Sophie says this pointing to the back where the store is and the bead artefacts are sold).

R: Joale mona o etsang? [Now what are you doing here?](Researcher asks what Lenah is doing, pointing to the artefact she is working on).

LM: Ngi enza igama le Manager wethu. [I am working on a name tag of our Manager].

R: Le etsa joang hore ho be straight? [How do you ensure that this part of the ornament you are making is straight?] (The researcher points to the straight part of the ornament in which LM is writing the name of the Manager at Lesedi Cultural Village. The Manager's name is Xolani).

LM: Indaba ise nhloko. O ya yazi [The matter is here in the head] (Linah says this pointing to her head. Later on, she further explains that they take two beads at a time) O I stopa kabini ngale [You take two beads that side].

SS: Si khetha umqamu o linganayo. [You choose beads of the same size](This is an additional explanation from Sophie about how the straight lines are made and maintained. Sophie then continues to explain how various shapes are made, for instance indic with a big bead to indicate the centre).

R: Le tseba joang hore mona ke bead e kgolo, mona ke e nnyane? [How do you know that here you put a large bead and here you put a small bead?].

LM: Si bona nga mehlo. [We can see with our eyes].

R: Manje, ni bona ka njane ukuthi ni fake esingakhi? [Now, how do you see that you must put so many beads at a particular point?].

SS: Si ya zi bala. [We count them].

R: Kanti ni yazi kanjani ukubala? Ni the a niyanga esikolweni. [How do you count them? You told me you have not gone to school].

SS: Si ya zibala.

R: Hai. Ni zi bala ka njani? Le mpoleletse hore ha le a ya sekolong. [No. How do you Count them? You told me that you have not attended school].

SS: Si ya zi bala. Sithi Kunye, Bili, Thato, Kune, Hlano. Ku hla ngapha ngi ya jika. Ngi bheke le [We count them. We say one, two, three, four, and five. Then we make a turn to move in the other direction].

R: So, kutsho ukuthi ufuna u ku yenzani. If o batla ho etsa ntho e e riling, o a bala then o jike [So it depends on what you want to do. If you want a specific artefact, you Count and then make a turn].

SS & LM: Yebo [yes] (The two ladies respond at the same time. The interview then continues to e Ndebele culture).

In this excerpt of the interview with the two ladies, it can be noted that they are using a variety of mathematical concepts that are part of the artefacts they are making. Firstly, they refer to straightness of lines in making some of the artefacts. One of the ladies attributes this to their sense of estimation and actually mentions that they just watch and get a sense that the line is a straight as they need it to be. She suggests that this is based on the experience they have gained in using this skill many times. However, Sophie further indicates that the straightness is also maintained through the size of beads that are used. In fact, she mentions that turns (angles) are made using different sizes of beads. Even though they have not attended school, they clearly demonstrate that they know how to count by counting from one to five. This counting is crucial in their activities as it determines the patterns and shapes they make. In the context of indigenous knowledge which is passed from generation to generation, the ladies indicate that the knowledge of working with beads is generally passed from mother to daughter, and such skills can actually be taught to others, and in this case the young are nurtured into these activities, ensuring that the skills do not die but are kept alive for the benefit of the greater society, in this case, of the Ndebele people.

Example Two: Analysis in String Figure Games

The excerpt reported below was conducted in a mathematics classroom in a school Mankweng Township, Limpopo Province. The learners who knew the Malepa Game (String Figure Games) were given an opportunity to give demonstrations to their fellow learners on any Gate that they had the knowledge of. Each learner who gave such a demonstration was given an opportunity to use the language that they were comfortable in and the learner giving the demonstration below decided to use the Sesotho sa Lebowa language which was understood by all the learners.

1. Tseang wa boseven le tsentsheng mo. Le ka go gongwe diang. [Take the seventh and put it in here. Do the same on the other one] {The presenter already had the string hooked on the thumbs and the small fingers, so she started with the very next step. One of the learners asked her to wait a bit before she continues so that they also hook the string on the four fingers. The learner has not started by numbering the fingers, and by the seventh she refers to the pointing finger on the right hand side. When she says that they must also do on the other side she points to the side by the pointing finger of the left hand}.

2. Ntshang wa bo five. [Remove the fifth] {She removes the thumbs at the same time}.

3. Le thieng ka fatshe. A kere le dirile so, e buseng ka mo, le e gogeng so, e be so. Haaa. Goga wa mo fatshe. Ka mo go o monnyane. [Let it pass underneath. You have done like this, turn it back this side, pull it, it must be like this. No. Pull the one underneath. On the small one.] {By passing underneath she is referring to the thumbs. As she starts to hook the string with thumbs the learners murmur to indicate that they are not following the demonstration very well. At this point the demonstrator reverses the thumbs and starts to explain what they have already done up that point. She then starts the step again. One of the learners then asks her to wait a little bit, and she does. Then she looks at what that learner has done and comments about her method, commenting about what she seems to be doing wrong, particularly as it relates to the string underneath. She illustrates to this learner without starting from the beginning nor moving nearer her for assistance but helps her still standing at the front}.

4. Go e ya bo seven gogang e enngwe, e, ye, e, e, e be so. [On the seventh, pull the other one, yes, yes, yes, yes, must be like this] {She first spends a few seconds looking

at her string and then talks about the seventh. Then a male learner asks her whether it is the seventh she is referring to. She points to the string on the seventh finger to be removed with thumbs, then she uses the thumbs to pull the string from the seventh, without specifying the use of thumbs. Then she keeps saying yes - on about three occasions - as the learners check with her whether they are doing the correct thing}.

5. Ntshang o monnyane. [Remove the small one] {Although she has actually referred to one small finger she removes both small fingers}.

6. Le goge e, e be so. Mo e tshwanetse le e dire so, e tshopagane so. E tshopagane so. [Pull it, it must be like this. Here you must do it like this, it must be entangled like this. Entangled like this] {By pulling the demonstrator refers to using small fingers, although does not refer to them explicitly. Another female learner calls her by her name - Sophy - and once more asks her to wait}.

7. Ntshang so, e, e megolo, e, ntsha e megolo, le sale ka one le seven. [Remove like this, yes, the big one, yes, remove the big one, you must remain with one and seven] {The learners ask her if she is referring to the big ones, and whether she means both big fingers, and she says yes on both occasions. She refers to fingers one and seven without clearly showing them but just looking at the right hand, and the learners do not ask her what she means by one and seven, giving an indication that they either see exactly which fingers she is referring to or that they know what she means by one and seven}.

8. Tatang ka mo so, le tla dia? Le ka go gongwe dia ka mouwe [Make a twist here like this, are you doing it? Do the same the other side] {She starts to make an anti-clockwise twist with the fourth finger on the right hand side. By doing it on the other side she is not referring to the left hand but to the small finger of the right hand side, symmetry between the fingers and not necessarily the hands as has been the case thus far. Before she asks them whether they will do it, she looks at them and seems to be getting a feeling that they are either not sure what to do or they are finding it difficult to do it. Unfortunately the video camera did not get focussed on the learners to verify their activities at this point. The presenter laughs a little about the difficulties experienced by the learners in this step}.

9. Le ka mo left diang. [Also on the left hand side do the same] {She then makes similar twists on the left hand side, although not exactly similar as the twists on the left hand side are actually clockwise twists i.e. the learner does not make a distinction between anticlockwise and clockwise twists, but still knows that the same activity must be done on the right hand side. However, this time she does not mention the twist to be done on the small finger but rather follows that doing on the left hand means doing it for both fingers. A female learner then asks ‘and then’? This suggests that she had made this step and was now eager to see what will happen next.}.

10. Tsentsha mo, go e mennyane. E, e be so. [Put it in here, in the small ones. Yes. Must be like this] {In this step the thumbs are used to pull the string on top of the small fingers, however no reference to thumbs but to small fingers, not even indicating which string on the small fingers as there are two. Another learner asks whether the figure must look like the one this learner had made, and the demonstrator says yes. However it is interesting that she does not say that it must be like this learner's figure but refers to her's (the demonstrators) i.e. responds to the learner's question by referring to the model being used upfront}.

11. Tsea o wa boseven, o e tsentshe mo. Le ka mo go o. [Take the seventh one, and put it here. And also on this one] {The demonstrator starts with the string on the left hand, whereas the twists at step number 8 were started on the right hand. This has important implications as it means that you do not always have to start with the right or the left hand all the time, but in many instance you can start with any hand, and don't have to keep to what you started with throughout the activity all the time. This is also different from the how the hands are used in step 12 below}.

12. Le ntshe so, le ka mo lentshe. [Remove it like this, also this side remove it]. {The demonstrator first removes the string at the back of the right hand thumb, then follows with the left hand thumb. At times, especially for beginners, this step is best done through the use of a mouth so that you don't lose the other string that must remain on the thumb. This illustrates how adept the demonstrator is about making the gates and the manipulations involved}.

13. Tsentsha mo, mo, e, wa boseven [Put it here, here, yes, on the seventh finger] {Female learner asks where, and the demonstrator points again at the triangles underneath the thumbs without mentioning the geometric figure involved}.

14. Ke ka moka lentshe e mennyane. [Then remove the small ones].

15. Le e goge. [Pull them]. {Pulling here means turning the Gate away from you to face the learners}.

16. The demonstrator then shows the learners how String Figure Gate 6 looks like. {This is done without an accompanying explanation}.

The researcher then asks the learners if they had managed to do it and finds out that only one learner had managed to make this Gate.

The analysis of the String Figure Gate 6 above reveals a number of mathematical concepts. It is important to indicate that the list of mathematical concepts specified below is not exhaustive. It is possible that other mathematical concepts may be found through further analysis or literature on String Figures. The following mathematical concepts were found in the analysis of String Figure Gates in general:

i. Identification of a variety of geometric figures after making the different String Figure Gates: triangles; quadrilaterals (depending on how the string was stretched, quadrilaterals also specified into squares and rectangles);
ii. Specification of relationships between various figures and generalisations drawn from these relationships;
iii. triangles and quadrilaterals: $y = 2x + 2$
iv. quadrilaterals and intersecting points: $y = 3x + 1$;
v. quadrilaterals and the number of spaces (spaces is given by the combination of triangles and quadrilaterals): $y = 3x + 2$
vi. Symmetry: Symmetry in terms of performance of some steps in making the gates - performing an activity on one side similar to the one done on the other side; Exploration of the different types of symmetries and the related operations in the different gates - bilateral (reflectional) symmetry, rotational symmetry (different folds), radial symmetry, translational (repetitive) symmetry, antisymmetry; Various properties of symmetries; Disentangling the string along a specific line of symmetry which ensures that the string does not get entangled.

MATHEMATICAL ANALYSIS IN ETHNOMATHEMATICAL ACTIVITIES: A REFLECTION

In the two examples given above, the researchers are influenced by their own mathematical knowledge. This means that they need to have an appropriate level of mathematical knowledge in order to recognize mathematical concepts, principles and processes in any activity (including indigenous activity) when they see one. Whenever they observe an activity taking place, they consider and interpret it in relation to a variety of mathematical concepts that are possible, and the dominant ones are most likely those they have been largely involved in or have taken an active part in. However, it is possible to do further and deeper analysis on a variety of mathematical aspects that a researcher has not necessarily been involved in, although this would be limited by the extent to which such mathematical concepts can be recognized or arrived at. It looks like a very important component in mathematical analysis is not to limit yourself to the amount and level of mathematical knowledge that you have been exposed to or have done before as this would reduce on the levels of mathematical analysis that is possible. It is also very useful to share your thoughts with other mathematicians and mathematics education researchers to help you consider other components that you may have not thought about.

Other arguments that have been advanced in relation to mathematical analysis are whether it is appropriate to impose your own mathematical thinking and knowledge on an activity that may have nothing to do with mathematics or rather those who are involved in the activity are not thinking at all about mathematical implications of their activity. This is a question that still needs further debates as there are many arguments for and against the notion of putting on a mathematical lens for the interpretation of indigenous activities. The ethnomathematics community needs to explore this further and provide further evidence from its research whether this is of greater benefit or not.

For the purpose of this Address I have focused on the mathematical analysis of various indigenous activities in order to provide mathematics educators and AMESA members with examples of how mathematical concepts and processes are derived from these kinds of activities. However, there are various methodological issues not discussed here that have a bearing on ethnomathematical research. Mosimege (2013) has identified a number of these methodological issues when research is conducted. Two are restated here for emphasis:

i. Research Language: How does language feature in ethnomathematical studies? What is the issue of language in the interview and the research process? Is it important to share a language with an indigenous community? What happens if the researcher can't speak the language of the community? Does it mean that the research can't continue with the research? Is it impossible for the researcher to continue with the research because of language problems?
ii. Who are classified as knowledgeable people in the area of research or the indigenous activity or artifact you are studying? If you are dealing with the elders, who is classified as an elder? How do you identify an elder? In the case of indigenous game, who do you go to ask questions about the game? Who is deemed knowledgeable in indigenous games? How do you mediate the fact that people have different knowledge about the games?

It is essential that these methodological issues are taken into account as they impact on the nature of research and the results emanating from such research. They bring to the fore ethnomathematical studies as a component of Indigenous Knowledge Systems (IKS).

INDIGENOUS KNOWLEDGE SYSTEMS AND ETHNOMATHEMATICAL RESEARCH

The National Curriculum Statement for Mathematics and Mathematical Literacy identifies a number of Principles which are intended to guide and direct interaction and learning in mathematics classrooms. One of the Principles is stated as 'Valuing Indigenous Knowledge Systems: acknowledging the rich history and heritage of this country as important contributors to nurturing the values contained in the Constitution' (DBE, 2011, p. 5). This Principle calls upon mathematics educators to have an understanding of IKS broadly and how it can be interpreted and enacted to guide classroom interactions.

Nkopodi and Mosimege (2009) have argued and shown how many of the definitions of IKS (not discussed in this Plenary Address) and some of the definitions of ethnomathematics given in this Address and elsewhere bear a very close resemblance in terms of the rich history and heritage of indigenous and local communities. They conclude that on the basis of the close resemblance and emphasis, ethnomathematical studies can be considered as one of the components of IKS.

I therefore want to argue further that in order for the mathematics educators to make real the principle of valuing IKS as specified in the Mathematics Curriculum and Assessment Policy Statement (CAPS), there has to be a consideration and integration of ethnomathematical studies into the various content areas. Without this effort, the Principle will remain a part of the other Principles in CAPS without being made meaningful in classroom interaction and activities.

SOME ETHNOMATHEMATICAL STUDIES IN SOUTHERN AFRICA

In South Africa most of the ethnomathematical studies have been conducted at Wits University under the guidance and supervision of Professor Paul Laridon. In 1996 Laridon, together with a number of mathematics educators and post graduate students undertook a study funded by the National Research Foundation on '*The place of Ethnomathematics in the Secondary School mathematics Curriculum in South Africa'*. (Purkey, 1998). Following this study, a number of students embarked upon post graduate studies in ethnomathematics. For instance Mogari (2002), Ismael (2002), Cherinda (2002), among others. Examples of other studies are Mosimege (2000), Dabula (2000), and others. Recent work in this area has been done by Mosimege (2012) and Mogari (2014). These studies are identified here as an example of the kinds of ethnomathematical studies that have been undertaken. They are also included here for AMESA members to take note and integrate, as far as it is possible, in their mathematics classroom practices. I also hope that these studies will provide mathematics educators with further research questions that can be investigated in their classrooms.

In other Southern African countries, the work of Paulus Gerdes and some of his students and colleagues at the Ethnomathematics Research Centre in Maputo, Mozambique is noted. Included in the extensive and elaborate ethnomathematical studies that he has undertaken (Gerdes: 1985, 1988, 1994, 1995), is the book

'*Geometry from Africa: Mathematical and Educational Explorations'* (1999). The importance of this book is its relevance to the Mathematics Curriculum and Assessment Policy Statement for the FET as it gives detailed mathematical analysis and examples related to the Theorem of Pythagoras, Hexagonal Weaving, Diagonally woven baskets, and twisted decahedron.

An additional feature of the book by Gerdes (1999) which would be of great help to mathematics educators who may want to explore similar studies is the explanation of the research methodology used. Gerdes says the following about the methodology:

We developed a complementary methodology that enables one to uncover in traditional, material culture some hidden moments in geometrical thinking. It can be characterised as follows. We looked to the geometrical forms and patterns of traditional objects like baskets, mats, pots, houses, fish traps, and so forth and posed the question: Why do these material products possess the form they have? In order to answer this question, we learned the usual production techniques and tried to vary the forms. It came out that the form of these objects is almost never arbitrary, but generally represents many practical advantages and is, quite a lot of times, the only possible or optimal solution of a production problem. The traditional form reflects accumulated experience and wisdom. It constitutes not only biological and physical knowledge about the materials used, but also mathematical knowledge, knowledge about the properties and relations of circles, angles, rectangles, squares, regular pentagons and hexagons, cones, cylinders, and so forth. Even though the methodology described by Gerdes above is largely applicable to ethnomathematical work that focuses on geometric shapes, it provides an opportunity to understand (and even trial) how one form of ethnomathematical work is undertaken. This is useful for mathematics educators who would be interested to explore ethnomathematical work in their classrooms related to Geometry.

RECLAIMING THE AFRICAN PRIDE THROUGH THE USE OF ETHNOMATHEMATICAL APPROACHES IN MATHEMATICS TEACHING AND LEARNING

In the Third Edition of her book *Africa Counts* Claudia Zaslavsky writes the following: 'The first edition of Africa Counts was dedicated to "those Africans who are now engaged in the formidable task of reclaiming their heritage". In recent years, Africans in ever greater numbers, as well as their colleagues in other parts of the world, have indeed, been engaged in this formidable task. But much still remains to be done. Scholars, including educators, must accept this challenge to continue the work of reclaiming the rich mathematical heritage of Africa' (1999, p. ii). I do not know whether the organizers of AMESA 2016 Conference had this statement from Claudia Zaslavsky in mind when they decided on the Theme or perhaps Zaslavsky decided to challenge us in South Africa and the rest of the African continent when she wrote the statement.

Writing further about African Mathematics, Zaslavsky (1999, p. 7) indicates that mathematics is in evidence in many aspects of African life, these include the use of numbers in daily life, mathematical recreations, patterns and shape, as well as geometric forms and symmetries in Art.

Following the challenge by Zaslavsky, I want to conclude by saying that when mathematics educators go into the mathematics classrooms to interact with the learners, it is not just about teaching the learners mathematics content as prescribed by the curriculum that matters, they should regard this activity as part of the Africans who are engaged in the formidable task of reclaiming their heritage and in the process restore the African pride in attaining mathematical knowledge. This would give a completely new meaning to mathematics teaching and learning.

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PREPARATION FOR MATRIC (NSC) MATHEMATICS STARTS AT CONCEPTION

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For some time South African mathematics educators have known that a focus on National Senior Certificate (NSC) at Grade 12 level was too late; and recently there have been grave concerns about the very poor Annual National Assessment (ANA) attainment at Grade 9 level. In this paper I argue that preparation for both Grade 12 (NCS) and Grade 9 (ANA) in mathematics commences long before either of these assessments. The paper is situated in relation to key literature on child development; neuroscience and theories of the embodied mind (Carey 2009; Dahaene 1997, 2003; Butterworth 2015; Lakoff & Nὐῆez 2000) where our knowledge relating to 'core cognition' - arguably recognisable as the mathematics of infants and very young children - has been a growing research focus internationally. To illustrate connections between Matric (NSC) and core cognition I start with a Matric mathematics examination question and trace back the foundational concepts which underlie successfully solving this question. I do this in order to stress our need for coherence and connections between our collective efforts in the South African mathematics teaching community.

Keywords: Educators, mathematics, matric, National Senior Certificate.

INTRODUCTION

We are all aware of the poor outcomes of our school mathematics system in South Africa. Both the quantity and quality of National Senior Certificate (NSC) passes in mathematics at Grade 12 level have been a concern since the dawn of our democracy. Data from international studies (eg Trends in International Mathematics and Science Study); regional studies (eg Southern and Eastern Africa Consortium for Monitoring Educational Quality); provincial systemic testing; and our national standardised assessments (Annual National Assessments) has shown that the problems evident at Grade 12 emanate from far lower in the schooling system. In this paper I take a question posed in the 2015 (Paper 1) matriculation paper; and trace back the mathematical conceptual development necessary for a learner to successfully complete this question. I focus on early development and go all the way back to the idea of core cognition (what human babies are born with) as I believe that building on what we know all human children have in common in their toolbox for mathematical learning is the most powerful starting point for mathematics teaching and instructional design.

METHOD

This paper is a theoretical piece which takes the form of a thought experiment. Working back from a Grade 12 NSC question, I conjecture the cognitive structures which are necessary to be able to solve the problem. In so doing I draw on my own mathematical problem solving approaches, and my reading of the mathematics education and cognitive development literature.

DISCUSSION

To find where the mathematical concepts necessary to solve the NSC problem first emerge, we start looking earlier in high school. I take you back to this child entering secondary school in Grade 8 at about 13 or 14 years of age: Is this where their mathematics starts?

But what about further back to this child's first day of primary school in Grade 1 or Grade R: Does mathematics start then when they go to big school? Is there any mathematics when the child is in a pre-school or an ECD (Early Childhood Development) centre, or sent to a Tannie (aunt), a playgroup, or a Gogo (grandmother) down the street? What about before they were two years-old? Before children can talk can they think mathematically? Or when they were born, when they took their first breath – did they have a starter kit from which their mathematics would grow?

In this long rewinding of time (maybe 19 or 20 years) where is the point at which we can first recognise that our child is mathematical? When can we say – now we see mathematics, now they are doing or learning mathematics? Or even now we see the beginnings of mathematics in this child's mind (in their way of thinking) and actions (in their way of behaving)?

In this paper I argue that we have evidence that mathematical thinking occurs from a very early stage: Preparation for Matric mathematics starts at conception. Our target is the NSC examination which is supported by learning mathematics at high school level (from age 13 in Grade 8 to 18 years in Grade 12). However to reach this (Phase 5) target we can see evidence of mathematical thinking at four earlier phases of development:

1. Our inherited human capacity (before conception);
2. The first 1 000 days (conception to 2^{nd} birthday);
3. The pre-school infant (from 2 years to 6 years); and

4. The primary school child (from 6 years in Grade R to 13 years in Grade 7).

Phase 5: High school (Target - The NSC Paper 1 question)

For the sake of my argument, I pick the first question in the Paper 1 of the NSC 2015 to be our target:

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 9x + 20 = 0$ (3)

Figure 1: Question 1.1.1 (NSC Paper 1, 2015).

This question is an algebraic equation which has to be solved for x. Most commonly learners factorise this trinomial to get two factors one of which must 0, for the equation to hold true. This reasoning is because any number multiplied by zero always gives 0. The memorandum outlines this as follows:

QUESTION/*VRAAG* 1

1.1.1	$x^2 - 9x + 20 = 0$ $(x-4)(x-5) = 0$ $x = 4$ or $x = 5$	✓factors ✓ $x = 4$ ✓ $x = 5$ (3)

Figure 2: Question 1.1.1 (NSC Paper 1, Memorandum 2015).

There are alternative ways to solve this problem (such as graphically, using a formula, trial and error, considering an area model, etc.) but at this level in schooling it is assumed that this is a simple recognisable procedure, and that learners will know several things immediately, and not work this out from 'first principles'.

As we know each individual approaches problems differently, I break down and describe the knowledge that I think I used to solve this problem.

At first I have to read and make sense of the question:

1. I read numerous symbols:

- x^2 represents x to the power of 2, or using an exponent of 2, and that means x multiplied by x;

- 9x represents 9 multiplied by x;
    - 9x represents minus 9x, or can refer to + negative 9x;
- + 20 represents plus twenty;
- = means 'the same as'
- 0 refers to zero

2. So I read the question '$x^2 - 9x + 20 = 0$'as follows: When you take 'my number' and multiply it by itself, then subtract 'my number' multiplied by 9 and then add 20; this is the same as zero. What's my number?
3. I recognised and labelled the question as a quadratic equation. I pictured that this would be a parabola when plotted as graph or could be sketched as an area of a rectangle.
4. I know that x can represent a variable (something that changes) and that 'solve for x' means find the value(s) of x that make this equation true.

Once I have read it, and re-posed the question I work out how to solve it and show this in writing:

1. The right hand side of this equation is 0 (which I know is significant). I can use the fact that 'the only way to get 0 as a solution when two factors are multiplied together is if one of the factors is 0'. Realising I can use factors gets me 1 mark.
2. I notice that the expression on the left hand side has three unlike terms (it is a trinomial) and know that I can manipulate the expression on the left hand side to be a product of two factors. So I factorise the trinomial.
3. To factorise the trinomial I consider the relationship between 20 and 9.
4. I break down 20 using factor pairs like 20 and 1 ($20=20\times1$), 10 and 2 ($10 = 10\times2$), 4 and 5 ($20 = 4\times5$). I have just try some numbers (and know my timetables).
5. I then add together the factor pairs to try to make 9. In this case the factor pair 4 and 5 has a sum of 9 ($4 + 5 = 9$).
6. I need to think about the signs to work with integers. I have to work with negative 9 and positive 20 as my targets. I know that (-4) multiplied by $(-5) = +20$ and that (-4) plus $(-5) = -9$.
7. Only when they have all of this information, can I write the first line of the memorandum and factorise the trinomial to give $(x + 4)(x + 5) = 0$ for one mark. This means that I have my number increased by 4 and I have my number increased by 5. When I multiply these two numbers ('my number + 4' and 'my number + 5') then I get 0. I picture a rectangle of side $x + 4$, and

side x + 5. The area of my rectangle must be 0. I know I could picture a parabola, but think I won't need it.

8. I know any number multiplied by 0 is 0. So either 'my number + 4' must be zero (making my number -4), or 'my number + 5' must be zero (making my number -5). I can then write the second line in the memorandum for another mark.

Remember that the National Senior Certificate is a timed written assessment. A learner has 2.5 minutes to complete the 12 steps in Question 1.1.1 which is worth 3 marks. By Further Education and Training Phase (Grade 10-12) we expect this problem to be a routine procedure which can be quickly recognised and solved efficiently.

When do we expect learners to become secure with the underlying concepts needed to solve this problem? This kind of problem first appears in Grade 10. But much ground work precedes this. In Senior Phase (Grades 7-9) learners first encounter the use of letters (x) to depict variables and unknowns. They would also learn to work with integers and exponents. Much of their mathematical time would be dedicated to manipulating algebraic expressions (including both factorising and simplifying) as well as solving algebraic equations. So there are clear building blocks for this question which are introduced at Senior Phase.

But is it really only 12 steps underlying this question? There are many concepts in the 12 steps which I have taken for granted. For example I did not make a step for reading and understanding numbers. I took it as given that the concepts of '9' and '20' and '0' were known. I took it as given that the concepts of multiplication (and factors) were known; and that adding/ subtracting; negative numbers and exponents did not require further explanation. There is even more conceptual development which underlies these 12 steps. To unpack the cognitive foundations for this problem I now jump all the way back to prior to birth.

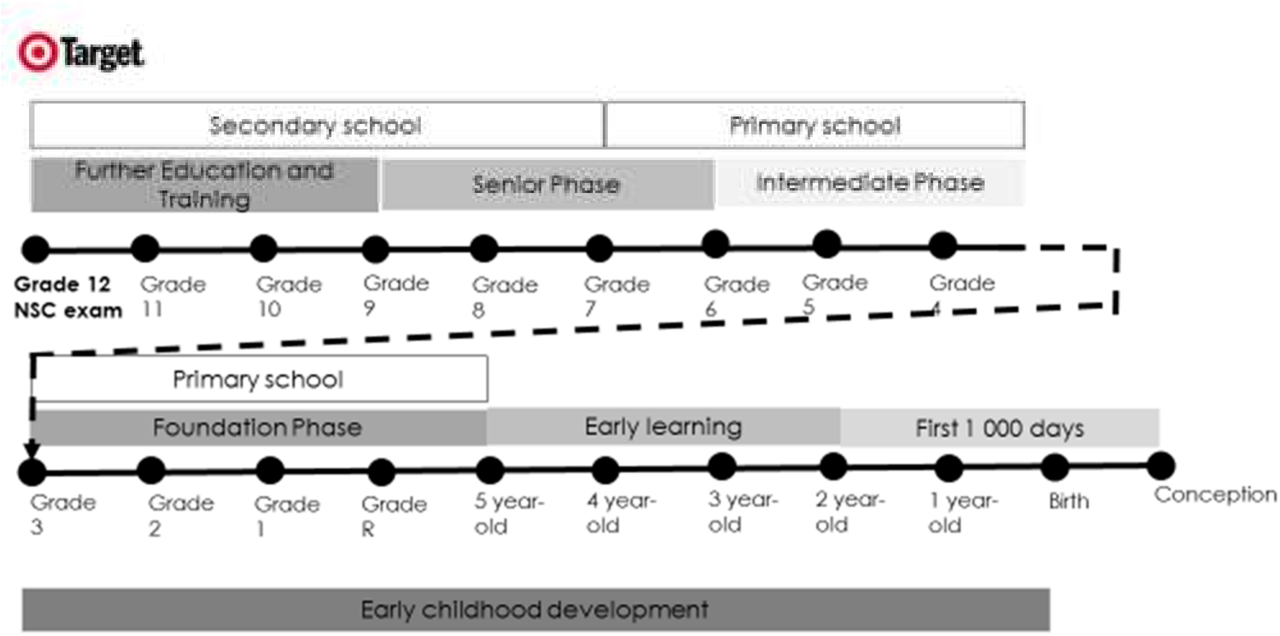


Figure 3: Working backwards from Grade 12.

Phase 1: Our inherited human capacity

There is strong research evidence that our mathematical capabilities are part of what makes us human. Mason argues that mathematical thinking is a natural human power (Mason 2007). Research on what makes us human centres on thinking about what it means to *not* by human. This means that we can better see our humanness by comparing humans to animals. There are numerous studies on the numerical abilities of animals (including chimpanzees, monkeys, raccoons, rates, parrots, pigeons and fish) (Lakoff & Núñez, 2000, Butterworth, 2015). For example rats can learn to perform an activity a given number of times. Lakoff & Núñez (2000) explain a study by Mechner and Guevrekian (1962) which illustrated this kind of research.

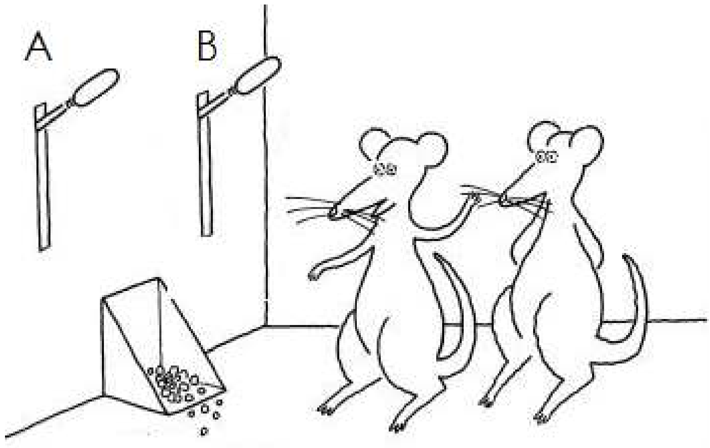


Figure 4: Rats can learn to count.

In this study the rats are deprived of food for a while. They are then placed in a cage with two levers. They will get food if they pull Lever B – but this will work only if they *first* pull Lever A, a certain number of times (say 4 times). They are punished for not pulling Lever A, or for not pulling it the correct number of times. The rats can learn to pull the lever the right number of times securely up to beyond 4 times but it starts to break down at 8 where if it goes up to 8 times, then the press close to 8 say 7 or 9 times. Later studies (Church and Meck 1984) showed that the rats could learn and generalise when dealing with numbers in response to different sensory formats (pulling a lever, looking at a flashing light, or listening to a repeated tone). There are similar studies with other animals such as crows, and fish which are designed to focus on the animal's actions and/or looking time. So researchers in cognitive and neuroscience have found some mathematics animals can perform. Our closest relative, the chimpanzee, has a 'non-trivial capacity for some innate arithmetic along with abilities that learned through long-term explicit, guided training' (Lakoff and Nunez, p. 23). This implies that if we humans share aspects of innate arithmetic with animals, that there is a common a shared human ability which human embryos inherit from their ancestors. Butterworth (2015) refers to this as the numerical 'starter kit' or toolbox of abilities which human children inherit (simply by being human) (Butterworth, 2015). We don't yet know the full extent of this starter kit, but the research work in child development and neuroscience is starting to shed some light on what the starter kit includes.

Phase 2: The first 1,000 days

There is a global campaign (see www.thousanddays.org), which is supported by our national Department of Health, and Department of Social Development that emphasis the first 1,000 days as critical to the child's life and development. This is a fascinating time in child development as it extends from conception through nine months of pregnancy to the birth of the child and the first two years of a child's life (roughly 280 days of pregnancy + 365 days in the first year + 365 days in the second year = 1,000 days). The campaign messages focus on reaching nutrition and health related milestones (such as vaccinations, monitoring weight, emotional attachment and gross motor development etcetera). Can we really see evidence of mathematical development during this time?

We can 'see' evidence that certain mathematical development takes place during pregnancy by observing what happens when the environment of the unborn child (the womb) is not healthy. We examine what is not normal, to better understand what normal development entails. For example, there is now compelling evidence that children who are exposed to alcohol (through their mother drinking during pregnancy) have 'a variety of cognitive deficits, most notably in mathematics and higher order processes such as abstraction' (Kapera-Frye, Dehaene, & Streissguth, 1996, p.1187). Further, children with Foetal Alcohol Spectrum Disorders (FASD) 'tend to have more difficulty with mathematics than with other cognitive areas, and mathematics is most highly correlated with the amount of prenatal alcohol exposure' (Rasmussen & Bisanz, 2009, p.259).

This research into FASD and mathematics is important for at least two reasons:

Firstly in South Africa we have an exceptionally high population of babies born within the spectrum. By way of example, Maya et al (2007) report that in a certain Western Cape community prevalence of FAS amongst first Grade children at 65.2–74.2 per 1000 (Viljoen et al., 2005). This means that in this community, about 50 children in the primary school of 700 would have FASD. By way of comparison the rates reported in general population of the United States are 0.33 - 2.3 children per 1000 (Maya et al., 2007).

Secondly that fact that we know that alcohol during pregnancy can interfere with a child's mathematical development means that their mathematics is developing during pregnancy. So we have established that mathematical development is taking place prior to birth.

Let us now assume that that pregnancy was healthy, and a healthy human child is born. The next two years of a child's life it is also hard to see or imagine mathematical learning taking place. The majority of this phase is pre-linguistic in that the infant is not yet talking. Nevertheless, like the research studies with animals, we now have research with young human babies which shows some numerical awareness at this very early age. These studies use data about what babies look at, and how long they look at particular things; or how babies reach for objects. There is a wide range of these studies which vary in method and detail. See for example the overview provided by Carey (2009) and examples of number discrimination by babies offered by Lakoff and Núñez (2000).

For the purpose of this paper, I will simply offer one example (Wynn, 1992 cited in Lakoff & Núñez, 2000 and in Carey, 2009). At four and half months old, a human baby 'can tell' that one plus one is two; and that two minus one is one. In the Wynn (1992) study a baby looks at an empty case. A hand comes in and puts an object into the case, a screen comes up and the hand can be seen to now place a second object behind the screen. The screen then drops. There are two possible outcomes in the experiment: the screen drops to reveal two objects (the expected outcome), or the screen drops to reveal only one object (the unexpected outcome). Wynn's study showed that babies look at the unexpected outcome (only one object is revealed) longer than the expected outcome. This is interpreted to show that the baby is aware that when one object is present, and another object is placed next to it, then one should see two objects (1 + 1 = 2). A similar process is adopted for the subtraction situation.

As with the work with animals, the studies with human babies and model arithmetic processes using concrete objects (such as dots on page or screen, puppets behind a screen and so on). Subtraction and addition situations are modelled and researchers find that children can reason about the expected outcomes. But is this awareness of expected outcome really mathematics? These looking-time experiments are a long way from a child reading 1 + 1 = ... or 2 – 1 = ... and writing down the correct answers. Researchers acknowledge that there is a substantial shift in mathematical cognitive development when human numerical capacities are symbolic (Butterworth 2015, Carey 2009). The capacity to use symbols (often numerals or number words) for numbers is more complex than number discrimination and reasoning about operations as modelled through a situation.

Before I proceed, let us first be sure we are using the words 'number', and 'number symbol' in the same way. When I refer to (1) the *number* five, I mean the idea/concept of fiveness (the numerosity of five). This might be a length that is five units long, a set of five objects, a duration of five heartbeats, or repeating an action like taking a step or clapping my hands five times. When I refer to (2) *number symbol*, I refer to the language symbols that are written numerals, and the language symbols that are both written and spoken. The *written numeral* (2a) is '5' when using Arabic numerals. It is ~~IIII~~ when using tally marks. The *spoken number name* (2b) is when I say the number name 'five' and the *written number word* (2c) is when I write the number word 'five'. The language symbols are language dependent. For example in Sesotho the *spoken number name* is saying 'hlano', and the *written number word* is writing 'hlano'.

This brain science research work is still in its infancy. It is now known that different parts of the brain are used for the processing of rote memorization of addition and multiplication and for algebraic work. We are long way from knowing exactly where and how mathematical reasoning happens in the brain, but small discoveries are slowly emerging which gives us a slightly better understanding. There are however at least three important findings from the cognitive science perspective which teachers ought to know that are relevant to this pre-linguistic phase. First pre-linguistic children are able to distinguish between more and fewer in a set using their Approximate Number System (ANS). Second they have an Object Tracking System (OTS) which allows a human infant to track moving objects (Henning and Ragpot 2014). Third Lakoff and Nὐῆez (2000) draw on this brain science research to put forward a theory of an 'embodied mind' which seeks to understand how much of mathematical understanding makes use of the same kinds of conceptual mechanisms that are used in the understanding or ordinary nonmathematical domains? (p.28). They argue that a great many cognitive mechanisms are not specifically or uniquely mathematical as 'mathematical ideas are often grounded in everyday experience'. They go some way in codify this 'everyday experience' by identifying spatial relationships and image schemas including a container schema and a source-path goal schema. Both of these schemas seem to be particularly relevant to mathematics pedagogy.

The innate Approximate Number System (ANS) allows for the (non-exact) distinction between different quantities and is seen as the primary foundation on which symbolic knowledge of number will develop (Henning and Ragpot 2014). Innately, humans and other animals are able to distinguish between 'more' and 'fewer' in a set. However, the ratio and proximity of numbers to each other play an important part in this ability. When sets are close together like 7 and 9, this is more difficult that when sets are further apart like comparing 2 and 9. The use of this system is evident in approximation tasks such as that shown in Figure 5.

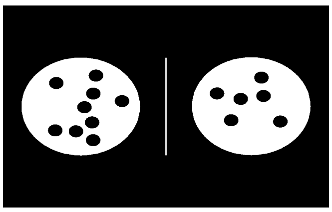


Figure 5: Which is the larger set? Image Source: (Butterworth, 2015).

This imagery is recognizable as making use of the Lakoff and Nὐῆez (2000) ‘container schemas’. This is a metaphor that numbers are objects, which can be inside or outside of a container. Lakoff and Nὐῆez (2000) argue that these container schemas are evident in the language structure of all human languages where the ideas of boundary, interior, exterior, *inside* and *outside* a container are used.

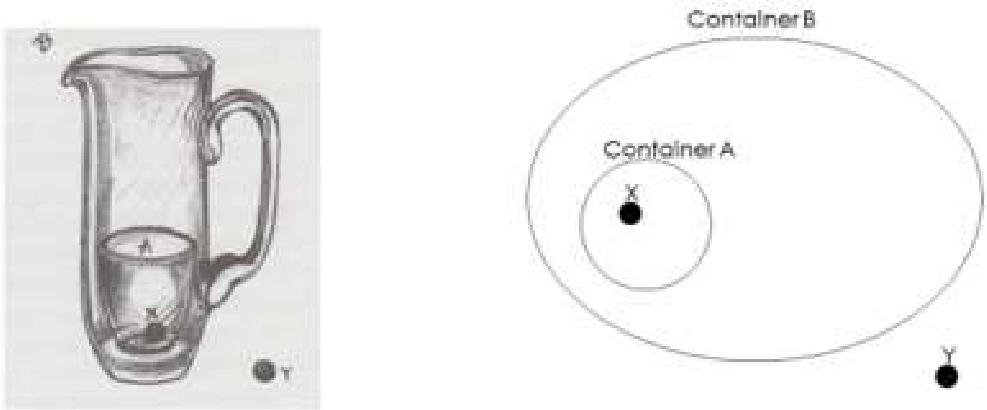


Figure 6: Container schema. Image source: Lakoff and Nὐῆez (2000).

In addition, Lakoff and Nὐῆez (2000) argue that in all languages, humans are able to reason that if Object X is *inside* Container A*,* and Container A is *inside* Container B, then Object X is also *inside* Container B. Similarly of Object Y is *outside* of Container B, then it is also *outside* of Container A. This container scheme is a grounding metaphor for the everyday experience of mathematics.

The innate Object Tracking System (OTS) which according to Carey (2009) may not be ‘numerical’ (and is indeed more geometric and spatial), allows human children to file and keep track of the position of up to three objects neurologically and so allows recognition of one, two, or three objects (Henning and Ragpot 2014). Tracking the movement of an object (as enabled by the OTS), is recognisable as connected to the Lakoff and Nὐῆez (2000) source-path-goal schema which is depicted as in Figure 7.

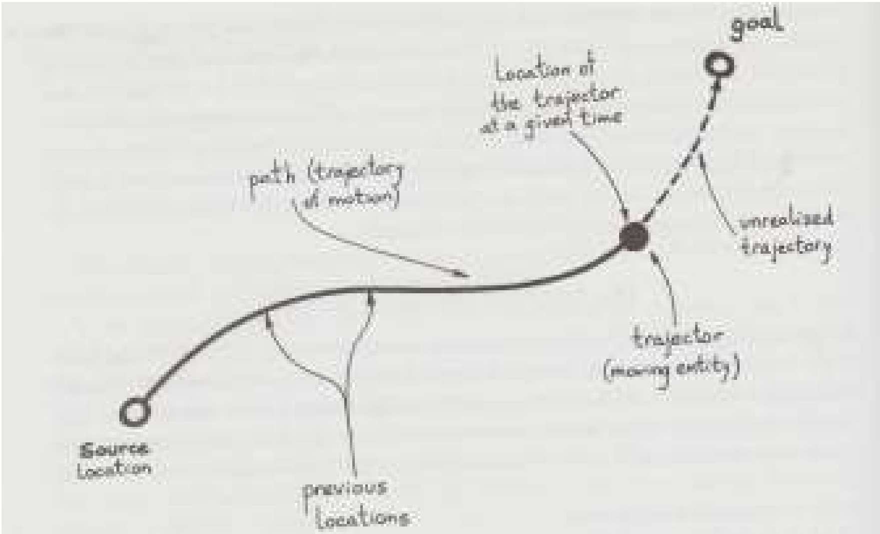


Figure 7: Source-path-goal scheme (motion as a creating line). Image source: Lakoff and Nὐῆez (2000).

This schema refers to tracking the movement of an object from its starting location (source) to the end of its journey (goal) and considering its change in position to be described by line. A line (either straight or curved) is thought about in terms of the motion of a point or object tracing the line (a point moving along the line creates the line depicting its path of movement). In everyday language this idea of a line as depicting motion if expressed in everyday sentences like: The road *runs* through the woods' and 'the fence *goes* up the hill'. This scheme is recognisable in mathematical sentences such as 'two lines *meet* at a point', or 'the function *reaches* a maximum at 0' (Lakoff & Núñez, 2000, p.39). It is my contention that both of these innate systems (the ANS and OTS) which are recognisable in the container image scheme and source-path-goal image schemes respectively are key resources on which teachers of mathematics can and should build.

Phase 3: The pre-school infant (from 2 years to 6 years when language develops)

We now move onto the third phase of children's development where we now have the benefit of their use of language. In this phase the children are starting to use symbols (such as number words) to represent things (a number of fingers, a number of objects) or events (a number of breaths, a number of jumps). Studies on learning and cognition suggest that the developmental process of constructing symbolic knowledge of number is not yet known. Butterworth asserts that 'the transition from approximations to exact whole number arithmetic is still mysterious' (Butterworth 2015, p. 24).

However research from mathematics education, and cognitive science coheres on the developmental process of learning to count where children first recite a ‘counting list’ from verbal memory without understanding the meaning of number words beyond three; gradually children begin to understand the concept of ‘the next number’ (in the counting list), and thereafter learn the principle of ‘one more than’ where their verbal knowing intersects with their conceptual knowing (Henning and Ragpot 2014). There are numerous frameworks which outline developmental progression in relation to counting (Wright, Stanger, Stafford, & Martland, 2006, Treffers, 2008, Fritz-Stratmann, Ehlert, & Klüsener, 2014; van den Heuvel-Panhuizen, Kühne, & Lombard, 2012). For the purpose of this paper I draw on Fritz-Stratmann et al (2014) as these colleagues at University of Johannesburg have developed and analysed Grade R level diagnostic tests using this proposed framework of four development levels, which have been empirically tested and validated using young South African learners in multiple languages:

- Level 1: Count number;
- Level 2: Mental number line;
- Level 3: Cardinality and decomposability; and
- Level 4: Class inclusion and embeddedness.

Level 1: Count Number

The first level represents a developmental stage where children realise that the number words (the language symbol) represent a quantity (Fritz-Stratmann et al., 2014). Numbers are considered in relation to their position (when they are said in a list of words) in relation to other numbers in the counting sequence. At first this is only in relation to number immediately before or immediately after (1 more than, or 1 less than), and later this extended to a bigger range.

We do not know precisely at what age ‘level 1: Count number’ takes place and when this development occurs seems to be vary considerably by social context/environment. For example Treffers (2008) working in the Netherlands locates awareness of the counting sequence to ten, and the ability to say ‘one more than’ a number (for whole numbers less than 10) at approximately age 3. Yet we have emerging evidence in South Africa of children not having this capacity when they enter Grade R (at age 6) and even as late as Grade 2 at age 9 (Roberts, 2016).

Level 2: Mental number line (ordinal aspect)

Ordinality is the capacity to place number words and numerals in sequence; for example, to know that 4 comes before 5 and after 3 in the sequence of natural numbers. Other aspects of ordinality include the use of ordinal names and symbols such as “first” and “1^{st}”. There is growing interest and investigation of number approximation tasks which use the ordinal aspects of number such as that shown in Figure 8:

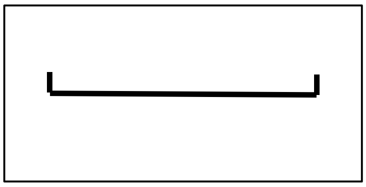


Figure 8: The line begins at 0 and ends at 10. Where is the number 6 located? Image source: Butterworth, 2015.

Fritz-Stratmann et al (2014) argues that the number word sequence (or 'the number word line') is gradually represented as an ordinal mental line, with succeeding numbers continuously increasing, explaining that this is a mental 'image'. She refers to this as 'Level 2: Mental number line'. Working in a Germany (and South Africa) Fritz-Stratmann (2015) asserts that 4- to 5-year-old children can compare numbers according to their position on the number word line or sequence: '9 is larger than 5 because I say 9 after 5 when I am counting'.

Phase 4: The primary school child (from 6-years in Grade R to 13-years in Grade 7)

Noting that the age of development seems to vary widely depending on context and environment, we move now into the primary school phase which spans from 2 to 7 years, which roughly maps to level 3 and level 4 of the four level framework.

Level 3: Cardinality and decomposition

Cardinality refers to the capacity to link number symbols to collections, e.g., to know that the number symbol "4" or the number word "four" is the correct representation to denote a group of four objects (Sinclair & Coles, 2015). At a very young age, children first use this cardinal aspect by utilising their OTS to keep track of up to three objects. Later (and again gradually) children learn to use the cardinal values of numbers more than three which means that if they count out a number of objects, they will attribute the numerosity of the set to the last numeral used (Henning & Ragpot 2014). Tasks depicting assessment of the cardinal aspect of number include the following:

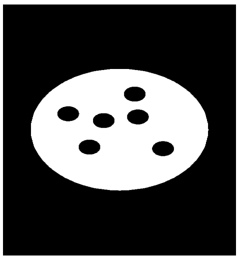
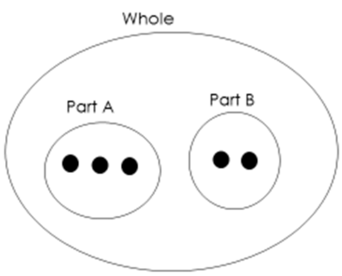
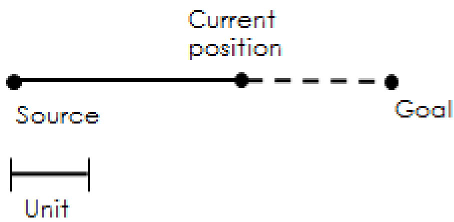


Figure 9: How many? Image source: Butterworth, 2015.

Fritz-Stratmann et al. (2014) define ‘Level 3: Cardinality and decomposition’ where they argue that *at the same time* as children become aware that the last number named is the numerosity of the set, they also become aware that ‘once composed together, numbers can be decomposed again’(p.141). This means that at this level children can now start to compose and decompose numbers in order to create parts and wholes for adding/subtracting and repeating a defined unit for multiplying/dividing.

Using the container and source-goal-path schemes for adding/subtracting may be depicted in Table 1.

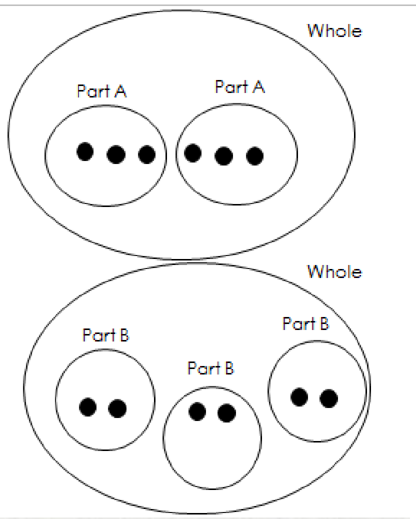
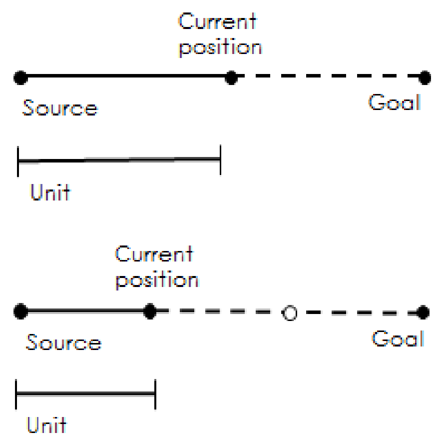
Table 1: Grounding image schemes for 5-3-2 additive relationship.

<table>
<tr><th>Container schema</th><th>Source-path-goal schema</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>A container with 5 can be split into 2 parts (one part with 3 and the other part with 2)</td><td>A journey of 5 units (like steps) can be broken into 2 parts (one part of 3 and the other part of 2)</td></tr>
<tr><td>5 = 3 + 2</td><td>5 = 3 + 2</td></tr>
</table>

(and so 5 – 3 = 2, and 5 – 2 = 3 and 5 = 2 + 3)	(and so 5 – 3 = 2, and 5 – 2 = 3 and 5 = 2 + 3)
Builds on categorisation, interior/exterior, structure, cardinality (making use of ANS?)	Builds on sequence, spatial and temporal event, movement through time, process/operation, ordinality (making use of OTS?),

Using the container schema and source-goal-path schemas for multiplying/dividing may be depicted as shown in Table 2.

Table 2: Grounding image schemes for 6-3-2 multiplicative relationship.

Container schema	**Source-path-goal schema**
	
A container of 6 can be split into 2 *equal* parts, with each part bring 3, or a container of 6 can be split into pairs (of 2 in each group), with 3 pairs.	A journey of 6 units can be split into 2 equal parts, with each part being 3, or a journey of 6 units is made up repeating an equal unit of 2, 3 times
6 ÷ 2 = 3 (and so 3 × 2 = 6, and 2 × 3 = 6 and 6 ÷ 3 = 2)	6 ÷ 2 = 3 (and so 3 × 2 = 6, and 2 × 3 = 6 and 6 ÷ 3 = 2)
Builds on categorisation, interior/exterior, structure, cardinality (making use of ANS?)	Builds on sequence, spatial and temporal event, movement through time, process/operation, ordinality (making use of OTS?),

There is contestation in the mathematics education literature as to whether the idea of adding/subtracting must necessarily precede the idea of multiplying/dividing. While some researchers (such as Fritz-Stratmann et al., 2014) argue that multiplication should be introduced as repeated addition and so adding must necessarily come before multiplying; other researchers such as Schmittau (2004) and Bass (2015), following the Russian curriculum of Davydov, argue that repeating a unit (the basis for multiplication) is the foundation for cardinality. They argue that both discrete objects and continuous measurement contexts should be introduced simultaneously and the centrality of *a choice of a unit* to repeat, is the basis for quantity. In this conception multiplication precedes addition.

It seems that the jury is still out on the debate of 'adding first' versus 'multiplying first'. What is clear is that there are two major mathematical ideas which are related (but distinct): (1) Adding/ subtracting which reflect a whole-part-part relationships and (2) multiplying/dividing which reflect a choice of unit that is repeated.

Level 4: Class inclusion and embeddedness.

Working with the assumption that addition precedes multiplication, Fritz-Strassman et al (2014) describes the fourth level of development to be 'the knowledge of quantities and their *relationships* [which] increasingly differentiates as children develop conceptually'. The indicator of this understanding is children's ability to find different decompositions for numbers. It is this level of awareness that is required to flexible break up 9 into 5 + 4 in the NSC Paper 1 as outlined above.

The cognitive science literature is not yet able to offer any further advice in relation to further core cognition building blocks for adding/subtracting or multiplying/dividing (or exponents, variables or integers). However the mathematics education (didactic/ pedagogic) literature has identified particularly powerful concepts and related representations for both operations (Askew 2012, Barmby et al. 2014). As the NSC question requires awareness of multiplication and factors, the array and area of rectangle models supporting multiplying/dividing are relevant.

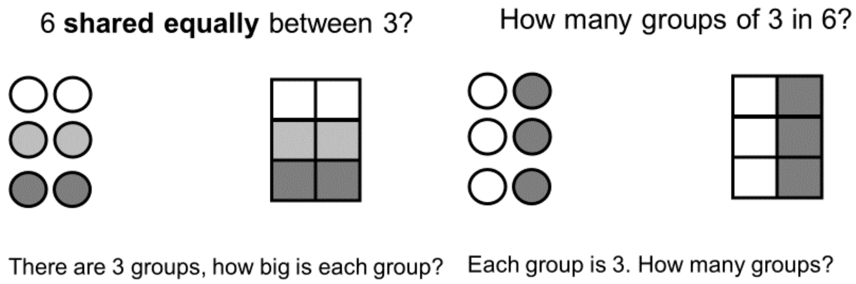
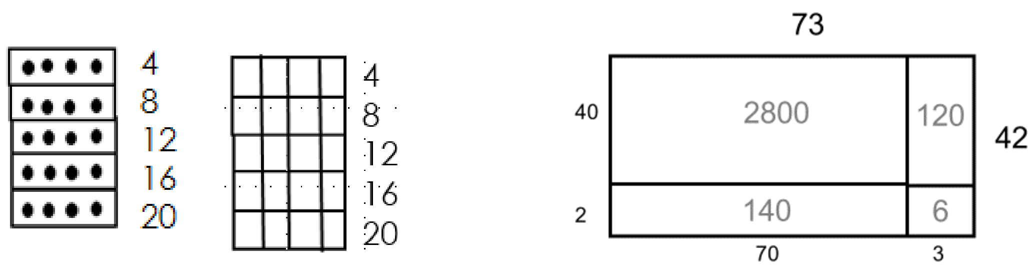


Figure 10: Two models for multiplying/dividing using array and area models.

The array model imposed a structural arrangement onto the container schema, and this is then reified to make use of grid structure with the area model. Flexibility in thinking about two possible models for multiplying/dividing and making use of array and area imagery, can be extended provide pedagogic support in coming to know the times tables, as well as procedures for multiplying/dividing bigger numbers.



Array or grid repeating the 'unit 4' a certain number of times.

Using an area model of multiplication to work out 73 × 42, making use of the distributive property. In this case breaking up 73 into 70+3, and 42 into 40+2 (other partitions are as effective).

Figure 11: Extending the array model for the timetables, and for multiplying/dividing bigger numbers.

Awareness of multiplication/division structures brings us closer to what the learner needs to know to solve the quadratic equation by factorising. Yet such imagery for multiplication was not included in the NSC question memorandum (which provided an algebraic solution).

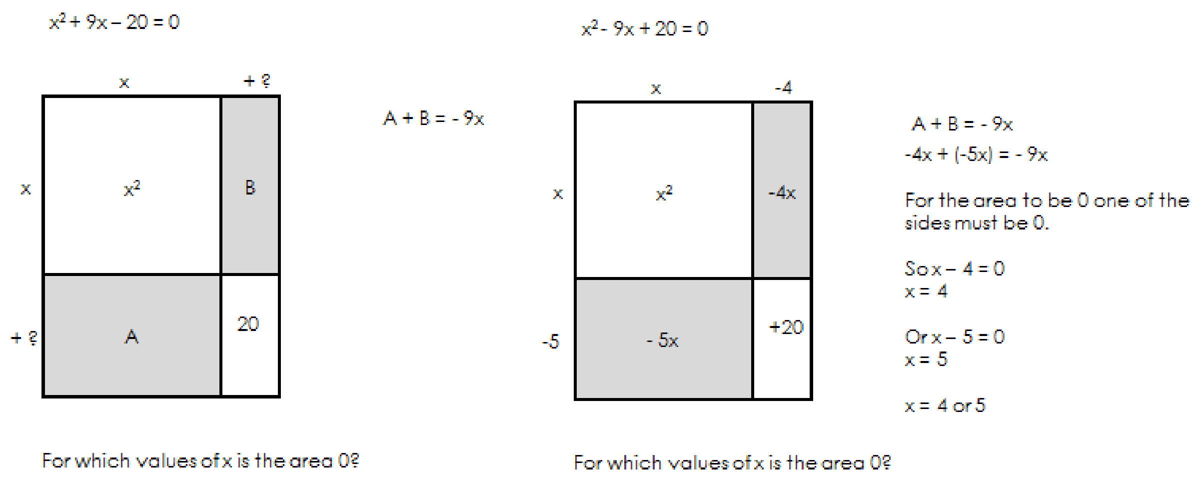


Figure 12: Using the area model to solve the NSC question.

I think that connecting the NSC problem to spatial imagery of an array or area model may be helpful to some Matric learners and teachers.

CONCLUSION

Taking an NSC mathematics question (Solve $x^2 + 9x - 20 = 0$) as a target for mathematical development, I have argued that there at least 12 steps involved in solving this algebraically, and that each of these steps can be broken down further and traced back to the introduction of exponents, integers, and variables at Senior Phase level. In turn, the building blocks for the Senior Phase mathematical ideas can be traced back to fluency with multiplying/dividing, factors and adding/subtracting in the Intermediate Phase. To master fluency with these operations and number relationships a four level development framework is considered in the Foundation Phase. I have presented evidence that prior to entering schooling foundational ideas relating to counting, number relationships and operations are already constructed by some learners, and that their origins are evident in all pre-linguistic children. Further we know that all children are born with a numerical starter kit which includes an Approximate Number System (ANS) and Object Tracking System (OTS), which are drawn in the grounding metaphors of mathematical thinking. These metaphors include the cardinal aspect of number as object (in the container schema) and ordinal aspect of number as a journey (in the source-path-goal schema).

Finally by considering development deficits we have research evidence that mathematical development takes place during pregnancy. As such, preparation for Matric mathematics starts at conception. I use this example of one NSC mathematics question, in order to demonstrate how what is taught (and required to be learned) at the endpoint of schooling is connected to earlier learning and development. All educators (in both secondary and primary schools and including ECD practitioners, caring adults, parents and mothers) have a role to play in helping the learner reach the NSC target. Teacher knowledge of our learners, of learning/cognition and of what we know our learners bring with them into class are significant resources in our collective effort to improve mathematical learning.

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ANALYSIS OF MANAGEMENT CONSTRAINTS IN THE DISTRIBUTION AND DEPLOYMENT OF QUALIFIED MATHEMATICS AND SCIENCE TEACHERS IN A POST-1994 EDUCATION SYSTEM OF SOUTH AFRICA

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The study analysed the management constraints in the distribution of qualified mathematics and science teachers in a post-1994 education system of South Africa. The study was qualitative and 14 participants were purposively sampled and semi-structured interviews were used to collect data from the identified participants. The interview transcripts were constantly compared and analysed and the data was classified into three main categories of management constraints and patterns: beliefs, experiences on management constraints and strategies for the elimination of management constraints. Turning vision into practice (TVP) framework was used to explain the relationship between its seven pillars of managing teacher recruitment and the links in the development, adoption, implementation, monitoring and evaluation of a teacher deployment system, focusing on mathematics and science. Findings of this study suggest that the current hybrid post establishment model is generic and focuses more on cost curtailment than on the supply of qualified mathematics and science teachers. The shortcomings of the model are exacerbated by the transgressions of the Employment of Educators Act. Contrary to the Employment of Educators Act, entry-level vacancies are not advertised in the province. In addition, the appointment and service conditions of qualified teachers are differential. While teachers from government bursary schemes are appointed immediately on permanent status and without probation, other qualified and long-serving mathematics and science teachers remain on temporary status for almost two years and without fringe benefits. The differential treatment leads to job insecurity and facilitates the exit of these qualified mathematics and science teachers from the profession. Moreover, schools horde and use qualified mathematics and science teachers in subjects they are not qualified to teach. It is recommended that through the suggested TVP framework, the current teacher recruitment and deployment strategies be revisited regularly to ensure effectiveness of teacher usage in mathematics and science. It is further recommended that school principals and other educational leaders should be provided with personnel management skills to ensure maximum effective recruitment and deployment of qualified mathematics and science teachers, particularly to the impoverished schools.

Keywords: change management, foreign teachers, *Funza Lushaka*, management constraints, mathematics and science, post establishment, redeployment, school governing body, senior secondary schools, teacher unions.

INTRODUCTION

The availability of qualified mathematics teachers in every school is an important and inseparable aspect of a quality education system. This was illustrated throughout the pre-1994 years of apartheid rule. Since 1953 when the National Party took control of all forms of education in South Africa, the provision of quality mathematics education programmes was stratified according to race to ensure socio-economic inequalities among various population groups. The then Minister of Native Affairs in the National Party government, Dr. H.F. Verwoerd, stated the major objective of this stratified provisioning of mathematics in South Africa:

> When I have control over native education I will reform it so that the natives will be taught from childhood to realize that equality with the European is not for them...What is the use of teaching the Bantu mathematics when it cannot use it in practice? (Lapping, 1987, p.1).

The above remark became the cornerstone of apartheid education in South Africa. Morrow (1990) defines the apartheid ideology as *"a form of oppression that has disempowered its victims by persistently treating them as objects of policy...Apartheid has dehumanized its victims, their dignity and self-esteem as persons, and their intellectual and moral confidence and autonomy has been damagingly undermined"*. These apartheid victims are Africans that have been confined to menial jobs the minority white population was burdened with the production of scientific and technological skills (Benson, 2011, p. 1). The result of institutionalised implementation of the apartheid ideology determined the demographics of a segregated education system. Highly qualified white teachers lived in affluent cities and towns, whereas the most unqualified teachers were largely Africans that were employed in the impoverished schools that are situated in townships and rural areas (Mda, 2009, p.209). However, the end of the apartheid Nationalist Party rule and the establishment of a government of national unity (GNU) resulted in the introduction of a new constitution with values that promoted principles such as *substantive equality* and *equity.* Substantive equality requires government to introduce remedial measures that are "geared to addressing both individual and group disadvantages created by a history of oppression and apartheid" (Henrard, 2002, p.24). In line with this principle, the racially stratified departments of education were abolished and a single Department of Education based on the values of non-racism and non-sexism was established (Mouton, Louw & Strydom, 2012, p. 1211). One of the major tasks of the newly established Department of Education was to implement the first post-apartheid White Paper which was introduced in 1995. There are important observations from this White Paper regarding deficits in the apartheid strategy of providing subjects such as mathematics and science. Some of these observations from the White Paper are that the attrition of mathematics and science in African schools is a special case because apartheid education has led to a very small number of African teachers graduating from colleges and universities in these subjects; and that “a cycle of mediocrity” was being perpetuated through the efforts of these poorly skilled teachers in the classroom (Republic of South Africa, RSA, 1995, p. 19).

In essence, the application of the constitutional substantive principle of equality and equity became imperative in the provision of qualified mathematics and science teachers to all schools in a post-1994 education system of South Africa. Therefore, the post-apartheid government implemented a series of policy initiatives to ensure both equality and equity in mathematics and science through an increased provision of qualified teachers to schools. The then Minister of Education, Naledi Pandor, made an undertaking to this effect when she stated that government will ensure that qualified teachers are deployed to all secondary schools and that:

> If we don’t have these teachers in our country, we must get teachers from outside...we can’t have any high school without these (Gadebe, 2007, p. 1).

The remark should be understood in the context of the strategies that the post-apartheid government embarked upon. These include the introduction of the National Policy Framework for Teacher Education and Development in South Africa which aimed at addressing teacher development and the apparent shortage of qualified teachers in the subjects of mathematics, science and technology (DoE, 2007, p. 13). The strategies of the framework are the following:

- Ensuring the establishment of a national electronic database and information service on teacher demand and supply in collaboration with Provincial Departments of Education [PEDs];
- Adjusting conditions of service to respond to challenges of recruitment including financial incentives to recruit and retain teachers in scarce skills areas, for top performing teachers, and for teachers in rural areas;
- PEDs are required to “make special provisions” with regard to mathematics, science, technology and language teachers in view of their scarcity.

Despite these post-1994 strategies and initiatives on the provision of qualified mathematics and science teachers; some of the researchers argue that the real problem of the shortage of these teachers does not lie in the numbers but in the distribution or deployment of the teachers. Mda (2009, pp. 201–202), for instance, argues that in terms of numbers, there may be enough teachers in South Africa and that the problem relates to distribution according to: geographic areas, provinces, regions/districts, grade levels, subjects, qualifications, skills, quality, race, and language. Similarly, the Centre for Development Enterprise (CDE, 2011, p. 2) notes that there is a tendency in some of the schools to allocate qualified mathematics and science teachers to subjects they are not qualified to teach. This problem of the absence of qualified teachers in some of the impoverished schools is aggravated by factors such as the inability of the education system to efficiently take up the *Funza Lushaka* graduates (DBE, 2011, p. 40). The result is the increasing offering of mathematics literacy instead of mathematics to most Grades 10 to 12 learners in several schools (South African Institute of Race Relations, SAIRR, 2013, p. 1). In this light, it necessary to investigate the recruitment and distribution of qualified mathematics and science teachers to both impoverished and affluent schools in a post-1994 education system of South Africa.

Although several studies have been conducted on the qualifications of teachers (Keevy, Green & Manik, 2014, p. 29) and their recruitment as well as deployment (Mulkeen, Chapman, Dejaeghere, Leu & Bryner, 2007, p. 19–20), this study specifically focuses on the factors that constrain education managers from ensuring equality and equity in the recruitment and distribution of qualified mathematics and science teachers to impoverished as well as affluent schools in a post-1994 education system of South Africa.

QUALIFICATIONS AND THEIR SOCIETAL RELEVANCE

The White Paper of 1995 notes both the insignificant numbers of teachers that graduated from colleges and universities under the rule of the apartheid government (Republic of South Africa, RSA, 1995:19). In addition, the qualifications that were offered to African teachers in the subjects of mathematics and science were inferior, thus perpetuating "a cycle of mediocrity" in the classroom (RSA, 1995, p. 19). In order to address this challenge of inferior teacher training and qualifications in a post-1994 South Africa, a new policy on the Higher Education Qualifications Sub-Framework was introduced to define qualified teachers including those in the fields of mathematics and science. According to this policy, qualified teacher must possess a Bachelor of Education (Bed) or an Advanced Diploma in Teaching which is an equivalent of the Relative Education Qualification Value (REQV 14) Keevy, Green & Manik, 2014, p. 29). Keevy at al. explain that teachers who qualified before the introduction of these new minimum requirements are deemed professionally qualified if they hold a qualification set at REQV13. However, a professionally under-qualified teacher can possess a qualification set up to REQV15 as an approved and recognised academic qualification. This teacher, however, does not qualify for permanent employment without a professional teaching qualification (Keevy et al. 2014, p. 30). It should be noted that no teacher can teach in South Africa without having registered with the South African Council for Educators (SACE). SACE is a professional council which was established through the South African Council for Educators Act (Act No. 31 of 2000).

It is also worth noting that reforms on teacher qualifications were accompanied by changes regarding responsibilities. Unlike apartheid government which tended to restrict teachers to classroom activities, the responsibilities of teachers in a post-1994 South Africa have been expanded to include roles that are beyond the classroom walls.

These roles, according to the Norms and Standards for Educators, include being mediators of learning, interpreters and designers of learning programmes and materials, leaders, administrators and managers, scholars, researchers and lifelong learners, community members, citizens and pastors, assessors, and subject specialists (Ngoepe & Kaino, 2014, p. 580). In essence, a post-apartheid South Africa expects teachers to be active in both the education and community environment. Therefore, mathematics teachers are also expected to be actively involved offsetting the legacy of apartheid which sees no value in teaching African children gateway subjects such as mathematics and science, thus burdening the minority white population with the production of skills in the fields of science and technology.

ENTRANCE TO THE EDUCATION SYSTEM FOR (MATHEMATICS) TEACHERS

There are two types of posts that teachers are recruited into. The first type is that of government (funded) posts that are created by the Member of the Executive Council for Education (MEC) in a province. The second type is that of School Governing Body posts. The filling of funded posts is the responsibility of the provincial Head of Department (HoD) who has to consult stakeholders represented in the provincial chamber of the Education Labour Relations Council (ELRC) regarding posts that have been budgeted for in a particular financial year according to the *post establishment* - a 1998 model used annually to determine the number of posts allocated to individual schools. The collective agreement reached in the ELRC allows the provincial Department of Education to proceed with the filling of posts. Every school is issued with a post establishment certificate. These certificates guide schools in determining whether they have shortage or excess (i.e. surplus) teachers. If the certificate indicates that the school has been granted additional posts, the school has a shortage and can recruit additional teachers. On the contrary, if the post establishment certificate shows less posts than the teachers currently at the school, the school is deemed to have a surplus and is required to initiate a process of identifying teachers in excess for transfer or redeployment to schools that are experiencing shortage. Details or profiles of teachers deemed to be in excess are submitted to the Departmental redeployment list for placement in other schools.

The first step in the placement of these educators, according to sub-section 2.4. (c) of the Employment of Educators Act (EEA (Act No. 76 of 1998), is that all vacancies that arise at educational institutions must be offered to serving teachers displaced as a result of operational requirements of the relevant provincial education department (Republic of South Africa (RSA, 1998, p. 75). The Act further states that all vacancies must be advertised and filled in terms of paragraph 3 (The advertising and Filling of Educator Posts). Provided that:

- Every attempt is made to accommodate serving educators, displaced as a result of operational requirements, in suitable vacant posts at educational institutions or offices; and
- A provincial education department may publish a closed vacancy list. In such an event, the procedures contained in the resolution dealing with the rationalisation and redeployment of educators in the provisioning of educator posts shall apply (RSA, 1998, p. 75).

Paragraph 3 of the EEA on the advertising and filling of posts provides that all vacancies in public schools must be advertised in a gazette, bulletin or circular whose existence shall be made public through an advertisement in the public media both provincially as well as nationally and that the gazette, bulletin or circular must be circulated to all educational institutions within the province (RSA, 1998, p. 76). In addition, Act prescribes that an interview committee (IC) must be formed in a school where there are posts to be filled. The school governing body has to convene the interview committee (IC) which appoints a chairperson and secretary from among its members. The interview committee must be constituted as follows:

- One departmental representative (who may be the school principal) as an observer and resource person;
- The principal of the school (if he/she is not the department's representative), except in the case where he/she is the applicant;
- Members of the SGB, excluding teacher members who are applicants to the advertised post/s; and One union representative per union that is a party to the provincial chamber of the ELRC.
- Union representatives shall be observers to the process of short listing, interviews and the drawing up of a preference list (RSA, 1998, p. 77).

In section 3(g-h), the Act mentions that all interviewees must receive equal treatment during the interviews and that the interviews must be conducted according to guidelines jointly agreed upon in the ELRC. The EEA explicitly states that the Head of Department (HoD) may appoint persons under sub-sections 2(a), 2(b) or 2(c). Appointments under sub-section 2(a) have permanent status. Those appointments under sub-section 2(b) have a temporary status, while sub-section 2(c) deals with contract employment. Furthermore, section 6(3) of the EEA states that the HoD should consider recommendations from the school governing body (SGB) before effecting an appointment, transfer or promotion of teachers in a provincial Department of Education. Section 3(i) of the EEA further prescribes that an interviewing committee should rank the candidates in order of preference. Prepare a brief motivation and submit these to the school governing body for its recommendation to the relevant employing authority (RSA, 1998, p. 77).

All the aforementioned processes should be guided by democratic values that are also found in several important laws such as the South African Schools Act (Act No. 84 of 1996) and, most importantly, in the constitution of the Republic of South Africa (Act No. 108 of 1996). Section 195(1) of the constitution, for instance, prescribes that public administration must be broadly representative of the South African people, with employment and personnel management practices based on ability, objectivity, fairness and the need to redress the imbalances of the past to achieve broad representation (RSA, 1996a, p. 105). These constitutional principles are encompassed in the EEA which provide that the Head of Department must give due regard to the values of equality and equity when appointments are made (RSA, 1998).

CENTRALISATION AND DECENTRALISATION IN EDUCATION

The application of the previously mentioned legislative prescripts involves the establishment of a healthy consultative relationship between Departments of Education and related stakeholders such as the school governing body and teacher unions. The compulsory nature of the consultations flows from the National Education Policy Act (NEPA) Act No 27 of 1996). The Act requires government to formally consult with unions and other education stakeholders as a prerequisite to passing any education legislation. Regarding the recruitment and distribution of teachers, Mulkeen, Chapman, Dejaeghere, Leu and Bryner (2007, p. 19–20) identify two systems that form the basis of a consultative process. These are the central authority and market systems of teacher deployment. A central authority system allows for national or provincial planning and teacher deployment. Schools are not afforded an opportunity to recruit their own teachers under this system (Mulkeen et al. 2007, pp. 19–20).

In contrast to the central authority, the market system allows for the decentralisation of authority to school-based management structures regarding teacher recruitment (Mulkeen et al. 2007, pp. 19–20). The characterised of a market system are that teachers deploy themselves by searching for jobs, and that it recognises the authority of each school in selecting its own teachers (Mulkeen et al. 2007, p. 18). According to research (Mobegi, Ondingi & Oburu, 2010, p. 408), the market system can contribute to endless disputes and delays in the recruitment process. In Kenya, for example, a study found that local communities demanded that their own people be appointed to the positions of school principal without the necessary qualifications (Mobegi, Ondingi & Oburu, 2010, p. 408). Equally, another Kenyan study found that local districts in were often accused of acts of nepotism, receiving bribes and keeping selection dates secret (Kipsoi & Anthony, 2008:7) while a study conducted in Pennsylvania found that some of the schools hired candidates with local ties, friends and relatives (Monk, 2007, p. 164).

In South Africa, education policy development has integrated the central authority and market systems. The post-1994 reforms were geared towards shifting education management from a heavily centralised apartheid administration to the decentralisation of authority to school-based management (SBM) structures such as the school governing body (SGB). These SBM structures are required to exercise their authority in partnership with government structures. The preamble of the South African Schools Act (Act No 84 of 1996), for example, states that learners, parents and teachers have rights that should be upheld and, in turn, these parties have to accept responsibility for the organization, governance and funding of schools in partnership with the State (RSA, 1996b, pp. 3-4). According to World Bank (2007, p. 2), a successful decentralisation takes place within an understanding that school-based role-players operate within policies that are determined by central government, and that the *"key to decentralization is to identify exactly what the government's role in decision-making should be"*. The absence of a clear role of government in the decision-making process of school-based management structures has attracted criticism from several researchers. They regard the inclusion of decentralisation aspects in the policies such as the South African Schools Act as an impediment to the creation of equality in a historically stratified society (Motala & Pampallis, 2005; Samoff, 2008).

Reddy (2003) and Whittle (2007, p. 157), for instance, assert that decentralisation of authority promotes a two-tiered system of education: one for the rich and another one for the poor. Similarly, Christie (2010, p. 707) posits that most of the post-apartheid education policies were developed for institutions that were already functioning well such as those in most developed countries, and that these post-apartheid policies fail to recognise that South African schools do not function equally and that many of them are dysfunctional. According to the World Bank 2007, p. 2), decentralisation or SBM principles include: choice and competition as well as school accountability. The *choice and competition* principle suggests that parents who are interested in maximizing their children's learning outcomes are able to choose to send their children to the most productive school (in terms of academic results) that they can find (World Bank, 2007, p. 2). This principle presumes that the demand-side pressure will improve the performance of all schools if they want to compete for learners. Furthermore, the World Bank indicates that the principle of *school accountability* compares schools to private firms or industry. Parents and learners are seen as "clients" while schools are viewed as service providers that should be held accountable for good quality and timely service (World Bank, 2007, p. 2).

In essence, the South African approach of blending of central authority and market systems in policies tend to create implementation challenges due to territorial contestations among education stakeholders. These contestations are more prevalent on matters that relate to the recruitment and distribution or deployment of teachers. The following examples illustrate contestation of authority between provincial Departments of Education and school governing bodies:

- *Settlers Agricultural High School v, Head of Department of Education, Limpopo Province, 2002*. The SGB at this school had recommended a candidate as principal but the Head of Department rejected the recommendation and appointed another candidate to the post in accordance with section 3(f) of the EEA of 1998. the SGB referred the matter to the court of law where they succeeded in overturning the decision of the Head of Department (Beckmann & Fussel, 2011, p. 567); and

- *Head of Western Cape Education Department v Governing Body of Point High School*. Similar to the Settlers dispute above, the Head of Department in the Western Cape had declined the school governing body recommendations for the appointment of a principal and deputy principal respectively. The school governing body approached the court of law and the judge ruled in favour of the school. The judge argued that the reasons of the Head of Department regarding equity were not relevant because all the candidates recommended by the SGB and those appointed by the Head of Department were all white. In such a case, the judge explained: *"[o]n the broad ground of unreasonableness as contemplated in s 6(2)(h) [of the EEA], in my view the HoD proceeded without a proper understanding of the discretion which he was called upon to exercise"* (Beckmann & Fussel, 2011, p. 567).

The aforementioned examples are aggravated by the involvement and apparent influence of teacher unions in the recruitment and deployment processes. Although the National Education Policy Act (NEPA) Act No 27 of 1996) requires government to formally consult with teacher unions, the South African Democratic Teachers' Union (SADTU) is often regarded as an impediment to the filling of vacancies in a post-1994 education system (Pattillo, 2012; Smit & Oosthuizen, 2011, Zengele, 2013). Some of the accusations are that the teacher union "unlawfully" interferes with the recommendations of SGBs to appoint teachers (Smit & Oosthuizen, 2011, p. 64); local SADTU leaders tamper or throw away applications in the district offices or solicit bribes in exchange of posts (Zengele, 2013, p. 88; Pattillo, 2012, p. 59). According to Pattillo (2012, p. 58), SADTU influences the filling of vacancies in the schools through its unwritten policy of patronage-based political appointments called cadre deployment which apparently allows the union to deploy its members to school leadership positions irrespective of qualifications (Pattillo, 2012, pp. 71–72). The 2030 vision of government, as contained in the National Development Plan (NDP), seems to support the views of the aforementioned authors. The NDP mentions, among others, that government intends to exercise full control on the recruitment and deployment of teachers:

> The administration of education (including appointment and disciplining of teachers) is the preserve of the government with unions ensuring that proper procedures are followed" (RSA, 2011, p. 266).

Equally, teacher unions appear as determined to retain the market system of teacher deployment which apparently allows them to influence the employment processes. SADTU, for instance, has adopted its own vision 2030 through which the union envisages a SADTU that "wields, through cadre deployment and influence, the established instruments of the state in line with the strategic objectives of SADTU" (SADTU, 2010, p. 8). The contrasting objectives of the two aforementioned vision 2030 signal continuous tussles among the different education stakeholders, namely, teacher unions, Departments of Education and school governing bodies regarding teacher recruitment and deployment.

RESEARCH APPROACH AND DESIGN

To gain an in-depth understanding of the management constraints in the recruitment and deployment of qualified mathematics and science teachers in a post-1994 education system, a qualitative research approach was adopted. The study was designed as a collective or multiple case which focused on policy implementation regarding the recruitment and distribution of qualified mathematics and science teachers in apost-1994 education system of South Africa. A case study is an object of study, as well as the product of the inquiry (Creswell, 2007, p. 73). In addition, a case study delimits the object of study: the case (Merriam, 1998, p. 27). This particular case study was limited to four secondary schools that are teaching mathematics and science as subjects to grade 10 until 12 learners. These schools are situated in two types of communities with varying economic demographics in the province of Mpumalanga. One of the two former Model C schools is situated in the town of Nelspruit where the head office of the Mpumalanga Department of Education is also found. The second school is situated in the town of Middleburg - about 200 kilometers from Nelspruit. Regarding the two historically disadvantaged schools, the first is found in a township which is about 35 kilometers from Nelspruit, whereas the second school is situated in another township which is about 225 kilometers from Nelspruit. The schools were purposively selected from the grade 12 schedules results of both the provincial and national Departments of Education. The selection criteria included the top 20 and bottom 20 positions of the schools for three consecutive years. Only fourteen (14) participants that were identified consented to participate in the semi-structured one-on-one, tape-recorded interviews. The participants included four principals, two education officials from the district and provincial offices, as well as eight mathematics and science teachers for grade 12 classes. Participants were guaranteed confidentiality and anonymity.

Constant analysis was used in working through the texts line by line linking raw data in the text to the research question by identifying relevant information that helped to answer the research question (Glaser & Laudel, 2013, p. 6). Data were indexed by attaching codes to the part of the text containing the relevant information. The indexing process was followed by another analysis of information from official publications and formal school documents. Salient data from these sources of information were constantly compared to data already obtained from transcripts. Subsequently, these data was included in the appropriate themes. All the themes were checked if they fit the research question and those that did not fit were revised to ensure that they link to the research question. Thereafter, descriptive paragraphs about the themes were written down in order to identify patterns (relationships) among themes. This process was useful in linking similar themes. The result is that plausible explanations were sought and interpreted for their usefulness in clarifying the research question.

Validity in the study was ensured through a pilot study which validated interview schedules in terms of pre-formulated questions. In addition, reliability was enhanced through the use of thick descriptors such as lengthy verbatim quotations from participants. Despite the aforementioned processes, the limitations of the study include insufficient information on the *Dinaledi* schools and classroom effectiveness of *Funza Lushaka* graduates. Furthermore, the study covered only four schools in two districts from the four that are in the province of Mpumalanga.

THEORETICAL FRAMEWORK

The focus of this study is largely on the change management in the post-apartheid education system, especially with regard to the process of the recruitment and deployment of qualified mathematics and science teachers to secondary schools. Rees and Hall (2013, p. 104) mention ‘managing change’ or ‘change management’ describes the application of systematic interventions to implement a planned change within an organisation in order to achieve a desired future state. This assertion is supported by Pryor, Taneja, Humphreys, Anderson and Singleton (2008, p. 2) who explain that organisational change is the movement of an organisation from the existing plateau toward a desired future. Moving an organisation from its current state or position to a desired future encompasses a set of abilities, techniques and disciplines through which complexity and specialisation are transformed into actions and results (Pisla, Irimias & Muntean, 2010, p. 166). However, research indicates that change management in organisations is often stalled because each incoming administration tends to launch a reform effort of its own which sometimes contradicts earlier measures (Moujaes, Hoteit, Hultunen & Sahlberg, 2012, p. 2). The result is often resistance to change and is attributed to factors such as fear of the unknown, lack of information, fear of failure, political undercurrents, the lack of benefits as well as the lack of cooperation in the organisation (Sanda & Sraha, 2011, p. 5). During a change transition, there are psychological adjustments that individuals must make, including letting go one’s old situation and identity as well as moving through a period of ambiguity and contradictions (Sanda & Sraha, 2011, p. 5). In the context of this study, effective communication is one of the important aspects that affords teachers an opportunity to move “through a period of ambiguity and contradictions” regarding the government’s vision which prioritises the subjects of mathematics and science through the deploying qualified teachers in these subjects.

Moujaes et al. (2012, p. 4) argue that education ministers and senior civil servants play a key role in setting the vision, providing resources and setting regulatory frameworks while principals, teachers, community activists, and entrepreneurs lead on the frontline (i.e. schools). It is the responsibility of education ministers and senior education managers to move the post-1994 education system from the existing racial imbalances toward a desired future which entails quality teaching in the mathematics and science programmes, particularly in disadvantaged schools. The aforementioned views resonate with Kotter's (1996, p. 61) Eight-Steps Change Model which suggests that real change is directed from the top (Webster, 2012, p. 15). In essence, post-1994 change management processes in the country necessitate that managers should adopt transformational leadership traits. The adoption of such traits would enable education ministers and senior education managers to create an enabling environment for the post-1994 vision which must be realised through practical steps that include the filling of all vacancies in secondary schools with qualified teachers, hence the development of the *turning the vision into practice* framework.

The aforementioned framework adopted and modified some of the steps in Kotter's (1996, p. 61) Eight-Steps Change Model in order to ensure that the framework provides useful practical procedures in the process of recruiting and distributing qualified mathematics and science teachers to secondary schools in a post-1994 education system of South Africa. Furthermore, the *turning vision into practice* framework explains the relationships between and among its management phases such as planning and policy development, the design of recruitment and deployment criteria, communication, implementation, monitoring, evaluation and feedback as well as policy review. The adoption of ***turning vision into practice*** framework is premised on the belief that systemic change management models such as that of Kotter, which comprises of sequential processes, can be modified through the introduction of an additional process at the end in order to provide a feedback step or phase (Rees & Hall, 2013, p. 113).

All steps of the ***turning vision into practice*** framework supports the four dimensions that are associated with transformational leadership - *idealized influence* or *charisma, inspirational motivation, individualized consideration* and *intellectual stimulation*.- These dimensions are helpful to education leaders that, presumably, have abilities of exercising influence over mathematics and science teachers' beliefs, attitudes and behaviours, motivating and inspiring teachers, showing concern for teachers' welfare while helping them to reframe problems and to approach old situations in new ways (Pastor & Mayo, 2006, p. 3). It is noteworthy, that these transformational leadership behaviours entail the aspects of teachers' job satisfaction and motivation that resonate with Abraham Maslow's Hierarchy of Need (1943) and Frederick Herzberg's (1959) Two-Factor or Motivation-Hygiene Theories. Maslow's theory arranges employees' needs in a hierarchy which entails four types of needs that must be satisfied before an employee can act unselfishly (Griffin, 2014, p. 125). Equally, Herzberg views responses about good feelings as relating to job content (motivators), while responses about bad feelings are associated with job context (hygiene factors (House & Wigdor, 2006, p. 370). In essence, tasks and responsibilities of leaders as well as managers in a post-apartheid education system encompass the reduction of job dissatisfaction, support and motivation for employees, ensuring job security, recognition of and providing opportunities for achievement, as well as improving the performance level of employees.

FINDINGS AND DISCUSSION

Post establishment model

The study found that the nationally developed *post establishment* model is the only strategy or tool that the Mpumalanga Department of Education (MDE) utilizes to annually allocate teaching posts to individual schools. This *post establishment* model encompasses the traits of both the central authority and market systems of teacher deployment (Mulkeen et al. 2007, pp. 19–20). The model also serves as a precursor for the recruitment of new teachers and of the redeployment of excess teachers (i.e. those declared as surplus in their schools). The findings suggest that the model seems focuses on cost curtailment regarding personnel expenditure instead of ensuring that each every learner has a qualified teacher, particularly in the prioritised subjects of mathematics and science. In addition, the apparent infusion of elements from the central authority and market systems of teacher deployment leads to disputes between the provincial Department and the schools. Regarding the redeployment of teachers, the model has no effect on the transfer of mathematics and science teachers from one school to the next because redeployment lists do not provide teachers for these subjects.

Transgressions of policy

Analysis of data indicates that vacancies for entry level or post level-1 teachers are not advertised in the Mpumalanga Department of Education (MDE). The non-advertisement of the vacancies is contrary to the provisions of the Employment of Educators Act (Act No. 76 of 1998). This Act prescribes that all vacancies must be advertised in a gazette, bulletin and circular and that the advertisements should be made public. Presumably, the advertisement of vacancies would ensure the promotion of the democratic values and principles of equality and equity in a post-apartheid education system. The apparent non-adherence to the provisions of the EEA potentially compromises the quality of teachers that are being recruited for mathematics and science subjects. The "headhunting" and "poaching" that disadvantaged schools utilise in their recruitment of teachers are not structured and criteria are not defined. It is noteworthy that while impoverished schools relied on informal networks to recruiting qualified teachers through informal networks, affluent schools continuously received CVs of qualified teachers without such informal networks because they "*have a constant flow of people handing their CVs…"* [Teacher-C1, Interview 8, 21/10/2013].

The role and influence of teacher unions

Another finding is that the relationship or consultative process between school governing bodies, education officials and teacher unions is often characterised by tensions and disputes, particularly with regard to the filling of vacancies in schools. The study shows that school governing bodies tend to resort to the courts of law in order to challenges decisions taken by the Head of Department (HoD) regarding the appointment of preferred candidates. With regard to whether teacher unions exert undue pressure on the school governing bodies, this study could not confirm these allegations. However, the study found that union representatives actively participate in the interview committee (IC) during interviewing. This active involvement is inconsistent with the provisions of the Employment of Educators Act that grant union representatives an observer status. In addition, the study found that representatives of Departments of Education in the interview committees condone the active participation of teacher unions' representatives. It is in this context that the dominant teacher union in the education sector, the South African Democratic Teachers Union (SADTU), has been singled out and accused of disruptive behaviour.

Evidence from interviews in this study suggests that the participation of union representatives should end at the level of the interview committee where union representatives observe the interview process. Unions are not represented during meetings of SGBs where recommendations and motivation from the interview committee are being considered. Therefore, the only plausible explanation to justify allegations against SADTU can derive from the union's 2030 vision which suggests that SADTU intends to influence established government structures for the next 20 years through cadre deployment and influence (SADTU, 2010, p. 8). In the absence of evidence to support previous research on the alleged unlawful interference of unions in the decision of the SGBs, this study can only confirm that union representatives do actively participate in a process they are required to observe its fairness.

Interviews and appointment process

Although policy requires interview committees to conduct interviews according to ELRC agreed upon guidelines, this study found that such guidelines did not exist. Furthermore, it was established during the study that interview committees do not adhere to the provisions of the EEA. Contrary to the EEA provisions, the interview committees submit their written recommendations of preferred candidates directly to the appointing authority instead of the school governing body. In addition, the study noted that ELRC processes sometimes contribute to the increase of constraints in the recruitment of qualified mathematics and science teachers. The collective agreement of 2001, for instance, encouraged education authorities to appoint under-qualified teachers where no qualified teachers can be found. In terms of Resolution No. 4 of 2001, these under-qualified teachers were to be appointed as from 31 December 2001 onwards (ELRC, 2001, p. 1). Considering that the shortage of teachers is largely in the subjects of mathematics and science, the implication of this collective agreement is that mathematics and science subjects will be taught by under-qualified teachers, especially in the schools that are situated in impoverished communities. In essence, education in such schools will only be reproductive. Education reproduction happens when the "education system merely serves to reproduce things as they are; children from poor backgrounds go to poor schools and then into poorly paid, low status jobs or unemployment" (Harber & Mncube, 2011, p. 234).

Preference of foreign teachers

Despite the MDE policy which requires that schools should first appoint excess teachers, followed by bursary holders before considering the appointment of foreign teachers, information from interviews suggests that the schools prefer the appointment of foreign mathematics and science teachers. This can be attributed to the belief of most interviewed principals and education officials that foreign mathematics and science teachers, particularly those from Zimbabwe, are very good and that most foreign teachers display better work ethics than their South African colleagues. One district official indicated that the schools' preference of foreign teachers emanates from perceptions that these teachers are more dedicated to their work because they teach during Saturdays, Sundays, holidays and the assumption that foreign teachers are passive regarding teacher unionism. During the study, it was further noted that the preference of foreign teachers has resulted in the transgression of the recruitment policy of the Provincial Department of Education which requires schools to first appoint excess teachers, followed by the placement of *Funza Lushaka* graduates before any other teacher could be recruited. Finally, the study has confirmed that some of the schools contribute to the apparent shortage by allocating qualified mathematics and science teachers to subjects they are not qualified to teach. This includes foreign teachers that were initially recruited to teach mathematics and science.

Placement of Funza Lushaka graduates

The study found that *Funza Lushaka* bursary beneficiaries are appointed in terms of section 7(2) (a) of the EEA because they enter the profession as permanent teachers and without probation. Their appointment under section 7(2)(a) enables these beneficiaries to immediately access fringe benefits such as medical aid, pension contributions and housing allowance. Regarding the placement of the graduates, some of the interviewed principals believe that the placement of *Funza Lushaka* graduates was not equally enforced between English-medium and Afrikaans-medium schools because most of the graduates use English as a language of teaching. Despite these concerns, interviews it was clear that all these principals support government's efforts in creating a pool of qualified mathematics and science teachers through the *Funza Lushaka* bursary scheme.

Teacher deployment in rural schools

Data analysis from the interviews shows that most mathematics and science teachers choose their workplaces based on family considerations. Five teachers indicated that they are not willing to relocate to rural schools without their families. One of the teachers in School D pointed out that the reason for his relocation from a rural school in another province was to be nearer his family. This particular teacher further stated that he will not in future consider returning to a rural school because it was *too depressing there is "no water, no toilets…you know it is just a matter of surviving but it's not nice"*. The findings of this study show that the movement of teachers is linked to the distance between the workplace and the teachers' family homes. Some of the participants had to request transfers from education authorities in order to work closer to their family members. Furthermore, once the teachers were reunited with their families, it became very difficult for these teachers to future transfers from education authorities because of harsh conditions in the rural areas.

CONCLUSION

The collapse of the apartheid government in 1994 led to large scale education reforms. In 1995 the first White Paper on education was introduced in order to reduce inequalities and to create greater access the provision of quality mathematics and science programmes. The post-apartheid government had realised very early that it was necessary to prioritise the supply of qualified and competent teachers for mathematics, science and technology to historically disadvantaged schools. However, the adoption of policies that blend centralisation and decentralisation traits have slowed down the implementation of the post-1994 priorities regarding the provision of quality mathematics and science programmes. The South African Schools Act (Act No. 84 of 1996) and the 1998 post establishment model for the distribution of teaching posts are some of the policies that seem to constrain the reduction of inequalities between affluent and impoverished schools. The post establishment model, for instance, does not link each post to an individual subject. The absence of a link between the model and the school subjects makes it difficult for the provincial Department of Education to determine the extent of teacher shortage in the fields of mathematics and science.

Furthermore, the concurrent implementation of centralisation (central authority) and decentralisation (market) systems increases uncertainty regarding the responsibilities of education stakeholders such as education authorities, school governing bodies and teacher unions, especially with regard to the fillings of vacancies. Finally, the decentralisation encourages competition and reinforces inequalities between affluent and disadvantaged schools. Affluent schools continuously levy high user fees in order to attract and retain best teachers in the subjects of mathematics and science teachers. The result is the perpetuation of "a cycle of mediocrity" through the efforts of poorly skilled teachers in the classroom of historically disadvantaged schools as noted in the first post-apartheid White Paper on education (Republic of South Africa, RSA, 1995,

p. 19). It is apparent that the provincial Department of Education has been constraint in making “special provisions” with regard to the filling of all vacancies in secondary schools with qualified teachers in the gateway subjects of mathematics, science, and technology in accordance with the National Policy Framework for Teacher Education and Development in South Africa (DoE, 2007, p. 13). Briefly stated, the constitutional principle of substantive equality and equity has been partially addressed through extensive policy formulation. There is however a greater responsibility of ensuring that both individual and group disadvantages created by a history of oppression and apartheid (Henrard, 2002, p. 24) are practically addressed through the recruitment and deployment of qualified mathematics teachers in a post-1994 education system of South Africa. It is only then that Africans, as victims of apartheid, can reclaim their African pride.

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TEACHING MATHEMATICAL PROBLEM SOLVING

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This paper presents some examples of mathematical problems used by the author to teach mathematical problem solving in several different contexts. These problems are all of an elementary, easily understandable nature, yet quite challenging. Emphasis is placed on the typical problem-solving process in a classroom.

Keywords: Mathematics, problem solving, teaching

INTRODUCTION AND MOTIVATION

Every mathematician will agree that mathematics can and should be much more than performing routine calculations according to prescribed algorithms: it should excite and engage and teach how to think in a logical and structured way. Mathematicians do not only want to discover laws, rules and patterns, but also understand why things are the way they are – the concept of a rigorous proof is unique to mathematics and distinguishes it somehow from other sciences.

But how are mathematical theorems and their proofs discovered? How does one approach a mathematical problem if one has not been told before what steps to follow? Teaching the skills required to tackle questions in mathematics that go beyond following a prescribed method is certainly a challenge, and this paper is not aiming to offer a simple solution. In fact, it is the author's belief that practice is the single most important key to success, and that the best an educator can hope for is to create a stimulating environment.

Instead, this paper provides some examples and case studies, based on the author's experiences in teaching mathematical problem solving skills on various levels:

- to high school learners (Grade 8 to Grade 12, occasionally also Grade 6 or 7) participating in biweekly classes offered by the Department of Mathematical Sciences at Stellenbosch University. The main aim of these classes is to prepare the participants for mathematical competitions such as the South African Mathematics Olympiad (SAMO) or the South African Mathematics Team Competition (SAMTC), but they also serve an important general goal: to expose the learners to interesting, challenging, but also entertaining mathematics, and to show them that there is interesting mathematics beyond the school mathematics syllabus that is sometimes perceived as dry.
- to the top performers in the South African Mathematics Olympiads, who are invited to an annual training camp taking place at Stellenbosch University in

December. Here, the goal is to select and prepare the teams that eventually represent South Africa at international competitions (International Mathematical Olympiad, Pan-African Mathematics Olympiad).

- to students at Stellenbosch University as part of the module "Foundations in Abstract Mathematics", a second-year course with the aim to teach abstract mathematical reasoning skills and in particular introduce the students to mathematical proofs. Each of the four terms is somewhat different in style, and the second term is usually dedicated to problem solving.
- to students at the African Institute for Mathematical Sciences (AIMS): these students come from all across the African continent to participate in a one-year "Structured Masters" programme. The course on mathematical problem solving forms an integral part of the first block of so-called "skills" courses, which are, as opposed to "review" courses, offered every year and compulsory to all students in the programme. This format of this course is largely due to Alan Beardon, who taught it for several years and also wrote a book on the subject (Beardon 2009): it aims to emulate the mathematical research process from formulating a problem to gathering ideas and experimental evidence, conjecturing and proving, and finally extending and generalising.

At first glance, it may seem that these would be very different experiences: surely, one would expect a major difference between high school learners and students participating in a postgraduate programme. Yet it turns out that there are many similarities, and in fact school learners are sometimes just as quick and skilled at solving problems as students who already have a degree in the mathematical sciences. When exposed to specific mathematical problems or the process of problem solving in general, mathematical talent and an open mind are often more important than advanced mathematical knowledge. Moreover, even though each of the aforementioned environments offers its unique challenges, some challenges (in particular diversity of backgrounds, talents and interests present in almost every classroom) are common to all of them. There is also always a common goal: to motivate learners and students to explore and experiment with mathematics, to find enjoyment in tackling mathematical questions and to look deeper by asking "Why is this the answer?" instead of just "What is the answer?"

NATURE OF THE PROBLEMS AND CLASSROOM SETUP

In the following, three selected problems will be presented as case studies. Each of them has been presented in at least two of the aforementioned contexts (often repeatedly over the course of several years). Generally speaking, the process in which they are approached and eventually solved is remarkably consistent and predictable in spite of the differences between the individual groups. It must be said, however, that the author can only provide anecdotal evidence that the presented examples are problems that "work well" in the classroom. This is based on personal experience, not a thorough scientific investigation.

All three problems are of an elementary nature – the term "elementary" does not mean "easy" in this context, quite the opposite. It only refers to the fact that no advanced mathematical knowledge is required to formulate and understand them. This allows for them to be used in a variety of contexts (as indicated earlier) without having to make assumptions on the target audience's background knowledge. They are mostly well-known classics, yet not so well-known that school learners or university students participating in a problem-solving course for the first time would normally have seen them before.

The classes in which these problems are discussed follow a fairly general pattern that the author of this paper found suitable: first, the problem is introduced to the class. Even though the problem statement is mostly quite short, it is important to make sure that everyone understands the question precisely, so the key parts are usually repeated. Then the class is encouraged to think about the problem, to experiment, to gather ideas, and to discuss the problem with others. The lecturer's main task is to observe, to moderate, to drop hints and sometimes ask the right questions.

After a while, patterns are observed and conjectures are formulated. To increase the exchange of ideas, the lecturer encourages selected students to share their thoughts and results with the entire class. Eventually, the ideas and observations are pieced together to a solution of the problem. Many of the problems are somewhat open-ended: they allow for extensions, modifications and generalisations. In addition to emulating the mathematical research process (where finding appropriate generalisations of results obtained is a core component), it allows interested students to pursue their work further even when the class is over. It also provides a lecturer who is teaching an inhomogeneous class with a simple way of keeping particularly fast students, who solve problems before everyone else, busy and interested by letting them work on extensions and generalisations.

Mathematical problems for the classroom – examples

Regions in the plane

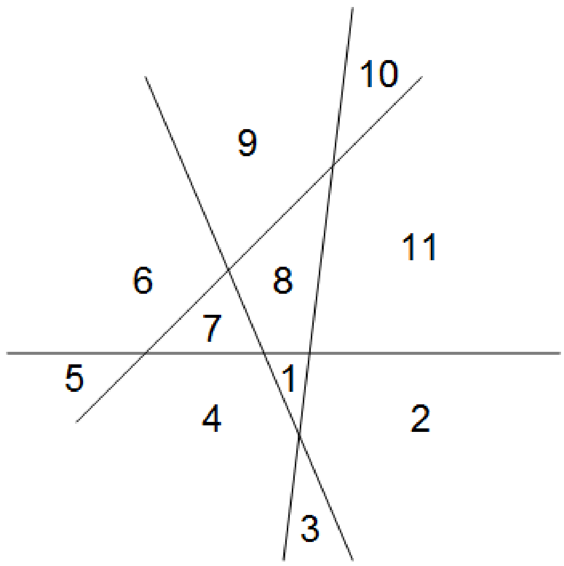
This is a typical problem for the first lecture, and it introduces the general setup of the course to the class. The problem statement only requires a few sentences, which are best accompanied by a simple sketch of the initial cases:

Problem: (Fomin, Genkin and Itenberg 1996, Ch.9, §2)

We draw n lines in the plane in such a way that there are no parallel lines and no intersections of three or more lines. These lines divide the plane into several regions; how many such regions are there?

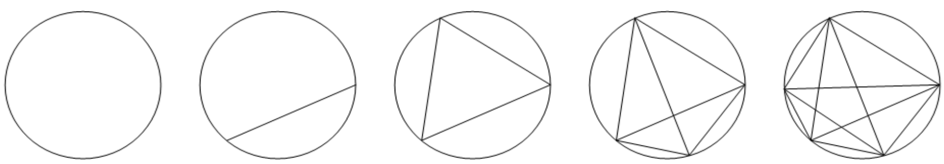
Drawing sketches for up to 5 lines (see the figure for the case of four lines); we find that the number of regions is 2, 4,7,11 and 16 respectively. It usually does not take long for someone to notice the pattern: when the n-th line is added, the number of regions increases by n. Therefore, the number of regions is

$$1 + (1 + 2 + 3 + \cdots + n) = 1 + \frac{n(n+1)}{2}.$$



The famous identity that we are using here would be enough material for a paper on its own right, but let us rather ask a different question: how do we know that the pattern continues indefinitely? How can we be absolutely sure? While learners and students are often quick to spot the pattern, explaining why it holds can be much more of a challenge, even if they have an intuitive idea.

It should be mentioned in this context that patterns, as useful as they are for mathematical investigation, can also be misleading, which makes the question for the reasons all the more important. An example of a very similar nature can serve as a warning (Conway and Guy 1996, pp.76–79):



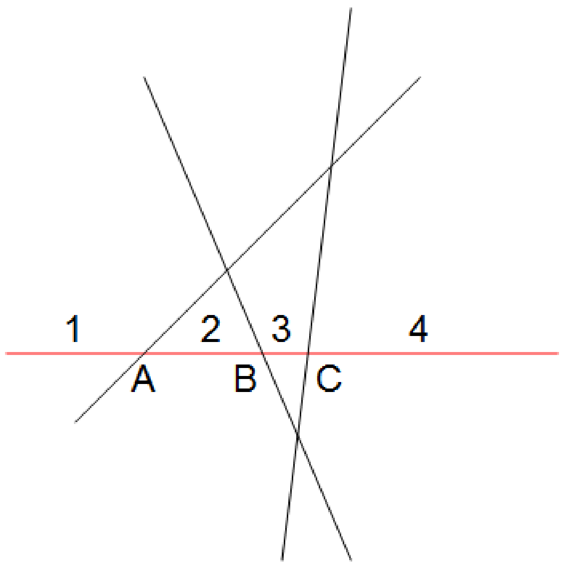
Problem

Let n points be marked on a circle, and draw all the line segments connecting them. In how many regions do these line segments divide the circle? We assume that no three of them pass through a common point.

One easily finds the answers for n=1, 2,3,4,5 to be 1, 2, 4,8,16 respectively. However, when six points are marked, the answer is 31! The "obvious" pattern breaks down, and indeed it can be shown that the correct formula is not 2^{n-1}, as one might expect at first, but rather

$$\frac{1}{24}(n^4 - 6n^3 + 23n^2 - 18n + 24).$$

With this in mind, let us return to our original question and argue why the observed pattern actually continues: If we add an additional line to $n - 1$ line in the plane, then we obtain exactly $n - 1$ new points of intersection (since there are no parallel lines and no intersections of more than three lines). These points divide the new line into n segments, see the figure below (in the case $n = 4$). Each of these segments divides one of the old regions into two new regions, while all other regions remain the same, so the total number does indeed increase by n.



This problem nicely illustrates the process from problem statement to experiment (trying out simple special cases) to observing patterns (and conjecturing a formula) to proving them. An additional advantage of this problem for classroom use is that it is almost open-ended: it lends itself to several natural follow-up questions and generalisations, for example:

How many of the regions are finite, how many infinite?

Can we adapt the formula to the case that some lines are parallel, or that there are intersections of three or more lines?

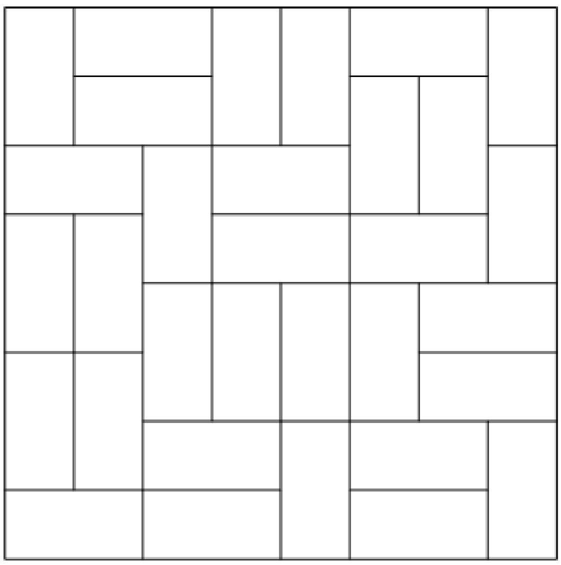
What happens if we use circles rather than lines? Or planes in three-dimensional space? Here, it also becomes important to introduce suitable additional conditions.

Tiling and colouring

Problems involving colouring have a long tradition, and are still popular in mathematical competitions (see e.g. Engel 1998, Ch.2). The most famous example is perhaps the following:

Problem

It is easy to divide an 8×8 chessboard into 2×1-rectangles without gaps or overlapping pieces, and in fact there are many ways to do it, as in the figure. Is this still possible if one corner is cut off? Is it possible if two diagonally opposite corners are cut off?



It is not uncommon that learners immediately answer the first question with "no" and the second with "yes", even without actually attempting it. Indeed, if one corner is cut off, there are 63 squares left, which is not divisible by 2, rendering the task impossible. On the other hand, it seems plausible at first glance that one can fill the 62 squares that remain when opposite corners are cut off with 31 2×1-rectangles. However, if one actually tries to do so, it soon becomes evident that it is actually impossible, and the question raises itself why this is so.

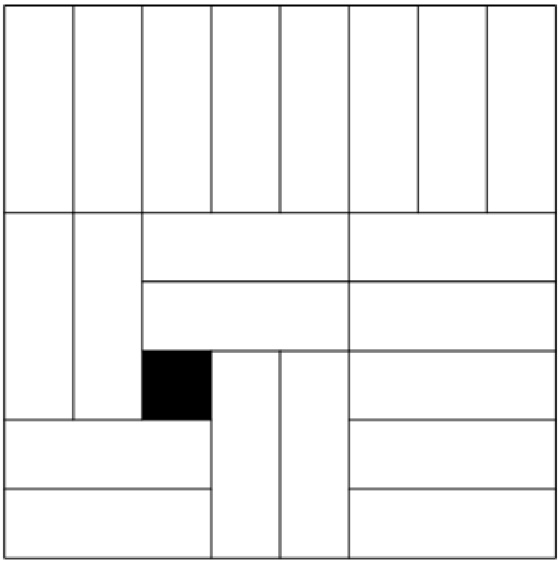
Now it can be surprisingly useful to draw the 8×8 square as an actual chessboard, with alternating white and black squares. Then one observes that, no matter how the rectangles are placed, two squares of the same colour remain uncovered at the end. One might also observe that every 2×1-rectangle, no matter how it is placed, will always cover a white square and a black square. Thus it is impossible to cover a region (such as the region obtained by cutting off opposite corners) for which the numbers of white and black squares are not equal.

It is important to make it clear to the class, especially an unexperienced class, that even a large number of failed attempts is not enough to show that something is impossible (unless the attempts actually exhaust all possibilities, which is clearly not feasible in this example). The colour argument, on the other hand, settles the question entirely.

This problem has several natural continuations, such as the following:

Problem

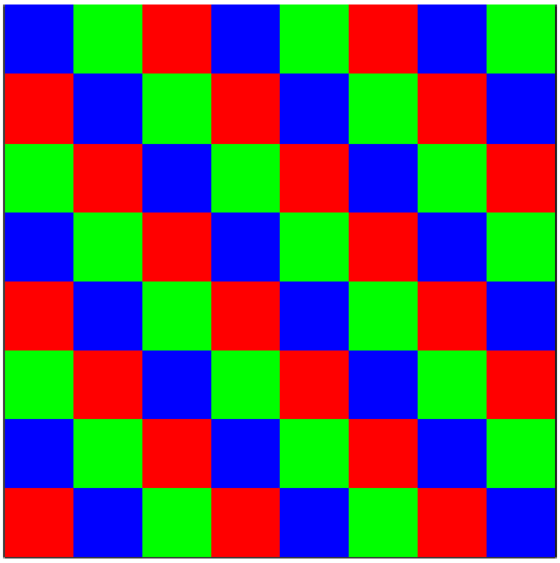
Is it possible to remove one square from an 8×8-chessboard and cover the rest with 3×1-rectangles without gaps or overlapping pieces? If so, which square should be removed? In particular, can the task be performed if a corner is removed?



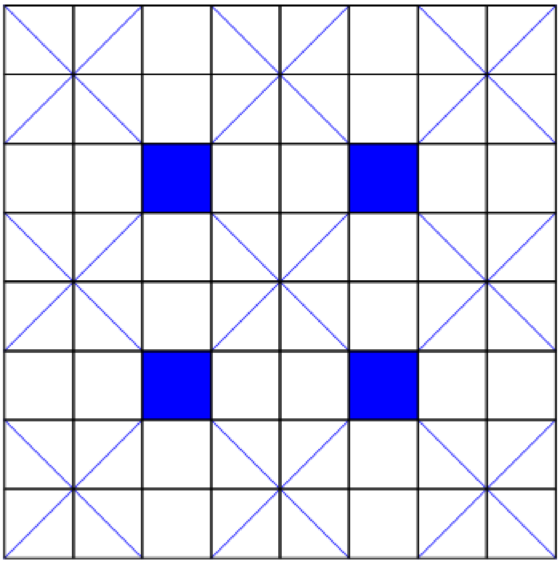
With a little bit of experimentation, it is indeed possible to find a solution as in the figure. However, the question remains whether other squares can be removed as well and in particular whether there is a possible way if a corner square is removed. Again, experiments seem to indicate that the task is impossible again.

A typical approach that many learners try at this point is to make the colour argument work for 3×1-rectangles, noticing that each of them covers either two white and a black square, or vice versa. However, this turns out to be a dead end after a while.

The solution requires a psychologically difficult step: everyone is familiar with the usual way a chessboard is coloured, so it needs imagination (and usually a few hints) to realise that the chessboard can also be coloured differently.



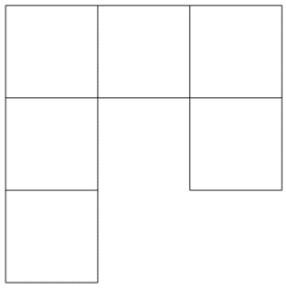
Using three colours and an appropriate colouring as in the figure, the argument can be repeated: since there are 21 red, 21 green and 22 blue squares, it becomes clear that a blue square needs to be removed. The idea can also be applied in two different directions, eventually only leaving four possible squares that can be removed.



Many more problems can be formulated and solved in the same way, involving tiling with general $k\times 1$-rectangles or other shapes and colourings of different types. The following question, to give one more example, was one of the hardest problems of the International Mathematical Olympiad in 2004 (see e.g. Djukić et al. 2006, p.327):

Problem

Define a ”hook” to be a figure made up of six unit squares as shown below in the picture, or any of the figures obtained by applying rotations and reflections to this figure. Determine all m×n rectangles that can be covered with hooks without gaps and without overlaps, and such that no part of a hook covers area outside the rectangle.



Camels and Egyptian fractions

School and university textbooks contain many problems that attempt – more or less successfully – to relate mathematical concepts and methods to real-world situations. The following problem can certainly not be called “real-world”, but whimsical stories like the following classic can actually help to arouse interest in mathematics.

Problem: (see Stewart 1992)

A long time ago in Egypt, an old man died and left a herd of 41 camels to his three sons. According to his will, the oldest son should get one half of the camels, the second son one third, and the youngest one seventh of the herd. While the sons were pondering how to follow these instructions, a wise man came along on his camel and solved the problem in the following way: he added his own camel to the herd, so that their number increased to 42 camels. Now the first son could get one half (21 camels), the second son one third (14 camels), and the third son one seventh (6 camels). The wise man's camel was still left, so he could take it back and leave, the problem was solved.

How is this possible? And by what other numbers could 2, 3, 7, 41 be replaced so that the story works out in the same way? What if the old man had four sons?

After being puzzled for a while, learners soon notice that the fractions in the father's will do not actually add up to 1, which is what makes the whole story possible:

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{7}=\frac{41}{42}.$$

Generally, the fractions need to add up to 1 – 1/n, where n is the number of camels (including the wise man's camel), so that one camel will still be left after the distribution. So if the fractions 1/2, 1/3, 1/7 are replaced by 1/x, 1/y, 1/z, then we have to have

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1-\frac{1}{n}$$

or

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{n}=1.$$

In other words, we are looking for four “unit fractions” (where the numerator equals 1) that add up to 1.

Once this has been established, the class is asked to find solutions, and soon enough many different solutions will be found, such as

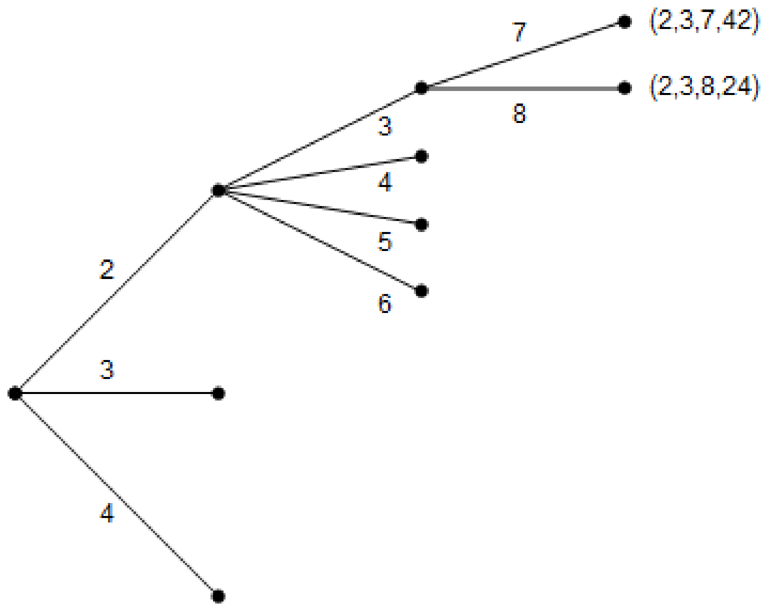
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}=1, \qquad \frac{1}{2}+\frac{1}{5}+\frac{1}{5}+\frac{1}{10}=1, \qquad \frac{1}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{6}=1.$$

The next step is the most difficult; as it requires realising that seemingly trivial observations are actually important. An innocuous question to the class, such as “are there any solutions where all denominators are greater than 100 can help to move things forward. Usually, this question is swiftly answered by someone pointing out that this would make the fractions too small to sum to 1.

As it turns out, this is the key observation: in fact, the largest of the four fractions can only be 1/2, 1/3 or 1/4, since the sum would otherwise be at most 4/5. This leaves us with three possibilities for the smallest number: 2, 3 or 4. In each case, we can repeat the argument.

Assume, for instance, that the smallest of the four numbers is 2. Then the sum of the remaining three fractions has to be 1/2, which implies that the largest of them is 1/3, 1/4, 1/5 or 1/6.

The procedure can now be continued in this way, providing an excellent example of what is called a “branch-and-bound” algorithm. This process is illustrated in the following figure, which displays part of the procedure.



In total, we have the following fourteen solutions (and their permutations) for the four denominators:

(2, 3, 7, 42), (2, 3, 8, 24), (2, 3, 9, 18), (2, 3, 10, 15), (2, 3, 12, 12), (2, 4, 5, 20),

(2, 4, 6, 12), (2, 4, 8, 8), (2, 5, 5, 10), (2, 6, 6, 6), (3, 3, 4, 12), (3, 3, 6, 6), (3, 4, 4, 6),

(4, 4, 4, 4).

Not all of them are actually solutions to the camel problem: for instance (2, 3, 10, 15) is not, since the first son can clearly not get one half of 15 camels. So we have to find those solutions for which n is divisible by x, y, z. With the only further exception of (3, 4, 4, 6), all other quadruples are actual solutions.

It is somewhat tedious to find all solutions by hand for more than four fractions, but a computer can find them easily. For example, there are 97 solutions for four sons, 1568 solutions for five sons, etc.

It is worth mentioning that there are problems of a very similar nature that are still unsolved. A famous instance is the Erdős-Straus conjecture, stating that for every integer $n \geq 2$, there exist positive integers x, y, and z such that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{n},$$

see e.g. Guy 2004, D11.

CONCLUSION

The aim of this paper was to present selected problems that have proven to be valuable in teaching mathematical problem solving skills in a variety of contexts, and to give some indication how the process of solving these problems typically evolves in a classroom context. The author strongly believes that the work on problems such as those presented has many benefits: they simulate the scientific research process (albeit of course on a lower level and on a smaller scale), they teach mathematical reasoning, and they help to arouse and nurture mathematical interest.

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USING THE *X-KIT ACHIEVE! MOBILE* APP TO ENHANCE THE TEACHING AND LEARNING OF MATHEMATICS IN GRADE 8

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Pearson: South Africa Wits University

The study sets out to investigate how Grade 8 mathematics teachers and learners in a public school in Johannesburg use and experience the X-kit Achieve! Mobile – a mobile application designed with the aim of improving the teaching and learning of mathematics. The study also explored how it (X-kit Achieve! Mobile) impacts (or not) on learner performance in mathematics. Both quantitative and qualitative means were used to understand the role of the X-kit Achieve! Mobile in five Grade 8 classes in this school. In terms of the qualitative results, a vast majority of the learners indicated that the application is easy to use, fun, motivating, and useful/educational. Some of the features that learners and teachers found useful include the availability of immediate feedback, the quiz, and an easy-to-use interface. Also, teachers indicated that in making the X-kit Achieve! Mobile an integral part of teaching and learning in their class, the learners became better participants in classroom discussions. In terms of the quantitative results, overall, the performance in the pre-test and the post-test items was very poor, with less than 1% of the learners achieving a mark of 50% and above. Nevertheless, statistical analysis of the performance of all the learners in the pre-test and post-test shows a statistically significance difference between the performance of the learners in the pre-test and that of the post test. What is worth noting is that the high achievers did not generally do better in the post-test. On the contrary, the low achievers in the pre-test generally performed better in the post-test. Given this result, it can be concluded that the intervention with the X-Kit Achieve! Mobile was more beneficial to the low achieving learners compared to the high achieving learners.

Keywords: Grade 8, learning, mathematics, mobile learning, teaching, X-Kit achieve

BACKGROUND AND RATIONALE

Due to the rapid growth in access to mobile phones around the world (UNESCO, 2012) and in South Africa in particular (Shiner, 2009; De Lanerolle, 2012; Noor, 2012), much attention has been given to the potential of mobile learning to improve teaching and learning. South Africa recorded 12, 6 million internet users in 2012 (UNESCO, 2012). Across the globe, 89 per cent of Grade 8-12 students have mobile device access through their smartphones and feature phones (UNESCO, 2012). According to UNESCO, the number of users who access the internet via mobile phones is expected to rise to 23 million by 2016 (UNESCO, 2012). The implication of these statistics is that majority of our learners in South Africa may be in a position of accessing mobile learning via their cell phones by 2016.

Mobile learning refers to any technology-enabled learning solution that allows learners to access educational content through any portable device such as a mobile phone, laptop or tablet (UNESCO, 2012; Brandt, 2012; Mckensey & Company, 2012). The use of mobile devices in learning is referred to as mobile learning (m-learning) (Sharples, 2005). According to Sharples, Taylor and Vavoula (2006), a first step in deciding to support mobile learning is to distinguish what is special about mobile learning compared to other types of learning activity. An obvious, yet essential difference is that it starts from the assumption that learners are continually on the move. We learn across space as we take ideas and learning resources gained in one location and apply or develop them in another. We learn across time, by revisiting knowledge that was gained earlier in a different context, and more broadly, through ideas and strategies gained in early years providing a framework for a lifetime of learning. As we learn, we move from topic to topic, managing a range of personal learning projects, rather than following a single curriculum. We also move in and out of engagement with technology, for example as we enter and leave mobile networks coverage (Sharples et al., 2006).

Based on the rationale discussed above and extensive research guided by a strong theoretical base, a product called X-kit Achieve! Mobile was developed by Pearson Holdings South Africa. In this study, we investigated whether the product delivers on what it was designed to deliver to teachers and learners. The study took place in a public school in Johannesburg, in five Grade 8 classes. In the following section, a detailed discussion of the product is given.

WHAT IS X-KIT ACHIEVE! MOBILE?

According to its developers, X-kit Achieve! Mobile is a Curriculum and Assessment Policy Statement (CAPS) aligned revision and practice tool which recognises the need for revision and practice anywhere at any time. The product covers all cognitive levels and aids learners in mastering content and application thereof. X-kit Achieve! Mobile focuses on helping learners' master difficult concepts through various exercises in order to prepare for examinations. X-kit Achieve! Mobile can be accessed by any high school learners using high end smart phones and low end feature phones, with access to the internet. The product can also be accessed on a laptop, tablet, iPad, and desktop, with access to the internet because it is a web based product.

Developers of the product also argue that the product provides beyond the classroom learning, quick practice for high school learners anywhere at any time, in mathematics. It aims at supporting the acquisition of content and subject specific skills through hints, theory, fully worked solution and tutoring, to build learners' confidence in their ability to apply and acquire subject knowledge. The product has been built with gamification and social messaging options, ensuring a cycle of user re-engagement and to motivate users and build confidence. As learners use the product, they will be able to collect badges for level completion.

The product has four quizzes: the pre-test quiz, know your basics quiz, apply your skills quiz and the master your topic quiz, each displaying 10 questions at a time, with a pool of 30 questions for randomisation if the learner wants to repeat the quiz. The learner cannot move from one level to another before achieving 80%. The questions are all multiple choice. Sharples et al. (2009) argue that although multiple choice questions have limitations to the opportunities offered to learners, if the questions are strategically developed, they may be very productive.

PURPOSE OF THE STUDY

The purpose of this study was to investigate how Grade 8 learners at a public school in Johannesburg use the mathematics X-kit Achieve! Mobile product to enhance their understanding of mathematics concepts. The study was structured to respond to the following key questions:

1. How does the use of the X-kit Achieve! Mobile impact (or not) on learner performance in mathematics?
2. What is the level of learner readiness to use mobile technology?
3. When do learners use X-kit Achieve! Mobile during the process of learning mathematics?
4. What is the general experience of learners who use the X-Kit Achieve! Mobile.
   a) What features of X-kit Achieve! Mobile do learners find useful in the process of learning?
   b) What do learners struggle with in the process of using X-kit Achieve! Mobile in the absence of a teacher?
   c) What do learners like the most and what do they dislike the most?
5. What assistance and support do learners and teachers require in order to use the product effectively?
6. How do teachers experience the X-kit Achieve! Mobile in their teaching of mathematics?

Technology in education

To tackle educational challenges, many attempts have been made to explore how the systemic integration of technology can help alleviate the effects of the crisis. A wide range of educational technology interventions initiated at institutional, provincial, national, regional and global levels focus on the enabling role of technology in improving the quality of teaching and learning, expanding access to learning opportunities, promoting social equity in education, and building inclusive ‘knowledge societies’(UNESCO, 2012). On a global level, UNESCO (2012) conducted studies in various countries and their findings were as follows: Initiatives like World Links, launched by the World Bank, and the Global e-Schools and Communities Initiative (GeSCI) established by the United Nations technology Task Force in 2003, provided support to local technology in education initiatives. At the regional level, World Links, SchoolNet Africa, and the New Partnership for Africa's Development (NEPAD) e-Schools Initiative were instrumental in developing networks of electronically-supported learning (e-learning) practitioners and policy-makers. Perhaps one of the most significant projects that emerged from these efforts was the NEPAD e-Schools Demonstration Project, which was a formidable public-private partnership involving the pan-African e-Africa Commission and five consortia, each led by a major multinational company (GSM Association & Kearney, 2011). The project rolled out 'end-to-end' technology solutions, which included personal computer laboratories (PC labs) equipped with curriculum content, teacher training modules and technical support, in six schools per country across sixteen countries in Africa.

At the national level, SchoolNet Namibia, Egypt's Smart School Network and the Jordan Education Initiative (JEI) were among the most prominent programmes. At the provincial level, notable initiatives included the Gauteng Online and Khanya projects in South Africa (Farrell and Isaacs, 2007; Farrell et al., 2007). Collectively, all of these initiatives involved significant financial, technological and human-capital investments, and worked to establish a global community of practice whose purpose was to catalyse a paradigm shift toward 'twenty-first century learning' and support the technology in education goals at various levels throughout the region's education systems (Hungi et al., 2011).

Africa is a continent where mobiles cell phones are affordable by majority of our learners. Therefore, mobile learning can be used to influence and enable learning more and more (Vosloo, 2012). Mobile learning can also be seen as a challenge to formal schooling, to the autonomy of the classroom and to the curriculum as the means to impart the knowledge and skills needed for adulthood. But it can also be an opportunity to bridge the gulf between formal and experiential learning, opening new possibilities for personal fulfillment and lifelong learning (Sharples et al., 2006).

THE STUDY

Research design and methodology

This study sought to understand the role of X-kit Achieve! Mobile in enhancing the teaching and learning of mathematics in Grade 8 classrooms. As indicated in the previous section, this study is important because the readiness of learners and teachers in using mobile learning in teaching and learning of mathematics is very crucial for any mobile learning product to help the learners and teachers successfully. It was therefore important for this study to seek to understand the role of X-kit Achieve! Mobile in enhancing learners' understanding of mathematics concepts as thoroughly as possible by using in-depth qualitative and quantitative approaches. The key question that the quantitative study seeks to address is: How does the use of the X-kit Achieve! Mobile impact (or not) on learner performance in mathematics? In providing answers to this question, quantitative means are used to understand the results obtained from the pre-test and the post-test results from five Grade 8 mathematics classrooms.

Sampling

The participants for the study were Grade 8 Mathematics learners (five classes of about 45 learners each) and 3 Grade 8 Mathematics teachers. In keeping with the sampling strategies of a qualitative study, purposive and opportunity sampling was used in this study (Cohen, Manion & Morrison, 2000). One school in Johannesburg with all facilities and resources available to the teacher and the learners was selected. The school was selected based on how many learners have cell phones which they can use to access the internet. The school selected was one where learners were using X-kit Achieve! Mobile already, so that there is no interruption in the school program.

Data collection

The process of data collection started off with a pre-test which was administered to learners at the end of the data collection process, a post-test was given to learners. It must be noted that the researchers were neither involved in determining the pre-test and post-test mathematics items, nor in the administration of these tests. The results of both tests were sent to the researchers for analysis by an independent body which specialises in testing learners.

A questionnaire was also given to learners to find out their readiness to use mobile learning in the process of learning mathematics, and in fact, their general experience of using the X-kit Achieve! Mobile. Two learner focus groups interviews were also conducted for 30 minutes. Eight learners were selected from all participants. Five learners were selected from those learners who use the product most frequently and the other 5 learners from learners who have never used the product. The first focus group of 5 learners focused on finding out what learners think about using the X-kit Achieve! Mobile in the process of learning mathematics concepts. The second focus group of 5 learners focused on finding out why some learners are not using X-kit Achieve! Mobile. Participants in focus groups were selected purposefully from the five Grade 8 classes. The focus groups and interviews were conducted after school in order to avoid interruptions in the school program.

Teacher interviews took 30 minutes each. The interviews were semi structured and, like the learner interviews, were audio recorded. The teacher interviews aimed at finding out the teachers' teaching experience and their beliefs on mobile learning as mathematics teachers. The interviews also was used to probe teachers on issues such as the way they use X-kit Achieve! Mobile as a teaching resource and the opportunities they give learners to access X-kit Achieve! Mobile during the lessons or after lessons. The researcher was involving in both the learner interviews and the teacher interviews.

Data analysis

The interviews and questionnaires were analysed qualitatively, while the quantitative data analysis was carried out by the researchers using the STATA software.

Permission was sought and obtained from the Gauteng Department of Education and all the relevant stakeholders in the school before the commencement of the research study.

FINDINGS

Results of the quantitative analysis data

In doing the quantitative analysis of all the classes, it was necessary to first look at the gains (or not) from the pre-test to the post-test for all the classes. It was also vital to specifically look at the performance of the top 10 learners and the bottom 10 learners from pre-test to post-test. It must be noted that generally, the results from the mathematics tests (both pre-test and post-tests) were poor with no learner achieving 50% or more in the pre-test, and only 4 learners in total achieving 50% or more in the post-test.

The table below (Table 3.1) provides a global picture of the performance of the all the Grade 8 classes in the pre-test and the post test. The statistical software STATA was used in the analysis.

Table 1: Paired t-test for all Classes.

Variable	Number of participants	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
Pre-test	221	17,10	0,68	10,07	15,76	18,43
Post-test	221	19,68	0,73	10,92	18,24	21,13
diff	221	-2,59	0,69	10,29	-3,95	-1,22
Ha: mean(diff) < 0	*Ha:mean(diff) != 0*	*Ha: mean(diff) > 0*				
Pr(T < t) = 0.0001	*Pr(T > t) = 0.0002*	*Pr(T > t) = 0.9999*				

The paired t-test deals with only the mean scores of students who participated in **both** pre-test and post-test exercises. From table above, it is observed that the mean score (in percentage) for the pre-test was 17.1% and the mean in the post-test was 19.68%. Overall, therefore, the mean gain in knowledge after the intervention using the X-Kit Achieve! Mobile is about 2.59. This difference in score is subjected to three different statistical tests: The first, which is the main interest to the intervention exercise, [Ha:mean(diff)<0] tests the hypothesis that the mean score of the post test is greater than that of the pre-test score. This is found to be statistically significant at 0.05 level. A result of 0,0001 as showed on the table above, indicates that there was a statistically significant difference between the pre-test and the post-test. The second [Ha: mean(diff!=0] tests the hypothesis that there is no difference between the pre- and post-tests. The third test is on the hypothesis that generally the pre-test performance is better than that of the post-test. As shown in the table this is not the case.

Summary of statistical analysis

It must be noted that due to space limitations, not all the statistical analysis is presented in this paper (see Essien, 2015 for full details). In the table below, we provide a summary of the t-test results for the top 10 learners and the bottom 10 learners in each of the Grade 8 class:

Table 2: Summary of statistical analysis.

	t-test results		
Grade	**Class (overall)**	**Top 10 in class**	**Bottom 10 in class**
Grade 8A	Significant	Not Significant	Significant
Grade 8B	Not Significant	Not Significant	Significant
Grade 8C	Significant	Significant	Significant
Grade 8D	Not Significant	Not Significant	Not Significant
Grade 8E	Not Significant	Not Significant	Significant

The above table indicates that in general, the bottom learners reaped greater gains from the intervention using the X-Kit Achieve! Mobile compared to the top performing learners. It must be added as a reminder that the overall analysis of the performance of all the classes put together indicated a statistically significant difference in the performance from pre-test to post-test.

Analysis of Learners' use and experience of the X-kit Achieve! Mobile

In order to engage with learners' use and experience of the X-kit Achieve! Mobile, the questionnaire items and learner focus group interviews were used. 215 learners provided answers to the questionnaire items while 8 learners were involved in two focus group interviews. In analyzing the questionnaires and interview responses, the questions in focus were:

- What is the level of learner readiness to use mobile technology?
- When do learners use X-kit Achieve! Mobile during the process of learning mathematics?
- What is the general experience of learners who use the X-Kit Achieve! Mobile.
    - What features of X-kit Achieve! Mobile do learners find useful in the process of learning?
    - What do learners struggle with in the process of using X-kit Achieve! Mobile in the absence of a teacher?
    - What do learners like the most and what do they dislike the most?
- What assistance and support do learners and teachers require in order to use the product effectively?

In ascertaining learner readiness to use mobile technology, the questionnaire questions which provided indicative information were questions regarding whether or not learners had personal mobile phone and if yes, whether the phone is a smart phone or a feature phone, and also what learners who have cell phones use their phones for.

Of the 215 learners who filled in the questionnaires, 201 learners, representing approximately 93, 5% of the learners indicated that they have personal mobile phones. Of the 201 learners who indicated that they have mobile phones, only 4 indicated that they have feature phones. This means that 98% of the learners who have phones use smart phones, which represents approximately 92% of the total learners in the whole of the five Grade 8 classes under investigation.

Of the learners who have mobile phones, the following data emerged.

Table 3: Learner use of mobile phones.

	Phone use: Whatsup	Phone use: Pers Internet	Phone use: Educ Internet	Phone use: X-Kit
Grade 8A	29	19	28	32
Grade 8B	32	20	26	34
Grade 8C	28	26	30	32
Grade 8D	25	19	24	37
Grade 8E	28	18	29	35
TOTAL	**142**	**102**	**137**	**170**

From the data above, it is evident that most learners use their phones not simply to make and receive calls, but also for social network, the internet and educational purposes. It is important to note that in each of the classes, the number of times the learners indicated that they use the phone to access the X-Kit Achieve Mobile application surpassed the times used for other purposes. With a total of 79% of learners accessing the X-Kit Achieve! Mobile, 63, 7% indicating they use their mobile phones for the internet, it can be argued that these learners are ready for the use of mobile technology in teaching and learning.

Analysis of the questionnaire with regards to what time(s) learners who use the X-kit Achieve! Mobile revealed that the mobile application was used more frequently in Grace 8C.

Table 4: Times when learners use the X-kit Achieve! Mobile.

	Grade 8A	Grade 8B	Grade 8C	Grade 8D	Grade 8E
In Bed	1	1	0	1	0
On way to school	2	1	1	2	0
At Break	5	11	24	4	1
During Lesson	9	15	37	3	25
At Home	20	43	37	20	39
For Homework	0	11	13	5	26
Never	2	0	0	3	0

On the other hand, the data above revealed that learners in Grade 8D used the X-kit Achieve! Mobile less frequently in class (during lessons). It is not clear as to whether this provides the explanation regarding the fact that there was no statistically significant difference from pre-test to post-test for all the variables in Grade 8D.

What is also noteworthy regarding the data above is that the X-kit Achieve! Mobile is used most frequently by learners at home. Using an educational mobile application like the X-kit Achieve! Mobile is a clear indication of and a support to the claim that learners are ready for the use of mobile technology in teaching and learning.

Learners general experience of using the x-kit Achieve! Mobile

Table 5 shows how the learners responded to the questions regarding their experience of using the X-kit Achieve! Mobile. As can be noted from the table, a vast majority of the learners indicated that the application is easy to use (111 learners), fun (135), motivating (87), and useful (115). Only few learners indicated that the software application is annoying, ordinary and time consuming. This is an indication that the learners generally find the application worthwhile to use.

Table 5: Learners general experience of using the x-kit Achieve! Mobile.

	Annoying	Confusing	Easy to use	Familiar	Fun	Difficult	High Quality	Motivating	Ordinary	Powerful	Straightforward	Time-consuming	Time-saving	Too Technical	Useful
Grade 8A	2	9	22	5	23	11	9	18	1	2	3	2	4	4	27
Grade 8B	1	6	23	4	27	15	4	18	2	4	3	0	12	2	26
Grade 8C	2	6	24	5	23	7	10	21	3	13	10	3	10	2	19
Grade 8D	3	5	21	1	29	10	6	15	1	2	4	0	4	9	23
Grade 8E	1	3	21	4	33	12	14	15	0	7	5	0	7	1	20
TOTAL	**9**	**29**	**111**	**19**	**135**	**55**	**43**	**87**	**7**	**28**	**25**	**5**	**37**	**18**	**115**

It is unclear whether the 55 learners who indicated that the X-kit Achieve! Mobile is difficult were alluding to the application itself or to the mathematical content which the application provides. The same applies to the 29 learners who indicated that the X-kit Achieve! Mobile is confusing. It is also unclear if they were referring to the navigation process involved in using the application, or to the mathematics content within the application. In terms of what learners dislike the most about the application, air-time 'wastage' was cited by most of the learners both in their responses to the questionnaire items and the focus group interviews that airtime. In fact, in the focus group interview of learners who do not use the App frequently, data consumption was cited unequivocally as the principal reason why these learners do not use the X-kit Achieve! Mobile.

Nevertheless, 183 out of 215 learners who provided answers to the questionnaire items (85, 1%) indicated that what they liked the most are that they find the application useful/helpful/challenging/stimulating and educational. In the focus group interview, Learner A captured this sentiment as follows:

Learner A	Any other final thing you want to say about your experiences of using the product?
Researcher	I think it [referring to X-kit Achieve! Mobile] can really help us to move on to the next grade because it gives us clear explanations, because sometimes in the class when learners are making noise and you can't hear what the teacher is saying, or can't remember what the teacher has said, you can go into the X-kit and read the theory and then remember what you need to know for the exams.

Teachers' experience of using the X-kit Achieve! Mobile in the teaching of mathematics

Of the 3 teacher educators interviewed, two (Teachers A and C) indicated that they used the product very frequently in their classes and have in fact, made the App an integral part of teaching and learning in their classrooms. The experiences recounted below are thus the experiences of these two educators.

Teacher A commented that one of the ways to improve the product would be to provide a platform in which the learners are able to ask teachers questions should the need arise in the course of learners attempting to work with the X-kit Achieve! Mobile. He also indicated that teachers should be given more control in determining what questions to give to learners (the school usually sent these questions to learners directly). Also, Teacher A noted that it would be worthwhile if teachers had a way of checking which learners are online at a particular time (as with the WhatsApp Application). The final suggestion for improvement by Teacher A was that it would be more worthwhile if teachers can view learners from 1st position to the last position instead of the current best/top 10 in the country.

Teacher C indicated that the product has been very useful in her class as more learners are now engaging better in classroom discussions and with the mathematics at hand. She indicated that the ability of the product to provide immediate feedback is very useful, and gives her confidence when she teaches. She indicated that the challenge for learners remain with learner access to the internet.

CONCLUSION FROM THE STUDY

Overall, the statistics from the study indicate that X-kit Achieve! Mobile has the potential to improve learner achievement in mathematics, especially among low achieving learners. The results from the qualitative data analysis also indicates that the learners are not only ready for mobile technology in the teaching and learning of mathematics, but also find the X-kit Achieve! Mobile very useful, user-friendly, educational and generally worthwhile.

Learners also indicated that when learning a new topic, the X-kit Achieve! Mobile did help them improve their understanding of mathematics because of the theory aspect in the different content areas. They also indicated that the competition aspect (on the dashboard) motivates them to work harder.

An overwhelming majority of learners indicated they find the quizzes, the different levels of the quizzes, and the exam practice quiz very important. Given the strong affirmative responses regarding the achievement of 80% before moving to the next level, the option to check ones progress on the App, the leaderboard, it is important that these features, as well as those pertaining to the quizzes be retained. This would be our first recommendation.

Some learners both in the questionnaire and in the focus group interviews complained about technical problems such as login error/problems and some others identified the level at which questions are pitched as some of the things they disliked the most about the App. Still, a few others cited 'data wastage' as what they disliked the most. Even though these are in the minority, their complaint should not be taken lightly. Hence, the second recommendation would be that problems associated with login be investigated and ways of ensuring a seamless login and logout of the Application are devised and implemented. Regarding data usage, we recommend a reward system in which top achievers could be given data rewards to encourage them.

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GRADE 12 LEARNERS' ACHIEVEMENT IN PROBABILITY

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Probability was recently introduced as a compulsory topic in Mathematics curriculum in the National Senior Certificate (NSC), South Africa. Many teachers experience constrain when they teach the topic, making it difficult for learners to grasp the concepts and effectively solve probability problems. The purpose of this study is to investigate the achievement levels of grade 12 learners in probability in Mhlophenkulu ward (in Nongoma educational circuit, KwaZulu-Natal). Four hundred and ninety grade 12 learners from seven different schools, categorised under four different quintiles (socioeconomic factors) were selected for the study using the stratified random sampling technique. A cognitive test was used to collect data. The study followed the survey research strategy and used Bloom's taxonomy as a framework of analysis. Results show that learners' achievement on the lower cognitive levels was better than on the higher levels. The repeated measures ANOVA was conducted to determine whether there were significant differences between the means of learners' scores at the various Blooms level. The result showed that Learners in this research had a better achievement in application cognitive demand than in comprehension cognitive demand questions although the difference between these two means is not statistically significant at α = 0.05. However the mean difference between all other consecutive cognitive demands of Bloom is significantly different. The achievement of learners decreased from the lower cognitive level to the higher cognitive level. The researchers recommend that teachers employ the problem solving teaching approach in teaching probability to help learners acquire deeper understanding of the ideas and process of the concept.

Keywords: Achievement in probability, Bloom's taxonomy, cognitive levels, grade 12 learners, mathematics and probability

INTRODUCTION

Many events cannot be predicted with total certainty. This makes an understanding of the concept of chance in decision making imperative. Probability can be thought of as a numeric measure of the chance or likelihood that a particular event will occur (Anderson, Sweeney & Williams, 1999). It is expressed as a decimal fraction between 1 and 0 or as percentage. An event with a probability of one or 100% is considered to be certain while an event with a probability of 0% is considered as impossible.

Probability finds application in almost every aspect of our lives, for example, in the medical field a medical practitioner may want to know the success rate of a surgical operation on a patient. This means knowing the chances of the patient surviving after the operation. In sports for example football, a coin may be tossed before play begins to determine which team starts the match. Before the toss both teams have 50; 50 chance of starting. Based on the number of years a group of people have lived, their ages can be used as guidance by entities like financial advisors to help their clients prepare for retirement. Predicting the weather is also another area of uncertainty. When planning an outdoor activity, people generally check the chances of rain. Likewise the meteorologist also makes predictions on based on the pattern of previous year's temperature and natural disasters are also predicted on chance.

Research studies in medicine and social science are often understood through statistical methods that have grown out of the probability theory (Huff, 1959).Knowledge of probability also helps one to understand issues in politics, insurance, gambling and so on. This is because probability terminologies and concepts such as unlikely, possible, fair, likely, impossible and many other are often used in these fields. While the measurement of chance in everyday life may not be realised consciously, but subconsciously it is present in almost every decision taking. Hence the knowledge of probability helps one to increase one's chances of making the right decisions (Newman, Obremski & Schaeffre, 1987). Probability connects many areas of mathematics particularly those based on counting and geometry (National Council of Teachers of Mathematics, 1989). Pugalee (1999) affirms this fact when proposing that learners' experience in probability can contribute to their conceptual knowledge of working with data.

Despite the usefulness of the study of probability some studies have revealed that many teachers find it a challenge to teach it (Watson, 1995). Other researchers have attributed leaners' challenges in the study of probability to the teacher (Fennema and Franke 1992; Papaieronymou, 2009). For example, Papaieronymou (2009) argued that a teacher's knowledge of the topic might also be a problem. His reason was that not all teachers had studied probability during their own school education. Fennema and Franke (1992) also noted that teacher content knowledge influences classroom instruction and the richness of learners' mathematical experiences. These researchers opined that learners develop a dislike of the topic of probability because their teachers present it to them in a highly abstract and formal way.

The language of probability has also been identified as a challenge to learners in comprehending the concept itself. According to Meaney, Trinick and Fairhall (2012), language can be either a support or a barrier to students learning of probability. Bennie (1998) noted in his work that the problem encountered in the teaching and learning of probability emanates from the fact that teachers and learners find it arduous to come to grips with the differences between the everyday concept of probability and the mathematical use of the language of probability. Learners bring their everyday understanding of certain words to the probability class only to realise that these words now have different meanings. When they find the language difficult, they reach a point where develop negative attitudes towards the topic (Paul and Hlanganipai, 2014).

Kaplan (2008) attribute students' failure to understand the concept of probability to their inability to handle rational numbers and the proportional reasoning used in calculation, reporting and interpreting the concept. Tso (2012) also attributes students' challenge with probability to their weakness in handling fractions, decimals and percentages. Moreover, probability concepts conflict with students' experiences. Fennema, Carpenter and Lamon (1981) noted that the intuitions and experiences that learners (and teachers) bring to the study of the topic at school could conflict with formal probability and intuitive knowledge of probability and that its expression could be misleading.

PROBABILITY IN THE SOUTH AFRICAN SECONDARY SCHOOL CURRICULUM

Probability is a relatively new topic in the South African school curriculum. In the National Curriculum Statement (NCS), the curriculum that was phased out in 2010, probability was examined in the grade 12 mathematics paper 3 (optional paper) but in the current curriculum, namely curriculum and assessment policy statement (CAPS) all mathematics learners are examined in the compulsory national mathematics paper 2. In the CAPS document for grades 10-12, probability and statistics form part of the 10 main topics (Department of Basic Education, 2011).

The CAPS document, which is simply an amendment of the NCS, stipulates that the grade 12 learners should demonstrate knowledge of the following aspects of probability: comparison of theoretical probability and relative frequency; use of Venn diagrams to assist in solving problem; identification of mutually exclusive and complementary events; identification of any two events A and B , P (A or B) = P (A) + P (B) – P (A and B); identification of dependent and independent events; contingency tables and tree diagrams to solve probability problems and fundamental counting principles. These areas of the topic should then be incorporated into probability problems.

Mathematics teachers are the implementers of the mathematics curriculum, however, research literature has revealed that a significant number of mathematics teachers in South Africa, encountered data handling and probability for the first time in 2006 when the topic became part of the school mathematics in Further Education and Training (FET) band (South African Broadcasting Corporation [SABC], 2013). Earlier probability was treated as only a component of statistics in tertiary institutions in South Africa (Makwakwa, 2012). This is an indication that most South African teachers who are teaching mathematics in the secondary schools have never studied the topic when they themselves were at secondary school or college of education.

Initiatives that aim to provide teachers with the necessary content knowledge and skills to teach statistics and probability have been organised by both non-governmental and governmental organisations (Makwakwa 2012, p.4). They include the South African Statistical Association (SASA), Statistics South Africa (Stats SA) and the Association for Mathematics Education of South Africa (AMESA). The Institute for Science and Technology Education (ISTE) of the University of South Africa (Unisa) has also offered workshops during winter vacations for mathematics teachers on how to teach difficult mathematics topics. These initiatives are intended to improve and upgrade teachers' knowledge of data handling and probability content, among other aspects. This is because teachers consider these topics to be among the most instructionally and conceptually problematic in mathematics (Atagana, Mogari, Kriek, Ochonogor, Ogbonnaya, Dhlamini & Makwakwa, 2010).

When a teacher lacks content knowledge in any subject or curriculum, the effect will be seen in the learners' performance (Ogbonnaya, 2014). Batanero, Biehler, Carmen, Engel and Vogel (2005) noted in their work that, using simulation to bridge teaching content and pedagogical knowledge on probability is increasingly an essential part of the school curriculum. However, most teachers have little experience of it and share with their learners a variety of probabilistic misconceptions. These authors used results from their simulation experience from studies on primary teachers in Spain and secondary and primary school teachers in Germany.

Learner average performance in probability and counting principles for 2015 examinations was 28% according to the National Diagnostic Report (Department of Basic Education, 2015). These topics were among those in which learners' achievement was relatively poor. The report indicates that although some aspects were answered adequately, other aspects were below standard. The report attributes learners' poor performance in answering probability questions to the fact that the topic is still new to a number of teachers. This affirms the point that most South African teachers find the task of teaching the concept of Probability in the mathematics classroom to be arduous and exceptionally demanding.

Knowing learners' achievement in terms of cognitive demand may be a way of identifying their weakness in order to address it .It is against this background that the current study was under taken with the objective of identifying the achievement levels of learners according to Bloom's taxonomy and establishing whether there is any statistically significant difference in probability in any of the cognitive levels of Bloom's taxonomy.

The research questions addressed are as follows:

1. What are the achievement levels of learners in probability according to Blooms taxonomy?

2. Does student achievement in probability increase or decrease from knowledge through to evaluation?
3. Are there any significant differences between the students' achievement means in the various cognitive level of Bloom's taxonomy?

CONCEPTUAL FRAMEWORK

The study is underpinned by the theoretical frame work of Bloom's taxonomy (1956).The taxonomy provides classification of educational objectives to assist teachers, administrators and researchers to discuss curricular and evaluation problems with greater precision. Bloom's taxonomy was initially described as a hierarchical model for the cognitive domain in 1956 (Bloom et al, 1956). The taxonomy is not a measure of difficulty but an indication of the type of cognitive process required to answer questions correctly. It consists of six cognitive skill levels; knowledge, comprehension, application, analysis synthesis and evaluation. The various cognitive levels increase in complexity beginning from the lowest level, knowledge to the most complex, evaluation.

Knowledge cognitive demand deals with the remembering of previously learned material; learners are expected to recall learned information. For example, what is the condition for two events, A and B, to be mutually exclusive? Among the keywords used in this cognitive demand are define, describe, identify and list.

Comprehension cognitive demand explains the ability to grasp the meaning of previously learned material. This may be demonstrated by translating material from one form to another, interpreting material (explaining or summarizing) or predicting consequences or effects. For example, give examples of mutually exclusive events. Examples of key words used under comprehension are as follows: comprehend, convert, distinguish, predict summarize, give examples and so on.

Application cognitive demand requires the ability to use learned material in new and concrete situations. This may include the application of rules, methods, concepts, principles, laws and theories and equations. For example: the probability that it will rain is 0.3 and the probability that it will not rain is 0.7. The question might be: Show that these two statements are mutually exclusive. Keywords that may be applicable here are applies, changes, complete, construct, demonstrate, discover, solve and show.

Analysis cognitive demand breaks down material into component parts so that its organisational structure may be understood. This exercise may include the identification of parts analysis of the relationships between the parts and the recognition of the organisational principles involved. For example: in a class of 30 learners 15 prefer to use a blue pen and 17 prefer a red pen. Each student prefers at least one pen. Draw a Venn diagram to illustrate the information. This question requires learner to be able to put figures at the correct regions on the Venn diagram before they will be able to solve it. The keyword used in this example could include analysis, break down, compare, contrast, diagram, and outline and distinguish.

Synthesis cognitive demand puts parts together to form a new whole; this may involve the production of a unique communication (thesis or speech), a plan of operations (research proposal). For example: Give an account of why the probability that it will rain today and the probability that it will not rain today are said to be mutually exclusive. Keywords used in framing questions include categorize, Combine, Compile, Compose, Create, Modify, Write, and tell. Evaluation cognitive demand judges the value of material for a given purpose that is judgments are to be based on definite internal and/or external criteria. For example: The probability that an event will occur is 1. Explain this probability. Keywords used in framing questions include compare, conclude, defend, explain and support. Furst (1994) as cited in Radnehr & Almolhodaei (2010) identified the assumption that the cognitive levels are ordered on a single dimension from simple to complex as a weakness in the original taxonomy. Krathwohl (2002) wrote that that a mastery of each of the lower cognitive was a prerequisite to mastery of the next higher cognitive.

Researchers like Smith, Wood, Coupland and Stephen, (1996) suggested a modification to the original Bloom's taxonomy. This led to a review of the model in 2001 (Anderson et al, 2001). The names of six major categories in Bloom's taxonomy were changed from nouns to verb forms in the revised taxonomy. The lowest level of the original taxonomy namely, knowledge was renamed remembering; comprehension became understanding; application became applying; analysis became analysing; evaluation became evaluating; and synthesis was renamed creating. Nevertheless despite the changes in the Bloom's taxonomy the authors of this paper have choose the language of the original version because it is well known and universally accepted across disciplines and national borders.

Many researchers have used the taxonomy to study student achievement or performance in various disciplines (Radnehr and Almolhodaei 2010; Karaali 2011; Ogbonnaya, 2014). Radnehr and Almolhodaei (2010) researched on the performance of student mathematical problem solving based on the cognitive process of the revised Bloom's taxonomy. Their result showed that there was a difference between students mathematical performance in each category of knowledge dimension according to the cognitive process of the revised taxonomy. Their findings also revealed that student mathematical achievement decreased from remembering through to creativity in each category of knowledge dimension.

Radnehr and Almolhodaei, (2010) pointed out that student achieved better in remembering mathematical objectives in each of the other five cognitive levels. They also achieved better result in applying questions than in understanding questions. These authors saw that there was no significant difference between achievement in analysis questions and achievement in evaluation questions. According to their study students were less successful in questions that demanded creativity. Karaali (2011) also pointed out that most text books rarely give examples of activities that involve the higher cognitive levels of, synthesis, analysis and evaluation. In the authors work on the topic namely evaluative calculus project when applying Blooms taxonomy to calculus, the author concluded that evaluative task have a place in the mathematics classroom and that teachers should incorporate such teaching and learning in their mathematics activities.

Ogbonnaya,2014 used a special case of the Blooms taxonomy to study the relationship between grade 11studnts' achievement in trigonometric functions and their teachers' content knowledge. His findings showed that there was a strong relationship between teachers' content knowledge and student leaning. In his conclusion he recommended that teachers needed regular content workshop to help them improve in their teaching of topics like trigonometry.

METHODOLOGY

Participants and the research approach

To address the research problem, a quantitative approach was undertaken to ascertain the grade 12 learners' achievement in probability. Survey research design was employed and data was collected using a cognitive test. The participants were a sample of 490 learners from seven schools sampled from a population of 12 schools in the Nongoma circuit of education. The sample used was the stratified random sampling technique (Frankel, Wallen & Hyun, 2012). This was used to ensure that the quintiles rankings were equally represented in the study.

There are 4 quintile groups in the Nongoma education circuit. South Africa uses the quintile classification system to address the uneven distribution of poverty across provinces. This they do by categorising government schools into five levels known as quintiles. Quintile allocations are made according to the poverty levels of the community where the schools are located; in addition certain infrastructural factors are taken into consideration. The most economically disadvantaged schools (the poorest schools) are categorised as quintile 1 and the least poor categorised as quintile 5. The ranking of the school determines the amount of funding it receives from the government. The first 3 quintiles are known as non-fee paying schools while the last two are the fee paying schools.

Instruments and the development of the test

The instrument for the data collection was a cognitive test on probability. The construction of the test was guided by Bloom's taxonomy. This taxonomy categorises questions into different cognitive demand. The questions tested learners on the following cognitive demands, knowledge, comprehension, application analysis, synthesis and evaluation. For example, on knowledge questions, learners were asked to write the addition formula for two events A and B given that these two events were mutually exclusive. On understanding questions learners were giving questions on Venn diagrams and were asked to identify which of them illustrated mutually exclusive. Inclusive and also independence.in each case they provided reasons to their answers. On application questions learners made use of the addition formula of probability of two events to solve calculation questions. On questions involving analysis learners were giving a story problem and were asked to put the respective figures at their correct regions of a Venn diagram. Leaners were giving simple proof questions to show. On evaluation questions learners were giving investigative form of questions and certain formula deduced. In all the various content of the grade 12 probability concept was covered.

Reliability and validity

Content validity was done by three experts in the field of mathematics education. These mathematics education experts moderated the questions, in terms of marks allocation for each question, the language used and also the content. They made inputs regarding the diction of the questions and the marks allocation. They judged the level of alignment of each question against the curriculum by using a 3 point rating scale (1 = not aligned; 2 = fairly aligned; 3 = much aligned). All questions were retained because they all had an average rating of 2, 5.Aftre the suggestions have been considered the requisite changes were made. The test was further piloted within a school at different circuit. After the learners had taken the test, some questions were found to be ambiguous thus they were reframed to bring about better understanding.

The test retest method was used to ascertain the reliability of the instrument. The Cronbach alpha formula was used in testing for the reliability and a reliability coefficient of 0.723 was obtained. This value was found to be an appropriate measure. Hof (2012) has documented that an acceptable value should lie between 0, 70 and 0, 90.

Data analysis

In the data analyses, frequency and measures of tendencies and dispersions were calculated to ascertain the learners' achievement at the various levels of Blooms taxonomy – the conceptual frame work of the study. The repeated measure analysis of variance (ANOVA) was used in testing the significance of the learners' different means achievement at the different cognitive levels.

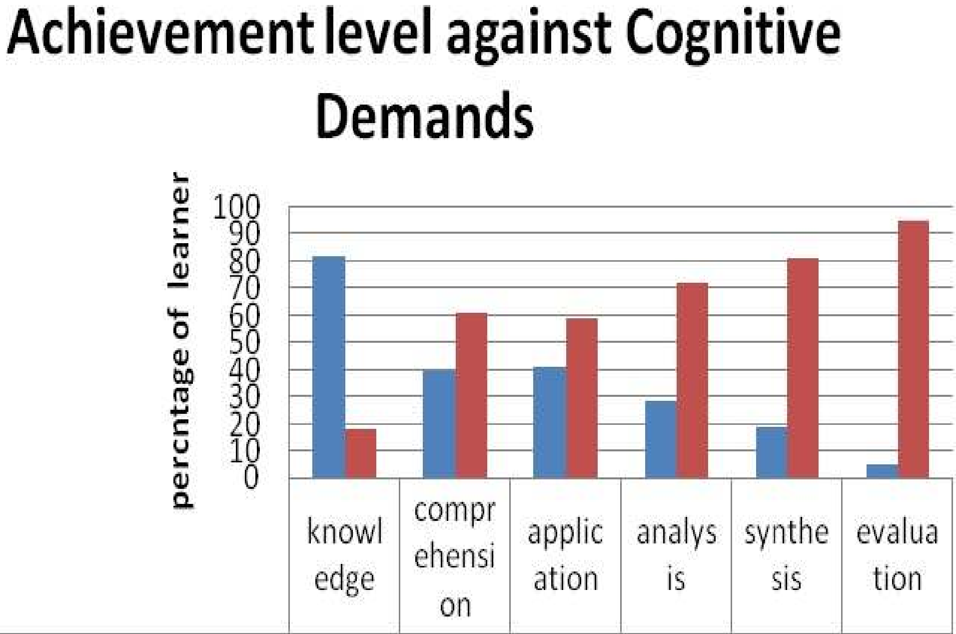
FINDINGS

Differences in cognitive abilities

Repeated measures ANOVA was conducted to determine whether there were significant differences between the students' cognitive abilities in terms of achievement means. Mauchly's Test of Sphericity indicated that the assumption of sphericity had been violated, $\chi^2(14) = 456.68$, $p < .001$, and therefore, a Greenhouse-Geisser correction was used. There was a significant difference found between the students' scores for each cognitive ability component, F (3915, 1914.48) = 601.35, $p < .001$. The pairwise, post-hoc comparisons revealed that, except for the comparison between the achievement in comprehension and application ($p = .656$), the students' displayed differences in their achievement in terms of cognitive abilities ($p < .001$). Specifically, their knowledge abilities were highest and their evaluation abilities lowest, suggesting their higher-order achievement were weakest (see table 1):

Table 1: Descriptive Statistics

Table 1: Descriptive Statistics			
Variable	M	SD	N
Knowledge	69.77	22.69	490
Comprehension	40.70	22.19	490
Application	41.28	26.12	490
Analysis	34.44	18.34	490
Synthesis	19.54	22.71	490
Evaluation	12.21	23.21	490



	knowledge	comprehension	application	analysis	synthesis	evaluation
50% achievemnet and more	82	39.3	40.8	28.3	18.8	5.2
50% achievement and less	18	60.7	59.2	71.7	81.2	94.8

Figure 1: Percentage of Learners versus Cognitive Level

Figure 1 illustrates the percentage of learners who achieved 50% and more and also 50% and less in the test. The number of learners who achieved 50 % and more decreased from knowledge cognitive demand to evaluation cognitive demand (skewed to the left). However the number of learners that achieved in application cognitive demand was slightly higher than in comprehension cognitive demand. The reverse can be said for the number that achieved 50% and less (skewed to the right).

DISCUSSION

The findings of this study show that learners were more successful answering questions in the knowledge category and least successful in answering question in the evaluation category. This might be due to the fact that most teachers tend to ask about 80% of knowledge questions during teaching (Fredericks, 2005) .Learners' achievement mean in the lower cognitive levels (knowledge, comprehension and application) in this study is higher than in the higher cognitive level (analysis, synthesis and evaluation).Possible reasons might be that most exercises and tests in the majority of introductory mathematics courses typically address the lower levels of the taxonomy rather than those that are higher (Karaali, 2011). Achievement on the higher cognitive level decreases from analysis through to evaluation. This is in consonance with Bloom et al (1956). However the achievement on the lower cognitive level, decreases from knowledge to application and then to comprehension. The result is a deviation form Bloom's taxonomy (1956) as well as that of Vidakavic, Bevis and Alexander (2004) but in consonance with Radnehr and Almolhodaei (2010). Reasons accounting for learners' higher achievement in application questions than in comprehension question could be that they might not thoroughly understand the concept but able solve questions using known algorithm.

The five year experience in teaching in this ward has revealed to the researchers that the most learners here find the English the medium of instructions for most subjects extremely difficult. Learners tend not to understand questions involving the higher level cognitive as asserted Meaney et al (2012). Lower achievement on the higher cognitive level is not a good sign for the future of learners in academia as these higher level questions are instrumental strengthening the brain (Fredericks, 2005).Moreover higher order level questions also facilitate the development of critical thinking skills and problem solving (Bloom et al., 1956).

The findings also show that percentage of the learners who achieved 50% and above in the knowledge, comprehension, application, analysis, synthesis and evaluation levels of the Bloom's taxonomy are 82% 39, 3% 40, 8% 28, 3% 18, 8% and 5, 2% respectively. These figures confirm that fact that learners were more successful in the lower cognitive levels, particularly knowledge cognitive demand, than in the higher cognitive demand. Though learner achievement in the synthesis and evaluation was poor, it is promising they obtained a substantial level of achievement in knowledge. Teachers should take advantage of the fact that learners are doing well in recall which could to improve their achievement in the higher cognitive levels. Although the mastery of a lower cognitive level does not guarantee that learners will do better in higher level (Aviles, 1999) teachers might find it expedient to use this success and employ questioning techniques to inspire higher level thinking in the classroom (Paul & Elder, 1997).

CONCLUSION AND RECOMMENDATION

The study has shown that most learners in this study were not successful in questions that had a higher cognitive demand, especially in evaluation and synthesis. From these findings the researchers suggest that teachers adopt a problem solving teaching approach in the teaching of probability. Teaching mathematics by problem solving is an enquiry based method. When teaching by this method, teachers should provide just enough information to establish the background of the problem leaving students to clarify, interpret and attempt to construct one or more solution process. This allows them to brain storm while teachers guide, ask insightful questions and share the process of solving the problem. This approach not only increases the interaction between learner and learner and that between teacher and learner it also teacher learner it also enhances the opportunity for relevant and vigorous mathematical dialogue between the students. Ultimately, the teaching by this approach helps the students to construct their own deep understanding of mathematical ideas and processes because they are actively engaged in creating, conjecturing, exploring, testing and verifying.

We also recommend that teacher workshops should be organised regularly to help teachers improve on both the content and the methodology of teaching the topic. This will go a long way to help the teaching and learning of probability. Textbook authors are also been advised to pay close attention to activities that involve comprehension questions and questions that involve the higher cognitive order, analysis synthesis and evaluation. This will help learners to familiarise themselves closely with the concept of probability.

In conclusion the study did not include independent secondary schools and quintile 5 schools because there are no such schools in the study area. Further study to include this category of secondary schools would be of inestimable value to education.

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MODELLING IN SOUTH AFRICAN PRIMARY SCHOOL MATHEMATICS CLASSROOMS

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This paper presents modelling as a teaching and learning activity for South African primary school classrooms. Modelling is presented in form of a modelling cycle while an example of a typical modelling task is included and discussed. Modelling is then explored in terms of its benefits for preparing learners for success beyond school in the 21st century. Results from two separate studies are presented where the findings are that modelling is a valuable activity for meaningful learning and teacher development. It is the aim of the paper to justify the inclusion of modelling tasks in primary school classrooms in order empower South African learners and to move mathematics education forward.

Keywords: Learning, mathematics, modelling, primary school, teaching

INTRODUCTION

South African mathematics classrooms are in need of support. Results on a local and international level are cause for concern with the media reporting regularly on this. The pass-rate for mathematics in the 2015 NSC (National Senior Certificate) was 49.1% while South Africa's Grade 9s performed below the international average in the 2011 TIMMS study (Reddy, Prinsloo, Arends, Visser, Winnaar, Feza, Rogers, Janse van Rensburg, Juan, Mthethwa, Ngema & Maja, 2012). Although much media emphasis is placed on the NSC results, Spaull and Kotze (2014, p. 2) insist that "the later in life we attempt to repair early learning deficits in mathematics, the costlier the remediation becomes". The results of the Annual National Assessments in the lower grades do not paint a better picture. The grade 4 to 6 mathematics results from 2012 to 2014 range between 27% and 43% (Van der Berg, 2015).

Research into, and development of, primary school mathematics classrooms is necessary. In fact, Van der Berg's (2015, 14) conclusion from a study of the ANAs is that "for most children, learning deficits are already so substantial by the middle of primary school that many doors have already closed for them". Since many mathematics classrooms are still traditional in terms of teaching and learning, this may be a factor in these results. Traditional teaching and learning is characterised by teachers who show students how to perform mathematical procedures and methods followed by exercises where students reproduce these methods. Learning is this type of classroom is largely passive and a burden on a student's memory. Very often gaps in understanding become evident when learners are required to apply their knowledge or to solve problems that are set in a context.

This paper proposes the integration of models and a modelling perspective to mathematics education. This perspective does not hold all the answers to the issues in mathematics education, but offers teachers and learners an alternative form of learning that goes beyond constructivism (Lesh & Doerr, 2003). It promotes productive problem solving strategies, higher-order conceptual systems, increasing mathematical competence and metacognitive functions (Lesh, Yoon & Zawojewski, 2007).

WHAT IS A MODELLING PERSPECTIVE?

A model-eliciting activity is a mathematically-based, real-life contextual problem. Modelling tasks go beyond "word-problems" and are more complex and integrated than traditional "problem solving". Very often modelling problems are designed for groups of students to solve. Lesh and Doerr (2013) see problem solving as a special case of modelling. They explain that the traditional view is to see "applied problem solving" as a sub-set of problem solving.

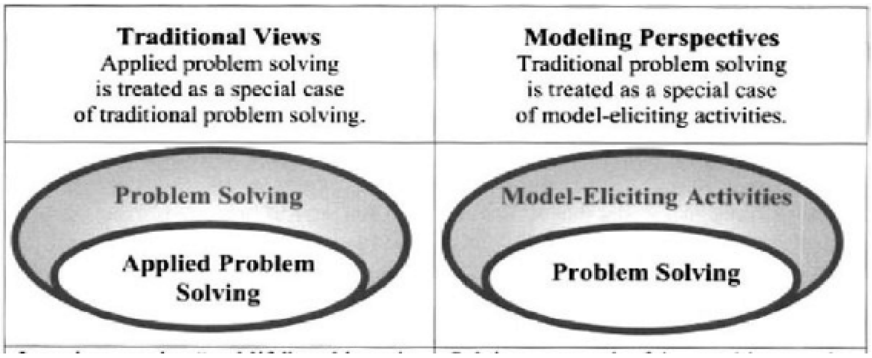


Figure 1. Traditional view vs Modelling perspective (Lesh & Doerr 2003a, p.4)

The focus of a modelling problem is not so much to reach a specific solution but rather to build on the existing mathematical ideas and concepts that students do have. However, Lesh and Doerr (2003b, p. 532) consider modelling to go "beyond constructivism" since modelling does not always involve building new concepts, but also allows students to sort out, refine and develop existing concepts. Modelling is also significant in that it gives a teacher a window on students' existing concepts.

Modelling problems typically follow a "modelling cycle".

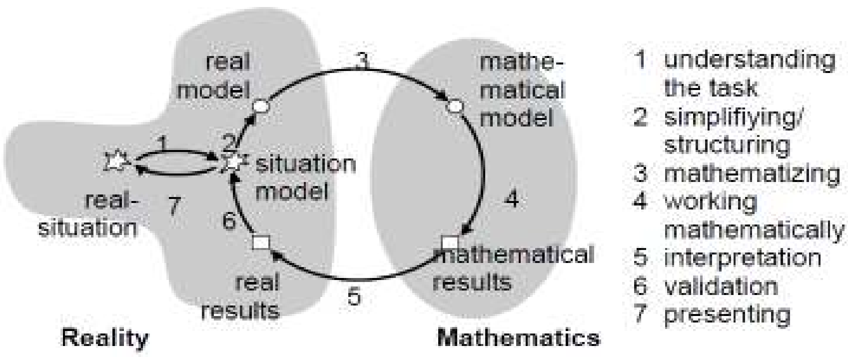


Figure 2: Typical modelling cycle (Blum & Leiss in Borromeo Ferri (2006, p. 87)

De Villiers (2007, p. 58) integrated modelling and technology. He integrates two phases a descriptive phase and an analytical phase. This conceptualization of the modelling cycle shows how descriptive models of a situation inform the analytical phase while the analytical model will further extend the descriptive model. This modelling cycle (as with Figure 2) also starts and ends with the problem context and not decontextualized mathematics.

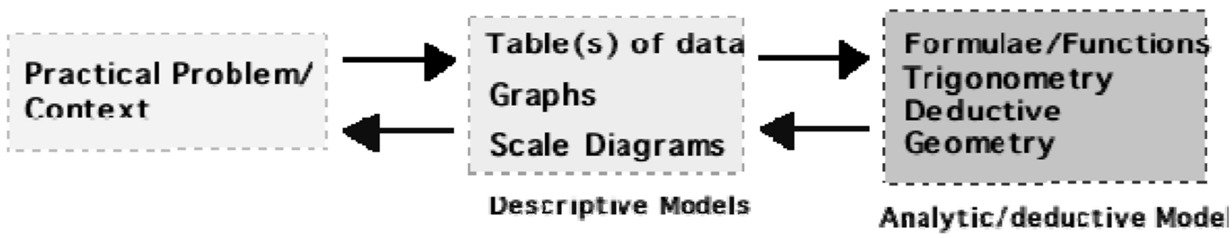


Figure 3: Modelling cycle with computing technology (De Villiers 2007, p. 58)

Students work between the world of mathematics and the reality within which the problem is set. They need to extract the mathematical components from the problem and to mathematise the problem. This is probably the most significant phase of the cycle. Once students know what mathematics they want to work with, they then produce mathematical results in the form of a mathematical model. This does not always happen in a single cycle. The nature of the problem is such that several iterations of the cycle may be necessary. Students may even move around the cycle in a haphazard fashion in their endeavor to sort out their understanding of the problem and to find a reasonable solution to the problem. Students then interpret their mathematical results in terms of the original real problem. They have to decide if their results make sense in the real-world context. In doing this, students create a model for the real situation or a situation model. The situation model solves the immediate problem they were given. Often, modelling problems will request a more generalisable model. The model can take the form of an explanation or description or structure that can be applied to similar situations. Generalising their results means raising the level of mathematical application.

The following example of a modelling problem may assist in understanding the nature of modelling tasks and the nature of learning that is possible through modelling.

Modelling example: Airplane Contest

A neighbouring school held their annual Paper Airplane Contest last week. Children took part in teams and each team that took part had to design and make a paper airplane. They then had to throw the plane from a starting point and aim to get it to a finish point marked on the school field. Each team was allowed to have three throws.

Problem:

The judges have problems in deciding how to select a winner for this competition. They don't know what to consider in determining who wins the award. The data and a description of how measurements were made have been included.

Table 1: Airplane problem (Lesh & Doerr 2003a, p. 7) https://engineering.purdue.edu/ENE/Research/SGMM/CASESTUDIESKIDSWEB/index.htm

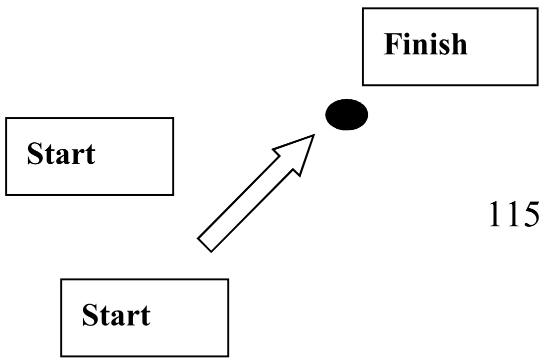
Measurements			
TEAM	Amount of time in air (seconds)	Length of throw (meters)	Distance from target (meters)
Team 1	3.1	11	1.8
	0.1	1.5	8.7
	2.7	7.6	4.5
Team 2	3.8	10.9	1.7
	4.2	13.1	5.4
	1.7	3.4	8.1
Team 3	4.2	12.6	4.5
	5.1	14.9	6.7
	3.7	11.3	3.9
Team 4	2.3	7.3	3.25
	2.7	9.1	4.9
	0.2	1.6	9.1
Team 5	4.9	7.9	2.8
	2.5	10.8	1.7
	5.1	12.8	5.7
Team 6	0.2	1.8	8.8
	2.4	10.1	4.6
	4.7	10.3	5.4

You will have to prepare a presentation for the judges of the contest. Explain a method that will assist them in choosing this years' winner. Your explanation must enable them to use your method future competitions also.

Amount of time in air – means the number of seconds from the time of throw to landing.

Length of throw – straight line distance from the start point to the landing point.

Distance from target – straight line distance from the landing point to the finish point



The task is designed for groups of students to solve. The task starts in the ‘real world’ with a problem that is unstructured. Students have to understand the conditions under which the competition takes place as well as the three criteria that will be used for judging the team’s airplanes. They then have the task of structuring the task. This involves understanding what the numbers are telling them. Does a higher number on the table always mean it is ‘better’? Although students may have been taught decimal numbers and be able to work procedurally with them, this task adds complexity by requiring students to understand what the numbers signify in the context of the problem. This is the process of mathematising the problem and understanding the mathematics within the context of the problem before they start with any mathematical calculations. Students will have to exercise some choices in this problem. They will have to rank the importance of the three categories before allocating a final winner. They must decide if using an ‘average’ for the results is useful in this case. This means that groups will not always reach the same result. However, they need to communicate their understanding of the problem in a presentation of their work and justify their mathematical choices. From the real problem, they will create a mathematical model that will describe, explain and structure the real problem. The groups will then be able to consider a general model for problems that may be similar to this one. Working at the level of generalization is mathematically both significant and complex. The type and level of understanding required for this type of problem goes beyond only a mastery of working with numbers. These problems allow students with divergent mathematical abilities entry into the problem. Furthermore, students can engage with the problems within their own scope of abilities. Very often, the same problem can be given to different grades within the school system. They will use different levels of mathematics to solve the problem.

WHAT ARE THE BENEFITS OF MODELLING?

Modelling as an activity has been described as “an advance on existing classroom problem solving” (English & Sriraman 2010, p. 273) in five significant ways.

1. The quantities and operations needed to mathematise realistic situations go beyond traditional school mathematics.
2. Modelling offers a richer learning experience than word-problems. Students have to elicit their own mathematics to solve the problems.
3. The explicit use of real world problems from several disciplines are used.
4. Students have to develop generalisable models – they have to extend their thinking to the structure of the problem.
5. They are designed for small groups to work in as a “local community of practice”.

The following diagram illustrates the modelling process where ideas are continually refined and extended. Switching between the world of reality and the mathematical world takes place:

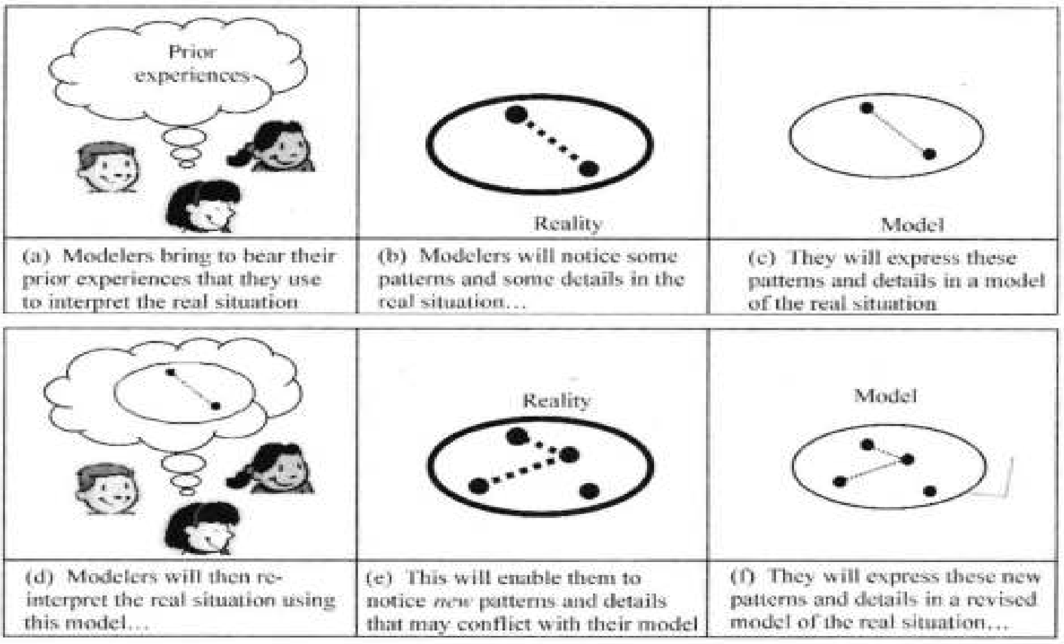


Figure 4: The modelling process (Yoon & Thompson 2007, p. 206)

As teachers of mathematics we should also aim at preparing our students for the demands of the workplace of the future. Lesh and Doerr (2003b, p. 521) accentuate the role of modelling for this purpose by stating that in an age of technology

> "preparation for success beyond school includes the ability to construct, describe, explain, predict, or manipulate complex systems" and that "making sense of complexity […] is at the heart of what is needed for success in the 21st century".

They conclude that "mathematical models and modeling abilities should be among the most important goals of mathematics instruction". The tasks that elicit this type of thinking meet the needs of the 21st century and are necessary in primary school mathematics classrooms. Model eliciting tasks encompass competencies that are important in meaningful mathematical learning. Lesh and Harel (2003), explain how there are similarities modelling cycles that students go through during a 60-90 minute activity and the stages of development that psychologists have observed that students go through during the natural development of constructs. It appears that the time spent modelling is beneficial to student development of concepts. Students of all ages and all abilities will benefit from model-eliciting experiences. Model-eliciting activities are even designed for very young children (English, 2012). Students will develop and use their own mathematics and not simply imitate what is presented to them by a teacher. This is what Lesh and Doerr (2003a) conceptualise as students moving beyond constructivism when involved in modelling.

By working through the modelling cycle, students may appreciate the value of mathematics in solving problems. They may also realise that their own knowledge of the context will provide important input into solving the problem. Too often, in traditional instruction, students are only required to work on a single part of the modelling cycle – working mathematically. The rest of the cycle is usually undertaken by the teacher. The teacher will: assist students in understanding the problem, structure the problem to make it easier for students, mathematise the problem by telling students what procedures to follow and finally the teacher may even take over the role of validating their responses. Traditional teaching makes teachers very good at mathematics, but does not always give students full access to "doing" mathematics.

Modelling involves structuring complex systems and organizing complex data in a meaningful way so that the model can be used to answer questions. Modelling does not only comprise one type of question. Thompson and Yoon (2007, pp.194-195) describe six situations that give rise to the need for model building. Their diagram shows the many dimensions of modelling in mathematics education.

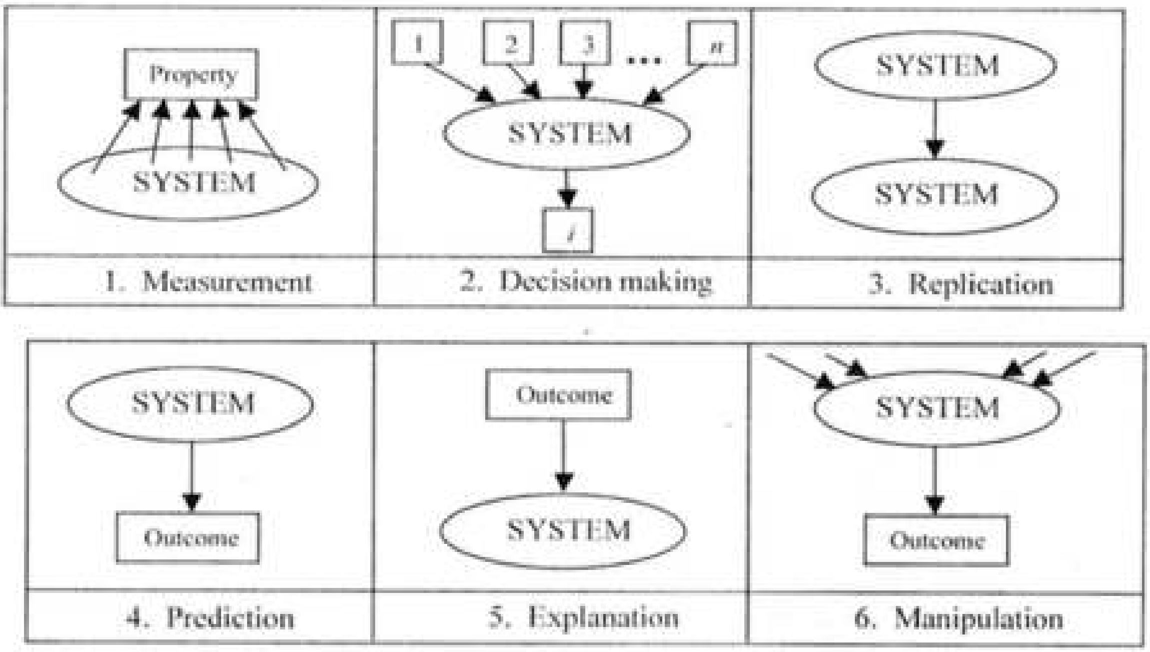


Figure 5: The need for model buildings

Thompson and Yoon (2007) describe these six situations as follows. The information sought through modelling is:

1. a measurement of some property of a system.
2. a way of deciding between alternatives based on the system.
3. a template that will allow one to replicate the system.
4. a prediction of the outcome of a system.
5. an explanation of some outcome of the system.
6. an understanding of how to manipulate a system so that it produces some desired outcomes.

The variety of model building situations allows teachers to select problems that meet one or more of these needs. It means that solving modelling problems will rarely become routine and predictable. It will extend and stimulate learners to use, refine and structure mathematical concepts. These six areas of modeling are not only important to develop mathematical competence, but also for necessary for skills needed in the workplace. Modelling and model-eliciting tasks encompass higher order and more complex thinking than traditional instruction. It is evident that these tasks should be integrated into a normal teaching and learning program and not relegated to the end of a section of basics if time permits. Research on modelling and model eliciting tasks needs to be based in South Africa. It is of interest to investigate the role of modelling in South African classrooms on two levels. One level should question how the tasks relate to the learning of mathematics and one level should question how the tasks could impact the teaching of mathematics. Although modelling is not a feature in many South African primary school classrooms, research interest in modelling in South Africa has previously been investigated (De Villiers, 2007; Julie, 2007; Wessels, 2009; Wessels, 2011). Wessels (2009) specifically proposed that modelling be realized in classrooms so that students would be exposed to a fuller mathematical experience as well as making them more internationally competitive. De Villiers (2007) strongly opposed to traditional teaching where students were teacher dependent and bound to senseless answers because of decontextualized mathematics experiences. This paper aims to build on this previous research in South Africa so that our students become independent mathematical thinkers.

In two separate studies by the author (Biccard 2010; 2013), modelling tasks were the basis of intervention programs. The first study dealt with student modelling competencies. This study purposively sampled students that were considered both mathematically ‘strong’ and ‘weak’ when judged by traditional assessment methods. It was found that ‘weak’ students displayed strong modelling competencies and although they were sometimes immobilized by weak mathematical concepts they used surprisingly complex ideas (Biccard, 2010). This study also showed that students’ competencies (cognitive, meta-cognitive and affective) improved through the duration of the program. These results tie up with other studies that have also show that significant concept development and powerful models of complex systems is achievable by students that are classified (by traditional school tests) as average, below-average and even students in remedial programs (Lesh, 2007; Lesh & English, 2005; Lesh & Harel, 2003).

The second study incorporated modelling tasks into a professional development program for primary school teachers. Modelling tasks enabled the teachers to transform their lessons from teacher-centred to more problem-centred. Early in the program the five volunteer teachers were asked to work on the airplane task. They were skeptical about using modelling in their own classrooms. The teachers then observed two groups of grade 7 students solving the Airplane task. They were asked what surprised them about their observations. Some of their responses (Biccard 2013, p. 189):

- The enthusiasm and participation of all taking part
- Understanding formulated by the learners
- The input given by all learners was surprising
- They needed very little assistance
- The could actually get on with the problem
- They interacted well
- They were very interested
- The learners worked out that everyone was supposed to share their ideas
- The learners did not read the whole activity thoroughly
- The learners were not in a hurry to complete the task
- Minimal off task talking was observed

In this study, modelling problems provided the cognitive conflict necessary for teachers to re-look at their own teaching and to reconsider their assumptions about problem-based learning. In the latter part of the year-long program teachers were showing signs of transforming many of the traditional practices within their own classrooms into more problem-centred practices.

MODELLING AND THE CURRICULUM (CAPS)

Teachers currently in South African classrooms need to be mindful of the curriculum document that prescribes the content and assessment of their teaching. The intermediate phase CAPS documents specify that:

> "solving problems involving whole numbers and decimal fractions, including measurement contexts" (DBE 2011a, p. 15)

These tasks also assist teachers in preparing the mathematics assessment tasks prescribed by CAPS. A project has to be set at least once per year with the following criteria given:

> Projects are used to assess a range of skills and competencies. Through projects, learners are able to demonstrate their understanding of different Mathematics concepts and apply them in real-life situations [...] Good projects contain and display of real data, followed by deductions that can be substantiated (DBE 2011a, p. 295).

Later, in the Grade 10 to 12 mathematics curriculum, modeling also plays a role. In the specific aims of the CAPS document, it states:

> Mathematical modelling is an important focal point of the curriculum. Real life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible (DBE 2011b, p. 8).

> In specifying examination requirements, it is stated that "Modelling as a process should be included in all papers, thus contextual questions can be set on any topic" (DBE 2011b, p. 55).

It is then necessary to introduce learners in the intermediate and senior phase to modelling tasks in order to meet the requirements of the curriculum and in preparing learners for career-related problem solving in the future.

CONCLUSION

Modelling tasks have much to offer primary school classrooms. They provide an avenue for students to see the value of mathematics in the real world. They enable students to refine and extend existing conceptual systems while offering teachers a view on these systems. The tasks allow teachers to gauge to what extent students are able to use their mathematical skills. Modelling problems are useful in teacher professional development programs in that they require that teachers revise their own thinking about student learning (Lesh, Hamilton & Kaput, 2007). Modelling also provides an avenue for both students and teachers to prepare for working life beyond schools where students will most likely be working in a team, working on a complex system that needs to be refined, replicated, explained or manipulated. In the pioneering days of model-eliciting task development, Lesh, Hoover and Kelly (1993 in Lesh and Doerr 2003a, p. 5) ascertained that the concepts that students invented or developed while modelling were "more powerful than anybody has dared to teach them using traditional methods" Once teachers have experienced modelling problems, they may think differently about how solving these types of problems can help students develop more complex thinking skills that are necessary in the 21st century.

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MEASUREMENT LEARNING IN SCHOOL AND OUTSIDE

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Learners from low income families in developing countries like India often have exposure to income-generating work-contexts through their participation in informal economy. Such exposure also entails measurement related knowledge. Experience of measurement learning in such out-of-school contexts are characterised by diversity as well as structural differences from school mathematical treatment of measurement. Outcome that school education aims for is distinct from the knowledge that is acquired in out-of-school settings. This paper unpacks measurement knowledge embedded in various work-contexts that middle-graders are exposed to and draws a possible connection of measurement learning in both the domains (school and out-of-school). Implications for classroom pedagogy is traced.

Keywords: Context, learning, mathematics, out-of- school, school

INTRODUCTION

Previous research has indicated that measurement learning and its requirements in school curriculum as well as in out-of-school contexts emerge in different ways (Bose & Subramaniam, 2015). While school textbooks envision measurement learning in terms of skill development for carrying out measurement tasks through activities and exercises given at the end of the textbook lessons, out-of-school settings present diverse measurement contexts which often make use of seemingly subtle underlying concepts of measurement. Such underlying mathematical concepts however remain hidden and doers/workers in the out-of-school contexts do not often realise the conceptual underpinnings or connections that arise from their measurement related experience.

The topic of measurement as covered in the textbooks reflects a disconnection with the out-of-school contexts and the separation is even more than that with arithmetic, which has some overlaps and connections with everyday calculation. The mathematics textbooks of Grades 5, 6 and 7 followed in the government-run schools in the state of Maharashtra, India where the field of this study was located, indicate that the current textbook treatment of measurement makes no references to the everyday world, and everyday experiences of measurement do not enter classroom discussion (Maharashtra Textbook Bureau, 2006).

A reflection of this is the fact that the textbooks faithfully implement the government directive of including only metric units and banning inches from textbooks, while in out-of-school contexts such directives are ignored. A consequence is that the school mathematics topic of measurement produces disconnects between itself and measurement in the real world.

The manner in which measurement occurs in everyday commerce or in work contexts, especially in the informal economy is significantly different from measurement in scientific or engineering contexts. Features of out-of-school measurement contexts lead to the use of a diversity of measurement modes in everyday contexts. In contrast, school curriculum emphasises scientific measurement which is based on precision and full quantification achieved within a system of units, with well-defined relationships between sub-units and between fundamental and derived units. Notions of abstraction vary between work-contexts and differ from abstractions handled at schools. For example, diverse measurement related work-contexts implicitly use abstract notions like construction of units and sub-units, chunking of measures, partitioning, unit iteration, covering, use of convenient units and modes (like templates) which are available to school students as part of their everyday mathematical knowledge. School curriculum, in contrast, treats learning of measurement as a skill development and then moves towards abstraction without building on the knowledge resource already available to the children from the work-contexts. Abstractions available to students in implicit form through their exposure and experience in work-contexts are potentially rich resources for building on measurement knowledge in the classrooms.

Other similar abstract notions such as conservation of attributes, transitivity and seriation that are foundation of comparison and hence of measurement knowledge are not sufficiently emphasised while handling abstractions in the school context. Thus, although experience at work-contexts or in the cultural practices helps in broadening children's learning potential, they are not leveraged in the formal learning situation. Exploration of the following research questions guides this paper:

1. What are the distinct features of the measurement learning that occurs in both the domains – school and out-of-school contexts,

2. What are the possible pedagogic interconnections between the two domains with respect to measurement learning?

Studies on measurement in the everyday context

Previous research on measurement within work-contexts or in other everyday settings was carried out alongside or within the research on everyday mathematics, with a particular focus on the alternative ways of thinking in different everyday contexts. Such research provided evidence of how mathematical ideas were developed and framed within work-contexts. Lave (1985) described the use of different units by Liberian tailors and dairy workers in their work-contexts. Her work showed that adults evolved techniques for mental estimation that were markedly different from the school learnt techniques (Lave, 1988). Millroy's ethnographic study (1992) with South African carpenters in their everyday woodworking activities noted extensive use of conventional mathematical concepts like congruence, symmetry, proportional reasoning and optimisation. Her study discussed the use of spatial visualisation and ways in which visual and tactile cues were incorporated. Nunes, Schliemann and Carraher (1993) studied how construction foremen applied multiplicative thinking in everyday work-contexts using proportions and inversion techniques that school students, despite having learnt to solve proportional problems, could not use as these tasks were not among the routine school-type problems. Scale-drawings used by the foremen are examples of how measurement knowledge and proportional reasoning come together in work-contexts. The recent work of Mukhopadhyay (2013) on “vernacular boat making” in the Indian state of Bengal, showed that builders of fishing boats made use of a blue-print that they saw once and subsequently drew on the knowledge of design and construction that they all possessed together as a community. The boat-builders were unschooled and used spatial visualisation and estimation skills and locally made indigenous tools.

Most of the above studies that focused on participants’ measurement knowledge involved adults in their singular work-contexts. There are not many studies that looked at the varied contexts in the everyday settings that students from low socio-economic backgrounds are exposed to and the affordances that these settings bear for school learning of measurement. The literature mentioned above has led to a cumulative understanding of the skills, procedures and strategies based on mathematical principles that are acquired in out of school work contexts. The focus has been on oral computation strategies, proportional reasoning strategies, visuo-spatial and geometric reasoning and estimation skills and strategies. Restricting the focus on the topic of measurement, this paper take a broader view derived from the funds of knowledge framework. It is not just interesting to note what the participants know or can do, but also what they have observed and are familiar with even if the mathematical knowledge associated with these aspects is partial and fragmented. The perspective is to explore what aspects can serve as starting points or building blocks for mathematical exploration in the classroom and how mathematical learning can strengthen the understanding of measurement practices in the real world.

The study

This paper draws data from a long ethnography conducted over two and a half years in a low-income urban settlement in central Mumbai city. The entire neighbourhood has a vibrant economy in terms of micro-enterprise and small scale manufacturing units dispersed in small tenements and workshops. The settlement is an old and established one with very high population density. Population here mostly consists of immigrants from different parts of India, mostly unskilled workers who find jobs in the workshops and some of them become apprentices in the small factories. The diverse work places and communities of work practice are resource-rich for creating varied opportunities for school going children to gather everyday mathematical knowledge. Children living here have an access to the community's funds of knowledge and gain exposure of the work-contexts happening around them. Some common micro-enterprises that students participate in are embroidery, *zari* (needle work & sequin stitching), stitching and garment-making, making plastic bags, leather goods (bags, wallets, purses, shoes), dyeing, button-stitching, making of *rakhi* (decorative wrist bands), stone-fixing work on ornaments and recycling work.

The study adopted ethnographic qualitative research design to explore students' everyday mathematical knowledge and the work-contexts that they had exposure to. The student participants and other members of the community that the author interacted with had a shared pattern of language and belief system. The study also involved classroom observation of Grades 6 and 7 mathematics lessons in two government-run schools (English and Urdu medium) located in the settlement. The schools ran in the morning shift till noon and all the learners came from the settlement. Many learners worked after their school hour or assisted their parents or family elders. The researcher (author) interacted with the learners which helped in knowing about the opportunities available to them to gather such knowledge and the extent of their involvement in economic activities. The initial phase of classroom observation and informal interaction with learners was followed by data collection through semi-structured interviews of a representative sample of 31 students (one-third of the two Grade 6 classes) to understand their family-background, socio-economic status, parental occupations, productive work done at home/elsewhere and student's involvement in them. In the third phase, a sub-sample of 10 students and an additional 7 students from the same grade who volunteered, were interviewed to obtain a detailed understanding of their work-context knowledge. Interviews were transcribed and transcripts were coded at first and second levels to review what they indicated about the nature of work students are involved in, and what they know about aspects of the work. Students have been designated with the letter “E” or “U” (for English and Urdu medium schools respectively) followed by a numerical subscript. The data used for this paper is drawn from the interviews for measurement aspects and from other phases of the study including informal visits to the house-holds, manufacturing units and discussions held with adults in these locations.

Diversity of out-of-school measurement knowledge

We highlight below some of the specific ways in which measurement knowledge is gathered from out-of-school contexts and ways in which such knowledge may be used as learning resources in the mathematics classroom. It was observed in this study that measurement experience in the everyday context is richer and more sophisticated than measurement experience that arises in the classroom context. This is due to the diversity of measurement modes and aspects of construction of units and tools that are often encountered in everyday contexts.

Construction of templates and units

Templates (called *farma*) are constructed and used locally for purposes of comparison. Units are different from templates in being used for measurement, i.e., for quantification of an attribute. A template may occasionally be used to iteratively cover an area. Although, iterative covering is the basic operation to quantify area in terms of a unit, the context in which children observe the operation may not have measurement as an overt aim. For example, a *farma* in leather work may be used to cut out pieces of the required size, or may be iterated to optimize the use of a large piece of leather. Further differences between templates and units are that units are chunked or partitioned in systematic ways to obtain larger or smaller units. Units are also typically generalized beyond the immediate context of application. Children in this study were familiar with the construction of templates to measure length, area and weight. They constructed units out of parts of the body to measure and estimate length. Familiarity with the idea of such construction is valuable in the learning of measurement. It also gives rise to questions and problems that can lead to fruitful mathematical work in the classroom: why is the construction of units or templates needed? How do we construct new templates or units from given templates? What attributes of templates allow them to be used as units (for instance, in measuring area)? In what contexts are units partitioned to yield smaller units? What quantities can be measured with a given combination of templates?

Non-transparent knowledge of measurement instruments

In the context of tailoring, length is measured using an inexpensive plastic tape that has both inches and centimetres marked on it. The researcher observed that almost every student was familiar with such tapes but lacked understanding of the different units present in the tape, the partitions of the units and the smaller sub-units indicated by the markings on the tape. Further, students often confused between the units from two different systems (inch and cm) and were not clear about the distinction. Knowledge about the underlying construction of measuring tapes and the relation to units remained unclear to most students, as evidenced in the teaching intervention. To cite an example, E6 (garment recycling work) during the interview clarified that the difference between an "inch" and a "metre" is that "*inch chhota rahta hai, metre bada rahta hai*" ["inch is smaller, metre is bigger"] but that he was unable to say anything further about the relationship between the two units. Similar were the responses from a few other respondents like E8 (mobile phone repairing work) to whom the difference between an inch and a centimetre/metre was not clear.

Despite not knowing the construction underlying a measuring tape, the students may be able to carry out measurements of acceptable accuracy by reading off the length from the tape. This instance was noticed among the sixth-seventh graders during the garment measuring activity in a vacation teaching camp (last phase of the study to build connections with school lessons). However, the measurement itself remains critically dependent on the integrity of the artefact. For example, it was noticed, some plastic scales bore different calibrations that led to differences in the measures they showed, that students did not think was a problem. During the classroom observation the researcher noted that some students found broken scales difficult to work with and some students made errors if the scale was used in a non-standard way (for e.g., measuring from a point other than zero). Some students counted the markings on the scale starting from the zero point. E6, for example, counted the finger-bands starting from the first mark on the finger and arrived at "four inches" instead of three inches corresponding to four "markings" on each finger. He described "*char inch itna hota hai/ yeh ek inch, do inch, teen inch, char inch*/ (showed finger-bands)" [four-inch is this much/ this is one inch, two inch[es], three inch[es], four inch[es]/].

However, in a formal classroom setup, unlike out-of-school practices, there is a single mode of quantifying an object – by its length/area/volume or its weight by noting the measurement readings from the calibrated, scientific scales. Classroom observation indicated that no other modes of quantification was put to use, such as, estimations or construction of newer and convenient units other than the formal ones mentioned in the textbooks. Measurement learning in the classrooms amounts to learning of the measuring skill which most of the students are anyway familiar with. What they are not familiar with are the conceptual underpinnings of measurement. Classroom learning also focuses on unit conversion and calculations, which do not connect to situations familiar to the students.

Some quantification were familiar to students but their origin was obscure, as in the example of shirt and garment sizes. The students were familiar with the garment sizes and U24 (ready-made garment selling) was skilled in connecting garment-sizes with age, but they were not clear as to what those numbers signified or how those numbers were arrived at. Similarly making of a measuring scale (inch-centimetre relation) and the quantification generated from it remain unclear to the students. It was observed that quantification was opaque not just to novice workers but even to the experts.

Fragmented knowledge of measurement tools

As discussed above, children in this study were familiar with common measuring tools such as the inch tape, but were unclear about the meaning and construction of the markings on the tape. Interviews with them further shows *demathematisation*[3] of the measurement tasks. Wherein, it is not required to understand the construction behind a measuring scale or the meanings and inter-connections between the different markings on it. What has become important now in the school curriculum is to be able to use the scale and be able to measure the length of a given object. For example, E6 admitted that he did not know the connection between an inch and a centimeter but nevertheless

3 Demathematisation is a process that devalues community's mathematical knowledge and carries forward the hegemony of academic mathematics or the hegemony of devalorising non-academic mathematics (See, Chevellard, 2007; Subramaniam & Bose, 2012). It leads to trivialisation of vocation and pushes for standardised knowledge.

he had a fair estimation of how much distance both signify. E6 gave a rough estimation of an inch and a meter to the researcher which was not accurate but nonetheless a close estimation. This is to argue here that such instances indicate that students from the neighbourhood favoured less use of tools and instruments and relied more on visual, tactile aids and on their own mental computations. The ability to estimate as a skill or reliance over mental computations devalue use of measuring scales and calculators and such propensity comes as a resistance to the increased prevalence of demathematisation processes in our contemporary society. Such unpacking of embedded mathematics in artefacts resists the demathematisation processes and stresses on the comprehension of the hidden underlying concepts. Such explorations therefore have strong potential to become effective pedagogic modes.

U2, who had started working as a helper (novice) in a tailoring unit discussed the use of *futta* (stiff canvas) in making *farma* by giving the specifications in “inches”. He was also aware of the use of different types of scales (regular and bent) depending on the context and was aware of the techniques of taking and recording measurements. U2 explained that inch tapes are used in tailoring work while *guj* (Hindi word for “yard”) is used as a unit in “qaaleen” (carpet) stitching work in which U2's father used to work before. However, U2’s knowledge about the connections between inch and guj in the same system of units or inter-connections of these units with units from different systems was not clear and U2 himself claimed that he did not know the relation between these units. His knowledge of the units was confined to the use of a particular unit in a particular work-context and hence fragmented. The diversity of units prevalent in the world of work, has underlying it historical and cultural reasons, which could also be explored in an archaeological[4] approach.

Limited opportunity to explore variations

As noted before, work practices do not call for understanding of the conceptual constructions underlying measurement activities or tools. Familiarity with the artefacts or tools, and knowledge and skill related to their use is sufficient. In work contexts, opportunities to explore variations different from those already contained in the traditional work processes is limited. Often work-contexts need to strictly follow the instructions and patterns that come with the work orders which limit opportunities for further exploration or “archaeology”. For instance, in E16’s stone-fixing work, the jewellery cards followed the array structure of 6 × 24 or 12 × 12 but there was no effort or requirement of exploring other arrays that could also amount to one gross, i.e., 144 – viz., 9 × 16 or 8 × 18 or 3 × 48 and so on. The work context did not give rise to questions like why the jewellery pieces were arranged in particular arrays and not in other ways. Engagement with such explorative ideas was not required in the work-

4 The idea of “archaeology” of embodied mathematics or mathematical ideas was developed through personal communication with K. Subramaniam, the author's doctoral supervisor. The author gratefully acknowledges the major contribution of Subramaniam in the formulation of this notion.

contexts. Mathematical learning in the classroom is different in that it contains the possibility of exploring variations outside those that are needed in practical contexts. This is important for understanding the general or invariant structures or concepts that underlie variations, and the affordances or limitations of such structures. In this important sense, school mathematics learning goes beyond the mathematical knowledge embedded in practical contexts. Though such instances of exploring variations on what students encountered in out-of-school contexts were not taken up during the teaching intervention, I claim that such opportunities in the work-contexts can enable a shift towards understanding which can illuminate both out-of-school experience and mathematics learning in school.

Prevalence of different units and systems

Students in this study used different kinds of units: international units, old Indian units, old British units and non-standard units. Besides suggesting the idea that units are purely conventional creations and are embedded in cultural and political histories, such knowledge is useful in exploring the relation and differences between different systems. Questions that can be fruitfully explored for example are, why do we need unit systems rather than just units? What are the different principles of subdivision and the advantages and disadvantages of the binary based and decimal based subdivisions? It was observed that students were more familiar and conversant with the words for binary fractional measures (half, quarter, half-quarter, etc.), than with the decimal fractions. Conversion between binary system to decimal system was a challenge. For example, in response to a question about how to express *pauna* (three-quarters) in decimal representation, students came up with alternative representations in the binary system itself and not in decimal, viz., *teen paav* (three quarters), *aadha aur paav* (half and a quarter), *ek se paav kam* (quarter less than a whole) and also *pachhattar* (seventy five). However, students had difficulty in arriving at the decimal representations in the class. Students were able to connect the multiplicative relations of the above binary fractions with their geometric meanings and arrived at their inter-connections. But, it is important to explore if similar inter-connections can be made between multiplicative and geometric relationships of the decimal units and between binary and decimal units. Such connections can facilitate building a pedagogic mode to connect school mathematics learning with out-of-school knowledge.

Quantification of various attributes

This study noted that the out-of-school work-contexts possessed a diversity of attributes that were quantified. The ways of quantification were diverse too. By drawing on students' familiarity with the range of objects and attributes that are quantified, students can explore questions such as what is common and what is different in how we quantify different attributes such as length, area, volume and weight? In what situations can one attribute substitute for the measurement of another attribute? What conditions or properties allow for such substitution? One can also raise questions about the quantification of other salient attributes. How is an abstract attribute like monetary (exchange) value quantified? How do we quantify different aspects of labour such as time, effort and expertise? Such questions are important to build an holistic understanding of the measurement concept that students get to handle in different domains of their lives. Students can also explore the need not only for units, but also for systems of units. In the first instance, each measurement system has its own units and sub-units. Within a particular measurement system, the same attribute can be measured in different objects using different units. For example, the attribute of weight is measured using different units for different objects – the units used to measure grain by weight are different from the units used to measure precious metals like silver and gold and the units used to measure quantities of salt and sand. However, quantification of such different attributes for different objects are not dealt with in the mathematics textbooks that have been analysed (for e.g., Maharashtra state mathematics textbooks). It is understood that measurement learning cannot be complete unless the curriculum engages with the complexity of different attributes (in the above case, weights) and measures prevalent in the world of commerce. Interestingly, the old textbooks reveal that different forms of tables with information about measurement units of various kinds followed in different systems and their inter-conversions. For example, four different systems of weight measures for weighing metals like gold and silver were presented in the math textbook written by Gopal Krishna Gokhale that was in practice in the last two decades of the 19th century: the system in the state of Maharashtra, the "old" system – prevalent in the city of Bombay and the system in England (Subramaniam & Bose, 2012). Ironically, other similar textbooks from a hundred years ago show a strong connection with life outside school, which have faded away from the contemporary textbooks, while educators worry about the lack of such connections in the modern textbooks.

Implications for classroom learning

Treatment given to the topic of measurement in school mathematics is structurally different from the measurement experiences embedded in the out-of-school contexts which are mostly characterised by diversity. However, at the same time it would not be correct to assume that there does not exist any connection between the content prescribed in school mathematics and out-of-school measurement experience or to assume that both the forms of measurement knowledge have no relevance for drawing from each other. The distinctness as well as the inter-penetration of out-of-school and school knowledge has been long recognized in the Vygotskian approach to education and psychology. On the similar lines, Moll *et. al* (1992) has argued that "everyday and scientific concepts are interconnected and inter-dependent", and draw on each other in their mutual development. Science is not something that stands over as distinct and apart from the everyday, but must illuminate, question and re-invent the everyday (Subramaniam, 2012).

Distinction between school and out-of-school mathematical knowledge is that both these forms of knowledge have different and distinct outcomes and goals (Resnick, 1987). Interestingly, mathematical knowledge that one acquires in the out-of-school contexts is distinct from what school mathematics recognises as its outcome. School mathematics aims for producing knowledge that is generalisable and not bound to specificities of particular contexts. In contrast to generalised knowledge, knowledge which are specialised are necessarily bound to contexts and embodied in individuals. Specialised knowledge can be effective in action and expertise driven only in limited domains and contexts. Knowledge that is generalisable on the other hand has wide applicability and generality but may not lead to expertise and efficiency in specific task contexts (Sfard & Cole, 2003). Generalised forms of knowledge however, also contain elements of situational and contextual knowledge and it has the ability to re-invent and shape the everyday knowledge. Generalised forms of knowledge are neither about abstraction without the concrete content, not are they about mere induction from a number of instances. Rather, generalisation is all about arriving at or holding an idea or a construct that can illuminate and be applicable in diverse instances. Valuing generalisability as an outcome of school learning in fact places greater importance to the diversity of out-of-school experiences, for such diversity actually creates contexts for school learning. From this standpoint, I understand that mathematical aspects are present in the work-contexts as hybridized and opaque embeddings and it would not be correct to look at such practices as reflecting mathematical thinking and understanding. At the same time, I argue that it would be fallicious to look for elements of school learning in a particular work-context or to expect school mathematics to increase proficiency in specific practices. It is claimed here that the formal mathematical learning can illuminate the diversity of practices as a whole and strengthen understanding, but not practice.

The funds of knowledge perspective illuminates how the connectedness of social networks gives rise to diverse and rich knowledge and experience that can be drawn on for the purposes of school learning. In this study, which is set in an urban, developing world context, I found that students often directly participate in work, or are closely aware of work contexts and practices. Experiences and knowledge of measurement drawn from such contexts are intimately familiar, with aspects of it even embodied in students and present in the classroom. All the students in this study were from two Grade 6 classes that were co-located in one school building. Such diversity of experience, within a school community hence presents a greater opportunity for learning that has largely been ignored in formal school education.

Educational thinkers in the developing world, and particularly in India, have recognized the value of work experience for education conceived in a broad sense. Policy documents on education have taken on board this insight. Gandhi's views on the role of work in education emphasises that modern education centred around work is different from the traditional education in the crafts. Its aim is a well rounded education and not just training in a particular craft (Gandhi, 1927). The learning of measurement in school must therefore be framed in broad terms. It is aimed at acquiring understanding and insight and not at practical training.

Existing curricula and teaching practices, in contrast to policy documents, serve to reinforce the separation of the everyday from formal school learning. Underpinning of this may be an implicit awareness of the structural differences between these forms of knowledge and learning. This may combine with an anxiety about the potential distractions caused by the contextual details of the everyday. Thus teaching practice typically keeps the everyday out of the classroom and creates school mathematics as a culture and practice that is not only distinct, but also disconnected.

One of the challenges before the teacher or the instructional designer is to imagine connections between school and out-of-school knowledge that can produce powerful learning. This paper points to two aspects of out-of-school knowledge that can lead to powerful connections. The first involves conceptual construction. Mathematics education researchers have pointed to the fact that in existing literature, the treatment of the topic of measurement largely ignores foundational concepts and emphasises the "physical act of measuring" (Sarama & Clements, 2009, p. 275). The notions of conservation of an attribute, transitivity and seriation form the foundation for comparison, which in turn, forms the foundation for ideas of measurement. Conceptual aspects of measurement such as equi-partitioning, conservation, transitivity, unit iteration and covering, structuring of unit coverings, accumulation of measure and additivity have been highlighted by researchers as critical to the understanding of measurement (Sarama & Clements, 2009). These aspects are not adequately treated in existing curricula in the Indian context.

From the point of view of the diversity of out-of-school experience, we need to go beyond critical concepts listed above to include construction of units and templates, equi-partitioning and chunking of measures and unit, construction of measuring scales, design of convenient measuring instruments and units. Further aspects critical to the understanding of measurement that have not been adequately addressed in the curriculum include the extensive use of comparison and estimation in real life contexts, the use of the body as a measuring instrument, the trade-offs between convenience and accuracy, the variety of purposes of measurement, the variety of modes of quantification and the limits of informal quantification, and the cultural-historical origins of units and systems of units. These aspects, with the exception of estimation, have also not received adequate attention from mathematics education researchers. Diversity of measurement experiences in out-of-school work contexts can be drawn on to illustrate each of these concepts and ideas, and for understanding the difference between comparison, estimation and measurement and their purposes.

Looking ahead

This study in informal work contexts reveals a situation characterised by a rich diversity of measurement units and modes of quantification. Besides informal units and units of convenience, older units may still be in use in the culture together with standard international units. As described above, Mathematics textbooks from a century ago reflect something of the diversity and richness of measurement units in the everyday world, but stress the arithmetic of conversion and computation rather than the concepts of measurement. In modern Indian school mathematics textbooks this diversity is not found and only standard units are taught using standard measuring instruments like a ruler or weighing balance. School curriculum designers do not consider it worthwhile to deal with a variety of units even though they may still be used. The framework of demathematisation helps explain why informal practices and contexts have disappeared partly from social practices and wholly from the curriculum and how their importance and value is diminished. However, in household based occupations, measurement in a diversity of modes and with a variety of units always plays a role. Emergence and survival of such informal mathematics can be seen as a counter-trend to the broad process of demathematisation.

When school mathematics textbooks adopt a restricted view of measurement, children may fail to see any connection between their classroom experience and the rich world of measurement outside school. Further, how an attribute is quantified may not be clear from classroom learning. Children also need to appreciate the fact that measurement as it occurs in the world of commerce or in work contexts in the informal sector may show characteristics quite different from precise, scientific measurement. The extent of precise quantification may be limited, and may just be sufficient for the purposes at hand. The quantification may be incomplete or if embedded in cultural artefacts, may be opaque. Even so, it may be embodied in the form of a skill at estimating quantities.

The prevalence of diversity in measurement units and modes in the culture suggests that more than teaching measurement as a skill, it is the conceptual aspects of measurement that are important to learn. Understanding how quantification is achieved in various modes may allow children to understand and make connections among the diverse ways of measuring that they encounter. It may lead them to appreciate the possibilities and limits of different kinds of informal measurement, and the distinctiveness of these from scientific measurement. Further, an inquiry into the history of older units still in use may provide interesting avenues of exploration and possibilities of connection with other curricular subjects. An inquiry into familiar measurement tools which embody measurement ideas in a “materialised” form, but where the process of quantification is obscure (like the inch tape or shirt sizes) can potentially become an important part of school learning. These ideas need to be explored further. It is likely therefore that more research about how measurement plays a role in the everyday world in diverse ways will have an impact on the school mathematics curriculum.

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AN ILLUSTRATION OF THE CONCEPT "FRAMING" USING A NUMBER FOCUSED LESSON IN A GRADE 2 CLASS

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In this paper we report on a number focused lesson drawn from a larger Masters study located within the Wits Maths Connect- Project (WMC-P) which investigated pedagogy across number focused grade 2 mathematics lessons in ten primary schools in Gauteng, Johannesburg. Bernstein (2000)'s notion of 'framing' and analytical literature has been used as in analysing pedagogy. Our analysis shows limited possibilities offered to learners for developing number sense, flexible counting skills, non-enhancement of number patterns that are essential for future mathematical proficiency. We argue for a more robust pedagogy that takes into account the development of mathematical proficiency in learners.

Keywords: framing, learners, mathematics, number

INTRODUCTION

The teaching of number is one of the fundamental aspects that learners need to be well proficient with as it forms the future basis for their mathematical learning. A considerable amount of literature has been published on the state of South African mathematics. These studies point to one thing in common- poor learner performance. Research data both local and international continue to show a nation that is in crisis in terms of performance in both the critical areas of literacy and numeracy. Spaull (2013) published a report that showed that South African learners acquire learning deficits early in their academic career. (Van der Berg, 2007) (Askew & Brown, 2003) ((NCTAF), 1996) (Anghileri, 2006) (Haylock & Cockburn, 2008)The highly internationally respected Trends in International Mathematics and Science Study (TIMMS) study showed that Grade 7, 8, and 9 maths scores where at the bottom of the maths club.

Considering that South African schools are relatively well –resourced compared to its African counterparts and yet continues to perform badly in international and local standardized assessment tests has led researchers such as, Fleisch (2008) to describe the education system as in a state of "crisis". Broader evidence points to connections between mathematical content knowledge and mathematics teaching practices (NCTAF, 1996). Extant literature also points to mathematical content knowledge being necessary, but not sufficient, for constructive classroom practice (Askew, Brown, Rhodes, William & Johnson, 1997; Ball & Bass, 2003). Given evidence of widespread gaps in primary teachers' mathematical content knowledge and literature detailing the important role of making the evaluative criteria explicit to learners, the authors' interest understood how these two issues were linked. The focus on early number in this study was motivated by literature which points to the importance of developing early number sense in the primary years (Anghileri, 2006; Askew et al., 1997; Haylock & Cockburn, 2008). The research question that guided this paper was:

1. What insights about early number teaching can be gained from observing a number focussed lesson in a grade 2 classroom?

Understandings of early number and its teaching from the empirical filed of this study; framing form the conceptual framework. Both of these features are detailed in the following sections.

LITERATURE REVIEW AND ANALYTICAL FRAMEWORK

Number sense

A student with number sense has a sense of what numbers mean, understand their relationship to one another, is able to perform mental math, understands symbolic representations, and can use these numbers in real word situations. Number sense some links between numbers are established through patterns in counting. Anghileri (2001a) noted that there is shift from teaching standard procedures for calculating, to enabling children to observe patterns and relationships, and make connections so that they develop an insight and a 'feel' for numbers. Similarly, earlier Skemp (1976) posited that teaching number sense needs to establish *relational thinking* where knowledge extends from how to do a calculation to *why* procedures work. Huckstep (1999) identifies different perspectives for teaching number sense viz-'a tool in everyday life', 'a means of communication', it's 'usefulness in providing mental training' and as a 'means of empowering people'. Of Huckstep (1999)'s first purposes suggest teaching standardized procedures for calculating but the other suggest that computational skills will provide a poor educational "opportunity to learn".

Reeves & Muller (2005) identified in the South African context as severely lacking in terms of sequencing and pacing. Coombe and Davis (1995) illustrate the difficulties posed by introducing public domain exemplars into classrooms. They investigated the use of a game by a teacher in a mathematics classroom in order to teach the properties of parallel lines. The game in other words, constitutes public domain text Dowling (1998) from which mathematical concepts can be drawn. Coombe and Davis show students potentially became distracted by the everyday associations of games and the logic of play, in such a way as to obscure (Munn, 1997) (Fuson, 1988) (Hughes, 1986) (Schaffer, Egglestone, & Scott, 1974) (Steffe, von Glaserfeld, Richard, & Cobb, 1993) (Tapson, 1995) (Piaget, The Child's Concept Of Number) (Gelaman & Gallistel, 1978) (Thompson, 1997) (Whitebread, 1999) (Bernstein, 1975) (Nunes & Bryant, 1996)e the mathematical purposes of the lesson. Having discussed the significance of number sense the next section details another key aspect of number learning-counting.

Counting

Counting forms an integral component of the discipline of mathematics and has played a role in the teaching and learning of mathematics. Nunes & Bryant (1996, p.22) describe counting as ‘a system which is partly an expression of universal laws and partly a bundle of convenient but arbitrary conventions’. Anghileri, (2001a) notes that learners bring to school a rich experience of numbers that relate to their personal lives, however this rich experience needs to be supplemented by making connections through consistent use of numbers in meaningful activities. In this paper we focus on Anghileri (2001a) notes that learners need to relate numbers to either quantity or location. Counting develops through a process in which a child can often take initiative and a little encouragement will help to establish some of the basic knowledge upon which counting is built. Munn (1997) talks about ‘ multiple overlapping developments in verbal, motor and cognitive activities that become integrated with each other over time’ She notes that ‘it is not until children acquire some experience of the mental activities involved in other people’s counting’ that they ‘internalize the cultural practices of counting’ (p.18). Because counting is powerful unifying idea in early number work Hughes (1986) showed that children can display a sound understanding of numerical relationships when there is a meaningful context. Children need to progress from practical experiences to working with imagery of those experiences and they will also need to identify for themselves appropriate images for tackling different problems. Reciting counting sequence, memorizing rhymes and songs is can provide a child’s first experience of counting forwards and backwards and using many different rhymes will help to establish consistency in ordering of numbers (Anghileri, 2001a). Extant literature shows that teaching needs to develop certain skills in counting. Learners need to be involved in activities that promote small number pattern recognition-this skill is called subtizing (Anghileri, 2001a). In subtizing there is no counting involved but a spatial arrangement.

Since, Piaget (1965) identified ‘conservation’ as the concept that enables children to realize that quantity is not affected by nay arrangement and demonstrated that young children are frequently misled by their intuitive judgments, Fuson (1988) has shown that children still relied on misleading length cues when comparing the sizes of two sets even when they could obtain information by counting. Pattern recognition is followed much later in children follow pattern recognition by recognizing and naming the whole set and this principle is called cardinality principle (Schaffer et al., 1974). By recognizing the spatial arrangement for naming a collection of two or three objects, and identifying this with the verbal count for one-to –one correspondence, children are able to deduce that the last counting word is the name given to the collection. Steffe et al., (1983) refer to a transfer that is made from the component nature of numbers to their composite nature. Understanding that ‘ten’ can be both ten individual items and at the same time a ‘ten’ that can be counted as ‘one ten’ is necessary for understanding place value. Making these links between the different uses of numbers and counting will give children powerful mental strategies for problem-solving. Gelmanon & Gallistel (1978) proposed the abstraction principle that deals with the definition of what is countable. Steffe et al., (1983) developed further the idea of what children can count and identified five different types of countable item of progressive difficulty:

- Perceptual units: items that can be identified as discreet 'things' to be counted as children would count a collection of buttons;
- Figural units: items that are not immediately available but are recalled and imagined (for example, the pets a child has at home);
- Verbal units: utterances like number words that can be themselves be counted (for example, saying'8, 9, 10' and knowing three numbers can be counted) and
- Abstract units: items that can be introduced by the child to 'match' count for that given number.

This progression takes children from seeing, touching, and counting 'actual' objects to understanding that the abstract number itself may be identified with the count that generates it, (for example, 'eight' by itself implies the sequence 'one, two... eight'). However school experiences are too often limited to static collections on work cards and some children fail to develop to the level of abstraction that is necessary to operate effectively with number (Anghileri, 2001a). the order irrelevance principle 'captures the way' different skills 'interact in contributing to a full appreciation of what counting is about' Gelman & Gallistel, 1978, p.141). This principle is demonstrated when moving objects, for example, children in a playground are to be counted and it may take several attempts to complete the task with different starting points and different strategies to keep track. By understanding the stages in learning to count and the range of skills to be mastered, teachers can monitor progress and provide activities that will address particular sub-skills that are not yet well established. Thompson (1997) provides a summary of the stages of learning to count as 'recitation' being able to recite the number words in the correct sequence, 'enumeration', assigning the correctly ordered number words in one-to-one correspondence with objects being counted and 'cardinality', realizing that the number of assigned to the final object counted tells how many there are in the whole. Thompson (1997, p.131) emphasizes that only" when children can successfully satisfy all three … criteria can it be said they are able to count". There are many diverse skills that children will master over an extended period of time. In supporting this view (Nunes & Bryant, 1996, p.41) conclude that is one thing to be able to count and answer the question "how many? "but quite another to understand the significance of the number uttered at the end of counting as a measure of set size." However, skills such as matching, ordering, and coloring need to be developed Whitebread (1999) questions whether the associated activities are adding to the children's number understanding and argues that they can serve to detach school work from useful purposes of numbers.

Framing

Bernstein's major focus was on understanding how education could be understood in its own terms, and not merely as a relay for social class and other inequalities. For Bernstein the key question was as he summed it up himself by asking, 'how does the outside become the inside, and how does the inside reveal itself and shape the outside?'(Bernstein, 1987, p.563, cited in Hasan, 2000). In other words he, was interested in how the social world structures consciousness, and how consciousness in turn structures the social world. He believed that cultural reproduction studies examined what is carried or relayed by education, such as class, gender and race inequalities rather than 'the construction of the relay itself' (Bernstein, 1996, p.19). He argued that these studies failed to focus on the internal logic of the discourse itself. He wanted to explicate the inner logic of the discourse and its practice. Specialization of voice refers to the way in which 'subjectivity is created through the socialization of individuals into categories of agents, knowledge and contexts that are distinguished (Hoadley, 2006)by the particularity of their voice' (Bernstein B. , 1971). Hoadley (2006) summarizes Bernstein's purpose of schooling as inducting learners into school ways of organizing experience and making meaning. This entails transmission and acquisition of context independent meanings. In order to describe how this happens, the code theory was developed, and the realizations of the elaborated code in institutionalized form were further conceptualized (Christie, 1999, p.3). The specializing of consciousness happens through two key mechanisms which are at the heart of Bernstein's theory: classification and framing, which refer, respectively, to power and control with the latter being adopted in this study. While classification operates at the macro level and is related to the social division of labour, framing refers to social relations within this social division. That is, according to Hoadley (2006) 'specific social relations in production/reproduction generate particular practices which we can talk about in terms of framing, or control relations'. Framing, therefore, refers to relations within (within boundaries). The concept of the frame "determines the structure of the message system" and refers to the "specific pedagogical relationship of teacher and taught" (Bernstein, 1971, p. 205). It is through interaction (framing) that boundaries between discourses, spaces and subjects are defined, maintained and changed.

Hoadley (2006) notes that at the micro level of pedagogic practice, framing refers to the location of control over the rules of communication. 'Framing refers to the degree of control teacher and pupil possess over the selection, sequencing, pacing and evaluation of the knowledge transmitted and received in the pedagogical relationship' (Bernstein 1975, p.88). In relation to pedagogy, framing has to do with the way in which the relationship between the teacher and the learner is set up, where strong framing refers to a limited degree of options for students, and weak framing implies more 'apparent' control by learners. Again, framing is expressed in terms of its strength or degree of control. According to Bernstein (1971; 2000) where framing is strong, there is a sharp boundary between what may be and may not be transmitted and the transmitter has explicit control over selection, sequencing, pacing, criteria and social base. Where framing is weak, there is a blurred boundary between what may be and may not be transmitted and the acquirer has more apparent control over the communication and its social base. It is important also to note that evaluation is a function of the strength of classification and framing, yet the strengths of the classification and framing can vary independently of each other (Bernstein, 1971) Strong framing argues, Hoadley (2006) would imply that students have limited control over the 'relations within' and a limited degree of control over the sequencing, pacing, selection and evaluation of the knowledge transmitted. In relation to framing Bernstein asserts that ‘control is double faced for it carries both the power of reproduction and the potential for its change’ (Bernstein, 1996, p.19).

Bernstein in his later work calls the regulative discourses or social order rules and these establish the conditions for conduct, character and manner of the school (Bernstein, 1975) or in the pedagogical relation (Bernstein, 2000; 2003). The regulative discourse also refers to the “forms of hierarchical relations in the pedagogic relation” and this can lead to the creation of either explicit hierarchical or implicit hierarchical relationships (Bernstein, 2000, p. 13; 2003). The instrumental order closely relates to the instructional discourse or discursive rules and both are concerned with how knowledge is transmitted and acquired (Bernstein, 1975), in fact it refers to the selection, sequence, pacing and criteria of knowledge (Bernstein, 2000; 2003). The regulative, social orders rules and the instructional discursive rules are a function and elements of framing with Bernstein defining framing as follows:

$$\text{Framing} = \frac{\text{instructional discourse ID}}{\text{regulative discourse RD}}$$

Bernstein (2000, p. 13) distinguishes between the instructional and the regulative discourse, with the former being “always embedded in the regulative discourse” and the latter being the “dominant discourse”. It is important to note that the strength of the instructional and regulative discourses and also the elements of the instructional discourse can vary independently of each other (Bernstein, 2000).

Research Design

One foundation phase teachers, Ntuli who teach Grade 2, from one of the ten schools participating in the WMC-P make up the sample of this study. Ntuli was selected based on the fact during our base-line data collection she was teaching a number focused topic in content and had expressed her willingness to participate in the study. By analysing her teaching of number we were able to see deeply how she facilitated the learning and teaching of number in her class, through analysing relative presences in the data drawn from her lesson episodes.

The data set worked consisted of video –recordings of a non-consecutive lessons (focused on number work) presented towards the beginning of 2012.Vidoe data were later transcribed to capture all teacher talk and teacher-learner interaction, writing on the board and learners' activities. A lesson overview was constructed wherein episodes were demarcated by tasks, activities and representations used within them. Teaching format changes demarcated episodes while tasks were determined by what was presented by the teacher as the focus of attention. Following, Mason & Johnstone-Wilder's (2006) distinction between task and activity, the activity outlined described what happened in the enactment of tasks. Data from the lesson overview were then analysed using the analytical framework formulated literature and the conceptual framework-framing from Bernstein.

Research findings

Episode 1: Counting

In Ntuli's video recorded lesson she starts by asking learners to count from 1 up to 100. She gives asks learners to refer to the 100 chart which she put up on the board. Whole class learners count in chorus. The teacher appears not to follow learners' counting as she get preoccupied with distributing worksheets. Learners are requested to strictly start counting from 1. Research shows that unitary counting in ones starts as a list of numbers that can be recited in a particular order and once children are confident to start at one they can be encouraged to start at any number (Anghileri, 1997). Counting that starts , for example at the number seven is more demanding than starting at one as it is rather like trying to complete the lines of a song or poem but starting in the middle. By asking the learners to always start counting from the number '1' the teacher missed an opportunity develop 'flexibility' in counting in children. Here Ntuli's choice of forward and backward oral count was thus a good choice. However, how Ntuli instructed the counting to be done did not develop mathematical proficiency and flexibility in learners (Anghileri, 1997). Thompson (1997) questions whether there is transfer of learning from such activities to situations involving numbers. Counting beyond ten will initially rely on memory as the number names between ten and 20 do not follow a clear pattern (Anghileri, 1997). Tapson (1995) notes that children need images that can be associated with the counting sequence beyond ten because 'teaching entails using representations' (Ball et al., 2005, p20). Fuson et al., (1997) describes there correct conceptions' of numbers used by children.

- Unitary conception, related to counting in ones up to 32;
- Sequencing conceptions, related to counting by tens and then by ones-10, 20, 30, 31, 32; and
- Separate tens and ones- 1 ten, 2 tens, 3 tens and 1 unit, 2 units.

Images associated with sequence conception include the number line or 'jumps on the empty number line' Beushuizen, 1999; Menne, 2001). In the lesson the teacher asked learners to count in ones only backwards and forwards, neither in tens, then by ones or separating tens and ones. Where number experiences involve a variety of situations and contexts, children will begin to develop a 'feel' for numbers that will be the foundation for calculating strategies and for using numbers effectively for problem – solving (Anghileri, 1997). Bernstein (2000) argued that successful learning depends to a great extent on the weak framing of pacing- that is, on conditions where children have some control over the time of their acquisition. Weak pacing is one of the characteristics that directly or indirectly allow the explicating of evaluation criteria. Morais (2002) argues that there are many ways of to make evaluation criteria explicit at the level of the classroom. Students can be led to produce the text legitimized by the school in both the transmission and evaluation context. As Bernstein (1990) says, there are criteria that students are expected to acquire and apply to their own practices and those of others. Criteria make the acquirer capable of understanding what is considered a legitimate or illegitimate position. We can therefore see that, since understanding of evaluation criteria contributes to the production of legitimate text, their acquisition is a factor that influences students' differential achievement. Literature notes the need for counting forwards and backwards in 1s, 2s, 3s, and 5s those learners develops an understanding of problems that assist in early addition and subtraction (Anghileri, 2006).

Episode 2: Addition: Word problem

Ntuli wrote up a word problem on the chalkboard and asked the learners to solve it:

> "Sipho has 9 sweets. Nomsa has 4 sweets. Mary has 3 sweets. How many sweets are there altogether?

Ntuli asked the learners to use an abacus and told the learners to count out '9', then ''4' and '3'. Learners counted form '1' up to '16'. Ntuli asked the learners the answer and the majority of learners simply chorused the number '16'. Literature suggest that if children are to appreciate number work as a pursuit that makes sense, in addition to having appropriate images , they must be clear about the reasons for calculating, Anghileri (2001a) suggest two distinct purposes for calculating" for solving individual problems and for exploring the structure of the number system. Ntuli seems to leading her learners towards a 'convergent' methodology where the main aim is to find and interpret the solution to a particular problem. We can infer that learners are not being exposed to the more analytical and 'divergent' approaches that are used in calculating for exploring structure, where the purpose is to focus on many possible approaches to a calculation in order to highlight the mathematical relationships and processes that can be involved. It appears Ntuli's focus was interpreting the information in the given problem, modelling the situation arithmetically, and applying the result to a real solution of the problem. However, literature seems to suggest that in addition to the solution of a particular problem, experiences for calculating for exploring structure may involve many different ways to tackle a single calculation and discussion to assess the effectiveness of different approaches (Anghileri, 1997). It is not clear how the learners got the correct solution. A possible explanation could be that learners used mental strategies for counting, if it is the case, these mental strategies were not explored and built on by Ntuli. Counting on and counting back could have been used by the learners but these need to be made more efficient by introducing' chunks' that relate counting to established number facts. The following example illustrates responses that could be elicited from children to explain the answers to mental calculations:

[9 + 4 = 13] '10, 11, 12, 13'

[13 + 3 = 12 + 4] 12 and 4 is 16.

There can be difficulties for teachers were where much of the calculating is done orally or little recording takes place. Brown (1997) reports that although she encourages mental mathematics in her classrooms, she admits finding it problematic with respect to 'assessment and some kind of monitoring of progresses. Where e there are no records of the mental strategies children are using, discussion becomes even more important as a way to monitor the children's thinking. Recording some of the ideas will be necessary for clear communication and this can be the focus of follow-up work, rather than the driving force in the calculation processes. There is research evidence to show that very young pupils can find solutions to calculations even if they lack sophisticated mathematical techniques, suggesting that the mathematics is developed out of problem-solving rather than learned separately and then applied (Hughes, 1986; Beushuizen, 1995).

Episode 3: Subtraction

After the addition task Ntuli announced that they were now going to work with 'minus', and asked for two numbers that can be taken away to make 16 and reiterates that there are many ways to make 16. The following sums were offered: 17- 1= 16 and writes on the board alongside the column of addition sums from the last task. '18- 2', is offered, and Ntuli asks class to verify answers offered by individuals by making on their abacus and counting. '20- 4' is offered; '19-3' offered and dealt with the same way. Literature notes that associated with every addition fact are two subtraction facts that are immediately available without the need for any calculation (Anghileri, 1997). If '9 and 7' together make 16' this also means that '16 is 7' more than 9', or that ''16-subtrcat 7' is 9' and '16 subtract 9' is 7'. Children who achieve success in addition and subtraction do so but recognizing that what is required the third number in such a number triple. Ntuli did not make explicit reference to the way number triples can be used and the way new facts can be derived from those that already known. Carpenter et al., (1999) states that teachers can help their learners develop understanding by introducing a wide range semantic structures and make these the basis of their class discussion. However, what Ntuli seemed to have been concerned with was the arithmetic calculations. We provide the diagram below to illustrate what can be done to identify number triples and explore different ways of to record the findings diagrammatically and with numbers.

9	7
16	

Figure 1: Diagram showing number triples.

This is powerful strategy and marks early differences between children who ‘just know the answer’ and those who continue to count. This finding is rather disappointing in that the teacher concentrated on calculating the algorithms only without offering learners opportunities to make links between addition and subtraction. Literature within the educational landscape points to the importance of connections being established between different representations forms and between different mathematical ideas and facets of the mathematics curriculum in the teaching and learning of early number (Askew et al., 1997; Haylock & Cockburn, 2008).

Subtracting can involve counting backwards and these needs to be practiced by asking particular number of steps which will also require children to maintain two counts at the same time. To calculate 8-3 involves starting at eight and counting back three steps: 8-7, 6, 5). this time one of the counts is backwards (7, 6, 5) while the other is a mental count5 forwards (1, 2, 3) to keep track.

FINAL CONSIDERATIONS

Do these findings suggest any possibilities for seeing improvement in the teaching of numeracy- a context dominated by strong framing of selection, sequencing, and pacing of valid mathematical knowledge? This study has demonstrated how Ntuli framed number content strongly resulting in limiting possibilities for epimistic access to number learning. It can therefore be assumed that Ntuli’s lesson did not assist learners to make meaning of number. Literature notes that specific aspects of pedagogic discourse are favourable to the development of the elaborated coding orientation to require by the school (Fontinhas et al., 1995).

It is the structuring of the message system in this case principles of teaching number that rendered the lesson of not much meaning to learners. The study by Morais (2002) stresses “explicating the evaluative criteria as the most crucial aspect of a pedagogic practice to promote higher levels of learning of all students” (p.568). Bernstein (1990; 2000) repeatedly argued that successful learning depends to a great extent on the weak framing of pacing- that is, on conditions where children have some control over the time of their acquisition.-Students from lower social class backgrounds are more likely to benefit from the explication of the evaluative criteria, and weak framing over the expected rate their acquisition, to promote epistemic access (Hoadley, 2006). Also as, Bernstein (1990) said, there are criteria which the student is expected to acquire and apply to his/her own practices and to others. We can therefore admit that because the understanding of the evaluation criteria contributes to the production of the legitimate text, acquisition is a factor which contributes to students' differential achievement. This confirms what has been characterized by Chick (1996) as "… teachers adopting authoritarian roles and doing most of the talking, with few pupil initiations, and with most of the pupil responses taking the form of group chorusing" (p.21).

The studies carried out so far point to explicating the evaluation criteria as the most crucial aspect of a pedagogic practice to promote higher levels of learning of all students. The studies have also shown the importance of other characteristics, such as weak framing of pacing. Hoadley (2012) in a study looking at working-class classrooms found that learning occurs at an extremely slow pace in poorer schools and was also undifferentiated. Slow pacing is extremely detrimental to working class children. Time wastage and slow pacing in poor schools is thus even more problematic given that the amount of time allocated to the task of enhancing these children's educational outcomes is already too little. Framing of selection and sequence at the macro level should be strong, although they are weak at the micro level, and both should be weakened as students acquire the recognition and realization rules for specific school contexts. The framing that characterize student relations should be weak.

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LEARNERS' MISCONCEPTIONS IN DEDUCTIVE GEOMETRY PROOFS AND REMEDIAL STRATEGIES

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The study sought to establish and describe the exiting Grade 11 mathematics learners' knowledge of and misconceptions in geometry proof and proffer remedial strategies to turn misconceptions into teaching opportunities. A content analysis of learners' responses to a circle geometry item in a National Senior Certificate Grade 11 mathematics paper 2 of November 2013 was conducted. The achievement test was taken by a cohort of 175 learners enrolled at a public secondary school in the Vhembe District of South Africa. Of these, 97 (55.4%) were girls and 78 (44.6%) were boys. Descriptive statistics was carried out on the quantitative data from the scoring rubric using SPSS version 22. Percentages, means, standard deviations, minimum and maximum of scores were used to indicate overall learners' performance and the disaggregation of scores by gender and degree of success in solving the proof problem. Collection of incorrect or partially correct proofs were analysed qualitatively to identify patterns of learners' misconceptions in the deductive geometry proof item. Only 1.7% of the learners performed well in the deductive proof item, while 98.3% evidenced misunderstandings or misconceptions which varied in complexity, suggesting proof development continues to be a problematic area for learners.

Keywords: deductive proof, deductive reasoning, inductive reasoning, misconceptions, proof

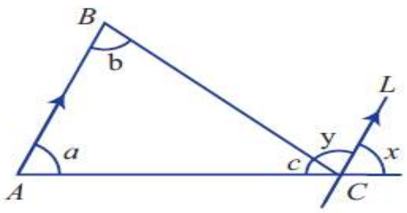
INTRODUCTION AND BACKGROUND

The role geometry plays in real life makes it a core component of mathematics that learners must understand and master (Luneta, 2015). For example, geometrical concepts such as triangles, circles, rectangles, lines, squares, areas and perimeters are used as traffic road signs meant to give control, information and danger warning signs (Siyepu & Mtonjeni, 2014). In South Africa geometry forms an important component of school mathematics curricula and its importance has been emphasised in the Curriculum and Assessment Policy Statement (CAPS). Also, from 2008 to 2013 geometry has been excluded from the core mathematics syllabus for Grade 12 and has been an optional topic tested in an optional paper (Mathematics Paper 3). Jansen and Dardagan (2014) concede that the issue of optionalising geometry was a way of marginalising South African learners from the development of advanced understanding of mathematics. Prior to the reintroduction of Geometry into the grade 11 core mathematics syllabus in 2013, it was a problem area to teach and learn in South Africa (Siyepu, 2012). Between 2009 and 2013, the majority of the students that opted for Mathematics Paper 3 did not do well in the examination (Department of Education, 2013). This suggest that the exclusion of geometry from the core mathematics syllabus for Grade 12 implied that most teachers were not qualified to teach Geometry. This is corroborated by Van Niekerk (1997, p. 112) who attributes the failure to teach geometry in South African schools to the fact that "... the majority of mathematics teachers are poorly trained".

The 2006 Trends in Mathematics and Science Study (TIMSS) report notes that mathematics teachers in South Africa were among the most frequently in-serviced (Reddy, 2006). This could be an indication of how low subject matter knowledge is among teachers of mathematics, which can have deleterious consequences on teachers' pedagogical content knowledge. Hence, teachers' knowledge of learners' misconceptions in deductive proof should go a long way in equipping them for effective delivery and guidance of proof activities in their classroom. Van der Sandt (2007, p. 2) concedes that in South Africa geometry is regarded as a 'problematic topic' at secondary school level. This implies that geometry education requires urgent attention now that it has been reintroduced into the core mathematics syllabus. Therefore, investigating the proportion of learners failing to solve a geometric proof problem and the misconceptions learners have in geometry proof item, a contribution is made towards the teaching and learning of Geometry in secondary schools.

The learning of proof has been a major goal of mathematics curricula in many countries and for many generations (Otton, 2007). The two reasons for teaching proofs are to teach deductive reasoning as part of human culture, and to serve as a vehicle for verifying and showing the universality of mathematical statements (Gfeller, 2010). As such, "... proofs help learners to develop logical or critical thinking skills that are useful beyond the mathematics classroom" (Dickerson & Doerr, 2008, p. 408). The geometrical proofs learners learn in high school can prepare them for higher education studies in the Science, Technology, Engineering, and Mathematics (STEM) careers. The axiomatic structure of Geometry requires teaching for understanding through the sequential process of exploration, inductive and deductive reasoning (Connolly, 2010). For example, in an exploratory lesson on the triangle sum property, learners would generate several different triangles, record each of the angle measurements and sum them in a table and notice a pattern. This would lead to a conjecture that all the angles in a triangle sum to 180° (inductive reasoning). This inductive reasoning process does not represent a valid proof. However, it suggests a possible mathematical truth worthy of further investigation. De Villiers (1998) propose that deductive proof is valuable to explain why the observed property hold true for all cases. The deductive proof is a step-by-step process of drawing conclusions based on previously known facts. Therefore, to prove that the angles inside any triangle sum up to 180° depends on the following known facts:

> If two parallel lines are cut by a transversal line, then alternate interior angles (alt. int.∠s) are equal and corresponding angles (corr.∠s) are equal. Angles on a straight line add up to180° (∠s on a line). The deductive proof might go something like this:



Given: ΔABC.
To prove: $a + b + c = 180°$.

Construction: Draw line L through C parallel to $\overline{AB}$.
Mark angles x and y as shown.

Proof.	$a = x$	corr. ∠s, $L // \overline{AB}$
	$b = y$	alt. int. ∠s, $L // \overline{AB}$
	$c + x + y = 180°$	∠s on a line
∴	$a + b + c = 180°$.	

Although deductive proof seems rather simple, it can go wrong in more than one way. The premises (known facts) used in deductive proof are the most important part of the entire process of deductive proof (Simon, 1996). If they are incorrect, the foundation of the whole line of reasoning is faulty, and nothing can be reliably concluded. Deductive proof is effective when all of the premises are true, and each step in the process follows logically from the previous step (Simon, 1996). Therefore, investigating the exiting Grade 11 mathematics learners' knowledge of and misconceptions in geometry proof and proffer remedial strategies, a contribution can be made to the improving of the teaching and learning of geometry proofs in secondary schools. Significantly, such information can inform teachers as to the instructional support in deductive proof that learners need, as they engage with geometry proof in Grade 12. The findings are also crucial to the Grade 11 teachers as the study delineates common conceptual and procedural misconceptions in geometry proof that they look out for when teaching the topic.

PURPOSE OF THE STUDY

The purpose of the study was to establish and describe the nature of Grade 11 mathematics learners' misconceptions in deductive geometry proofs and proffer remedial strategies to turn the misconceptions into teaching and learning opportunities.

RESEARCH QUESTIONS

The study responded to the following research questions:

1. What is the proportion of learners failing to solve a deductive geometric proof problem?
2. What misconceptions do learners have in deductive geometry proofs?
3. How can these misconceptions be turned into teaching and learning opportunities?

SIGNIFICANCE OF THE STUDY

Investigating the nature of learners' misconceptions in deductive geometry proofs, a contribution can be made to the improving of the teaching and learning of deductive geometry proofs in secondary schools.

THEORETICAL BACKGROUND

Ding and Jones (2006) support Piaget (1971) that children's geometrical understanding develops with age and that for children to create ideas about shapes they need physical interaction with objects. Van Hiele (1986, 1999) on the other hand tried to analyse the various aspects involved in the learning of geometry and space. Van Hiele's (1986) theory of geometric thought describes five different levels of understanding through which a learner progresses when learning geometry. The basis of the theory is the idea that a learner's growth in geometry takes place in terms of distinguishable levels of thinking. In the first level (visualisation and recognition), learners can identify a shape, but are not able to provide its properties. The shape is judged only by its appearance. The second level (analysis) is descriptive: learners are able to identify particular properties of shapes, but not in a logical order. The third level (abstraction and relationships) is informal and deductive: learners can combine shapes and their properties to provide a precise definition as well as relate the shape to other shapes. The fourth level is formally deductive: learners apply formal deductive arguments such as in proofs. The fifth level (rigour and axiomatics) is characterised by formal reasoning about mathematical systems by manipulating geometric statements such as axioms, definitions, and theorems. Van Hiele's levels provide teachers with a framework within which to plan geometric activities (Lim, 2011). Besides, the levels are also a good predictor of learners' current and future performance in Geometry (Lim, 2011). As such, Van Hiele's levels of geometrical thought were the guiding principles for studying exiting Grade 11 learners' knowledge of geometry proof and for determining the level at which the learners in the sample operated.

METHODOLOGY

Design

The study sought to establish and describe the exiting Grade 11 mathematics learners' knowledge of geometry proof, determine the nature of misconceptions in geometry proof and proffer remedial strategies to turn around the misconceptions into teaching opportunities. For that reason, an explanatory sequential mixed methods design was used in this study (Creswell, 2014). This design is a mixed methods strategy that involved collecting quantitative data, analysing results, and then using the results to inform the sampling procedure for qualitative data collection (Creswell, 2014). Collection of quantitative data for investigating the degree of success of learners in solving the geometric proof problem and qualitative data for the analysis of learners' misconceptions provided an understanding of the Grade 11 learners' challenges in geometry proof.

Participants

Participants in the achievement test were 175 Grade 11 mathematics learners enrolled at a public secondary school in the Vhembe District of South Africa. Of these, 97 (55.4%) were girls and 78 (44.6%) were boys. Participants were learners between the ages of 15-22 years whose language of instruction was English as a second or foreign language.

Instrument

Table 2: Scoring rubric used for the analysis of learners' responses.

(Adapted from Ndlovu & Mji, 2012)

Mark allocated	Description of performance
0	Lack of basic geometrical knowledge and vocabulary or lack of appropriate geometrical frame of reference.
1	Recognition of some helpful facts but inability to make a logical deduction.
2	Ability to notice helpful facts and make an inference, but inability to organise information in coherent chain of arguments from givens to conclusions.
3	Ability to notice helpful facts and make some inferences, but inability to be economic or precise, excess facts and/or imprecise labelling used leading to circuitous or clouded chains of argumentation.
4	Ability to notice helpful facts, make inferences and coherent chain of arguments from givens to conclusions efficiently.

DATA COLLECTION

Using the scoring rubric (Table 2), a quantitative content analysis of responses to the following circle geometry item (Figure 1) in a National Senior Certificate Grade 11 mathematics paper 2 of November 2013 was conducted.

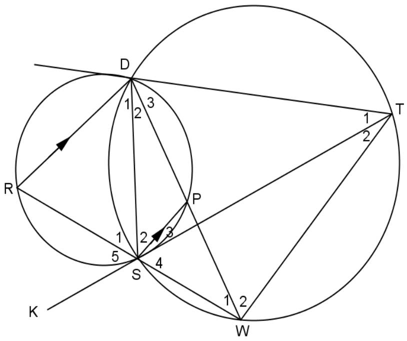


Figure 1

Figure 1: $SWTD$is a cyclic quadrilateral; $KST$is a straight line and; $TS$is a tangent to circle$RSPD$. Prove that$TW \parallel PS$. (In Figure 1, some angles are numbered to make alternative labelling possible, e.g. $D_1 = R\hat{D}S$).

Also, a qualitative content analysis of 50 proofs that were incorrect or partially correct was done to identify a particular pattern of responses. Those proofs that did not fall in any pattern of responses were discarded. From that collection examples were selected for discussion. The test item was selected in order to theorise about the learner's level of geometric thought development for purposes of suggesting remedial strategies.

DATA ANALYSIS

Descriptive statistics was carried out on the quantitative data from the scoring rubric (Gall, Gall, & Borg, 2007). These data were analysed with the help of SPSS version 22. Percentages, means, standard deviations, minimum and maximum of scores were used to indicate overall learners' performance and the disaggregation of scores by gender and degree of success in solving the proof problem. Data were illustrated using tables in order to show the key features of the data in a more interpretable manner (Johnson & Christensen, 2008). The collection of 50 incorrect or partially correct proofs was analysed qualitatively and categories of misconceptions were developed inductively out of the learners' responses (Mayring, 2014).

ETHICAL ISSUES

Permission to carry out the study at the selected site was not necessary since the author was the Grade 11 mathematics teacher at the time of data collection. During the process of data collection and processing anonymity and confidentiality were observed. Also insincerity and manipulation were guarded against.

RESULTS AND DISCUSSION

Quantitative results

There were more girl learners (55.4%) than boys. However, this difference was not statistically significant at $p < 0.05$ because the Chi -square value was 1.077 for 1 degree of freedom. In addition, Table 3 shows that the mean score for girls was higher than that for boys. However, this difference was not statistically significant. The overall standard deviations indicate greater dispersion among scores for boys than those for girls.

Table 3: Overall learners' performance.

Subject	Mean score	Standard deviation	Minimum	Maximum	Total	%
Girls	1.577	0.797	0	4	97	55.4
Boys	1.551	0.872	0	4	78	44.6
Overall	1.564	0.389	0	4	175	100

Table 4 shows a further disaggregation of scores by gender and degree of success in solving the deductive geometric proof problem.

Table 4: Performance analysis by score and gender.

Score	Girls	%	Boys	%	Total no. of learners	%
4	1	1.0	2	2.6	3	1.7*
3	4	4.1	3	3.8	7	4
2	59	60.8	42	53.8	101	57.7
1	19	19.6	20	25.6	39	22.3
0	14	14.5	11	14.2	25	14.3
Total	97	100	78	100	175	100

From Table 4, only 1.7% of learners evinced the ability to notice helpful facts, make inferences and coherent chain of arguments from givens to conclusions efficiently. These 3 learners (1.7%) can be classified as operating at Van Hiele's level 4 of geometry thinking. 98.3% demonstrated misunderstandings or misconceptions which varied in complexity, indicating that the performance of learners in geometry proof item was low. This suggests that 172 learners are possibly operating at Van Hiele's level 3 or less of geometry thinking. As a result, proof development continues to be a problematic area for learners as evidenced by this study. Suggestions are that: learners have to acquire a body of geometric content knowledge and; the activation and the utilization of this knowledge during the construction of proof need to be guided by general problem-solving and geometry reasoning skills.

Therefore, geometry teaching needs to consider the interactive role of the three knowledge components (geometry content knowledge, general problem-solving skills and geometry reasoning skills). This will help learners develop higher levels of competency in the development of geometry proofs. However, the design and incorporation of such knowledge components into a learning support environment are important issues for future research.

Qualitative results

Three categories of misconceptions were identified in the investigation. Two-column proofs, one giving the step-by-step of the proof and the other providing the reason, was evident in learners' layout of the proofs as is eminent in the following examples.

A misconception that 'writing known concepts/theorems and properties of given figures equals a proof'

This misconception reveals lack of knowledge of what a proof of a mathematical statement entails. This leads to the inability to choose relevant concepts and properties that explain how that statement follows logically from them.

Type I misconception:

Learner 1 (Opa)'s response.

Given: (1) $SWTD$ is a cyclic quadrilateral

(2) TS is tangent to circle $RSPD$

RTP: $TW \parallel PS$

Statement	Reason	Comment
L_1: $D_2 = T_2$	∠s subtended by chord SW	(True)
L_2: TS is a tangent	Given	(Restating a given)
L_3: $\angle PST = D_2$	Tan-chord theorem	(True)
L_4: $\angle DSP = D_1$	Alternate $\angle s$	(True but irrelevant)
L_5: $\therefore TW \parallel PS$		(No reason given)

(**NB:** L_n refers to Line n of the proof and pseudo names are used in this article).

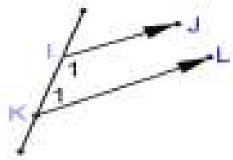
Analysis of Opa's proof

Opa's solution evidenced serious lack of knowledge of what a proof of a mathematical statement entails. She could accurately identify the equality of angles in the same segment (L_1) and the tan-chord theorem (L_3). This implies that she effectively put to use the given facts that $TS$is a tangent to circle $RSPD$ and that $SWTD$ is a cyclic quadrilateral. Although the learner could not combine these facts towards the conclusion, there was evidence of knowledge of theory (properties, definitions, theorems). The ability to make conclusion was therefore evidently absent. This is so because she did not know that for her to conclude that $TW \parallel PS$, she needed to either show that $\angle PST = T_2$ or$\angle PSW + (W_1 + W_2) = 180°$ ($alt\ \angle s\ equal$or$co-int\ \angle s\ supp$, respectively). So combining L_1andL_3, it can be seen that $\angle PST = T_2$. Failure to know where to start and end with the proof can be attributed to the lack of teaching emphasis on the meaning of proof (Gagatsis & Demetriadou, 2001). Opa's level of response was/could be classified as Van Hiele's Level 2 (Van Hiele, 1986), signifying a set of relevant facts known by the individual.

Remedial strategy of Opa's proof

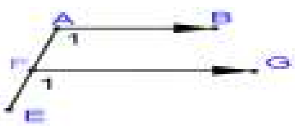
The seemingly lack of knowledge of what a proof in the geometry of the circle, parallel lines or a combination of both entails calls attention to the need to emphasis on *what to show* when proving [for example] that: a quadrilateral is cyclic; a line is tangent to a circle; two lines are parallel; two line segments are equal in order to help learners like Opa. This initial step (*what to show*) is frequently overlooked as a starting point to build learners' prior knowledge on how to do geometry proofs. The "what to show" when proving that two line segments are parallel is:

If $\hat{I}_1 + \hat{K}_1 = 180°$,**then** $IJ \parallel KL$ ($co - int\ \angle s\ supp$) [**U** pattern]



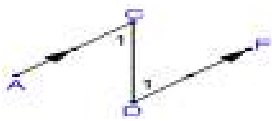
<u>OR</u>

If $\hat{A}_1 = \hat{F}_1$, **then** $AB \parallel FG$($corr\angle s\ equal$) [**F** pattern]



<u>OR</u>

If $\hat{C}_1 = \hat{D}_1$, **then** $AC \parallel DF$, ($alt\angle s\ equal$) [**Z** pattern]



Any of these three conditions constitutes the "what to show" when proving that two line segments are parallel. This suggests that remedying similarly affected learners would mean familiarising them with the "what to show" before the "how to show" and the "conclusion".

A misconception that 'a set of properties or relationships observed true in the figure represents a proof'

This misconception indicates an obstacle to building up proofs and, consequently, to learning to prove.

Type II misconception:

Learner 2 (Sofia)'s response.

Given: (1) $SWTD$ is a cyclic quadrilateral

(2) TS is tangent to circle $RSPD$

RTP: $TW \parallel PS$

Statement	**Reason**	**Comment**
L_1: $SW = SW$	Common side in ΔPSW & ΔTWS	(True but unhelpful fact)
L_2: $W_1 + W_2 + D_2 + D_3 = 180°$	Opp ∠s of a cyclic	(True)
L_3: $\angle DSP + \angle PST + \angle TSW + T_1 + T_2 = 180°$	Opp ∠s of a cyclic	(True)
L_4: $D_3 = \angle TSW$	∠s subtended by chord TW	(True)
L_5: $D_2 = T_2$	∠s subtended by chord SW	(True)
L_6: $D_2 = \angle PST$	(Tan-chord theorem)	(True)
L_7: $\therefore TW \parallel PS$		(No reason given)

Analysis of Sofia's proof

Sofia correctly deduced that, if a side is common to two shapes, then it is the same length (L_1). This was true but unhelpful given the demands of the question. She was also correctly aware that: opposite angles of a cyclic quadrilateral are supplementary (L_2 *and* L_3); angles in the same segment (L_4 *and* L_5) and; tan-chord theorem, but never used these facts as possible points of departure to build up a proof. Being able to identify properties or relationships in the geometric problem without connecting them in deductive arguments does not equal to a proof. Kim & Hannafin (2010) point out that learner difficulties emanating from limited prior knowledge and experience can lead to cognitive overload. The learner is aware of the properties or relationships in the geometric problem but did not know what to do with the facts, which was an obstacle to building up a proof. However, I posit that the learner's failure to connect them in deductive arguments to be a part of such prior knowledge and experience of what entails a proof. In this instance, she did not know that for her to conclude that $TW \parallel PS$, she needed to show that $(W_1 + W_2) + \angle PSW = 180°$ $(co - \text{int} \angle s\ supp)$. From$W_1 + W_2 + D_2 + D_3 = 180°(L_2)$, substitute D_2 with $\angle PST$ (L_6) and D_3 with $\angle TSW$ (L_4), to give$W_1 + W_2 + \angle PST + \angle TSW = 180°$, i.e., $W_1 + W_2 + \angle PSW = 180°$ (since $\angle PSW = \angle PST + \angle TSW$).

Sofia's level of response was also classified as Van Hiele's Level 2 (Van Hiele, 1986), signifying the products of thought are relationships among properties of geometric objects.

Remedial strategy of Sofia' proof

Sofia appeared to be a more redeemable case than Opa in that some of her statements were true. A remedial programme that starts with exploring what can be deduced from the *givens* to identify a collection of choices (frame of reference) could be appropriate in supporting such learners. For example, assuming that TW//PS, questions to generate a collection of choices could be: Which straight lines (transversals) intersect/meet with both parallel lines? What facts do we know or can we deduce about angles formed at the intersection of the parallel lines and the transversal(s)? This would entail refreshing the learners' knowledge bank of properties and relationships between objects/properties (Van Hiele's level 2). Scaffolding existing bank of geometrical knowledge of learners should not be done by conveying a ready-made deductive proof. Instead, effort should be made to provoke sense-making through questioning what can be deduced from the *givens* in order to identify frame of reference.

A misconception that 'in a proof the word given justifies any statement derived from the figure (geometric problem)'

This misconception suggests a poor set of properties related to the geometric problem leading to an inability to make a logical deduction. This lack of awareness prevents the learner from connecting facts in the process of building up proofs.

Type III Misconception:

Learner 3 (John)'s response.

John presented the following as his proof:

Given: (1) $SWTD$ is a cyclic quadrilateral

(2) TS is tangent to circle $RSPD$

RTP: $TW \parallel PS$

Statement	Reason	Comment
L_1: $\therefore TS$is a tangent	Given	(A given stated as a conclusion, odd to start with 'therefore')
L_2: $SWTD$ is a cyclic quad.	Given	(Restating a given)
L_3:$D_2 = T_2$	Given	(True but it is not a given)
L_4: $\therefore D_2D_3$		(True if $SW = TW$: equal chords subtend equal angles)
L_5: $\angle TDS = \angle TWS$	Given	(True if TS was given as a diameter)
L_6: $W_1 + W_2 + D_2 + D_3 = 180°$	Given	(True but it is not a given)
L_7: $\therefore TW \parallel PS$		(Nothing to show how this is arrived at)

Analysis of John's proof

John apparently did not use any of the *givens*. He simply restated them as conclusions (L_1 and L_2) and thus could not develop his proof in a meaningful way. For John, anything he perceives correct in the geometric problem is thus *given* information. No statement is true simply because it appears to be true from a figure. Developing his proof, John concluded that D_2 was equal to D_3 (L_4 which wrongly implied that $SW = TW$ and claimed that $\angle TDS$ was equal to $\angle TWS$ (L_5). The deduction and premise were incorrect. Using these incorrect facts he inferred that line segments TW and PS were parallel (L_7) without showing background knowledge of [for instance] relationship between parallel lines, transversals, and alternate angles. Earlier studies of how learners prove have also stressed the importance of maintaining the connections between proving and knowing (Herbst, 2002a). With these shortcomings John was unable to score any marks. His response could be classified as Van Hiele's Level 1.

Remedial strategy of John's proof

John's imprecise designation of angles implied a weakness in communication skills as an obstacle to effective handling of deductive geometry proof. Learners such as John who seem to have some correct ideas in some instances, but cannot express themselves accurately on paper need to be assisted to gain precision in their references to geometric objects. They need to be encouraged to reflect on the meanings of the symbols they use and to search for unintended interpretations that may arise from them. John's failure to justify statements with correct reasons (for example, L_5) together with the failure to link steps of a proof implied a lack of effective argumentation skills for proof execution. Learners encountering such difficulties need to be encouraged to read their sequence of statements again and again and to critically pay attention to the coherence in their argument – a critical metacognitive skill.

CONCLUSION

The article focused on reporting the degree of success of Grade 11 learners in performing geometry proof and their subsequent misconceptions. The findings of this study revealed that proof development continues to be a problematic area for learners. Suggestions are that teachers' knowledge of learners' misconceptions should go a long way in equipping them for effective delivery and guidance of proof activities in their classroom. As such, the phases of Van Hiele's levels of geometric thinking have to be considered when designing instructional activities. These phases, as adapted from Fuys, Geddes, Lovett and Tischler (1988) and Presmeg (1991) are useful in designing activities in the following manner:

- Information: The learner gets acquainted with the working domain/ field of exploration by using the material presented to him/her, for example, examines examples and non-examples. This process causes him/her to 'discover' a certain structure.
- Guided/Directed Orientation: the learner explores the field of investigation using the material, for example, by folding, measuring, and looking for symmetry.
- Explicitation/Explanation: A learner becomes conscious of the network of relations, tries to express them in words and learns the required technical language for the subject matter, for example, expresses ideas about the properties of figures.
- Free Orientation: The field of investigation/network of relations is still largely unknown at this stage, but the learner is given more complex tasks to find his/her way round this field, for example, a learner might know about the properties of one kind of shape but is required to investigate the properties for a new shape, for example, a kite. The tasks should be designed so that they can be carried out in different ways.
- Integration: A learner summarises all that s/he has learned about the subject, reflects on his/her actions and thus obtains an overview of the whole network/field that has been explored, for example, summarises properties of a figure.

I can conclude that formative assessment, like homework, can be used to locate mistakes, to figure out why they were made and how to provide support to learners by way of explanation and tutoring (Fang, 2010). This approach can help teachers learn some pedagogical lessons from exploring the content of learners' procedural knowledge and understanding (Pedrosa De Jesus, Neri De Souza, & Watts, 2005). In this case it is the procedural knowledge and understanding of deductive geometry proofs vis-à-vis a knowledge base of theory and procedural etiquette in proving. Since proof development continues to be a problematic area for learners as evidenced by this study (98.3% operating at Van Hiele's level 3 or less), the investigation of knowledge components that learners bring to understanding and constructing geometry proofs could provide important insights into the above problematic area (Battista, 2007). Therefore, research has to identify the processes and associated domain knowledge that learners activate and bring to the solution context (Chinnappan et al., 2012).

This article has limitation. As evidence of trustworthiness, this type of study relies on credibility. The findings from this approach are limited by their inattention to the broader meanings present in the data. This is so because learner debriefing was not done to shed more light on the thinking behind their deductive proofs advanced. Nevertheless, the multi-step character of the geometric proof item and the duality of the related concepts required (parallelism and circle theorems) served to illustrate learners' deductive geometry proofs in high school geometry.

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USING INDIGENOUS LANGUAGES IN MATHEMATICS TEACHING: THE CASE OF ISIXHOSA USE IN SOUTH AFRICA'S SECONDARY SCHOOL CLASSROOMS.

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This paper reports on teacher language practices and their implications on the efficacy of isiXhosa as the language of teaching and learning (LOLT) in rural and township secondary school mathematics classrooms in Eastern Cape, South Africa where the majority of learners learn through a second or third language. Data were obtained through observing five lessons and interviewing three grade 11 mathematics teachers purposively selected from three schools in three districts. Results show that teachers predominantly use borrowing strategies which, as this study concludes, still dominated by English mathematics register except for the isiXhosa prefixes attached to the terms. Loan word borrowing (LWB) was commonly practised by all participating teachers. This paper recommends that teachers should use indigenous African languages systematically and transparently, for the conceptual teaching of mathematics to reclaim our African pride.

Keywords: Indigenous languages, IsiXhosa, mathematics, secondary schools, teaching

INTRODUCTION

The teaching of mathematics in South Africa's township and rural secondary school classrooms is conducted using a mix of languages spoken in a given community. This mix of LOLT and indigenous languages occurs through code switching which is an alternate use of two or more languages in a conversation or an utterance (Adler, 2001; Farrugia, 2006). Code switching in this paper refers to all the ways in which teachers orally use South Africa's indigenous languages during the teaching of mathematics.

Code switching practices that incorporate indigenous languages are a feature of many South African classrooms where teachers and learners share a common home language, while the language of learning and teaching is English (Probyn, 2009). The use of an indigenous language by the teacher provides a communicative resource to facilitate learning when students lack proficiency in the LOLT (Zuma & Dempster, 2008). Teaching mathematics concepts in multilingual classrooms using an unfamiliar language presents teaching and learning challenges as students struggle with both the language and the mathematical concepts. To reduce linguistic and conceptual complexity in the learning of the subject, mathematics teachers are forced to employ diverse ways and strategies to assist their students. Inability to understand the language used for instruction or the concept being presented limits learning opportunities and eventually constrains the teaching of mathematics.

The goal of this paper was two-fold: first; was to explore the use of isiXhosa (an indigenous language) by teachers in multilingual mathematics classrooms at secondary school level and, secondly; to ascertain the readiness of isiXhosa to function as LOLT side by side with English in the teaching of mathematics in these classrooms. In pursuing this goal, the study was guided by the following research questions:

- What teacher language practices exist in multilingual secondary mathematics classrooms?
- What do these practices imply on the efficacy of indigenous languages to function as LOLTs at secondary school?

BACKGROUND TO THE STUDY

Language landscape in Eastern Cape Province

The advent of democracy in 1994 in South Africa led to the institution of 11 official languages up from two that had enjoyed official status during the apartheid era. While this was a positive move in making sure that African languages are recognized and used for official purposes, it came with its own challenges. Although all the eleven languages are official and equal under the constitution, their status in education is not equal. IsiXhosa, for instance, is spoken by around 80% of South Africans in the Eastern Cape (SA Statistics, 2012) which implies that it is spoken as a first language by more than three quarters of the population in the Eastern Cape.

English is spoken as a first language by 5.6% of the total population in the province (SA Statistics, 2012). The majority of rural and township schools in the Eastern Cape Province use the first language of the minority, which is English, as the LOLT at secondary school level. Use of an unfamiliar language in a way continues to perpetuate inequalities through education. Owen-Smith (2010) argues that:

> Any child who cannot use the language which he/she is most familiar with (usually the home language), is disadvantaged and unlikely to perform to the best of his/her ability. (p. 31).

The perpetuation of such unfortunate situations in South Africa is a result of a complex interaction of historical, political, economic and social factors. Some of these factors are highlighted in the ensuing sections.

Language policy in education

The new Language-in-Education Policy (LiEP) of 1997 mandates schools to decide on their own language policies in consultation with parents. The LiEP of 1997, in essence, affords all students the right to be taught in the language of their choice. This legislative framework was enacted with an understanding that all indigenous African languages are capable of functioning as LOLT. In most schools in South Africa, the LOLT chosen by schools from Grade 4 is English despite the fact that the majority of learners have little exposure to English outside the classroom. Similar observations were made in the Eastern Cape (Probyn, 2009), KwaZulu-Natal (Zuma & Dempster, 2008), Limpopo (Setati, 2008) and North-West (Carnoy et al., 2011). Based on what Mafela (2009) found, the official language policy of any region and position on the use of indigenous languages through code switching go a long way in setting a context for the acceptance, rejection, systematic use, or lack of use of African languages in educational settings.

The use of indigenous languages as a means of teaching and learning in South African schools is currently a much debated topic. Preference by both parents and learners of English as LOLT, driven by various factors from within and outside school settings (Probyn, 2009), show that English is likely to remain the LOLT at upper primary school, secondary schools and in teacher education institutions. This is so because decisions on language policy in South Africa have had to accommodate many competing historical, political, economic, social and educational factors (Setati, 2008). Consequently, English is likely to remain the chosen LOLT in many South African schools. As such, teachers have developed coping techniques and strategies that incorporate indigenous languages during teaching in order for these teachers to remain functional and reach their intended goals and objectives. Such strategies include code switching in the learners' home language.

Current teacher practices

The teachers' responses to LOLT chosen by the school has been to try and use both pupils' home language and the LOLT through code switching. In the Eastern Cape, most schools, including those identified for this study, have English as their LOLT despite the vast majority of the teachers and learners in these schools being isiXhosa first language speakers. Alidou & Brock-Utne (2011, p. 160) observe that "studies related to language of instruction issues in post-colonial Africa unanimously suggest that the maintenance of languages such as English, French and Portuguese as dominant or exclusive languages of instruction creates teaching and learning problems in African schools." This is mainly because of majority of learners, and to some extent teachers in these communities, are not fluent in the LOLT resulting in communication, teaching and learning problems in mathematics classrooms.

Wildsmith-Cromarty (2012), while acknowledging that teacher language practices such as code switching in science and mathematics classrooms appear to be an already established practice, concludes that:

> The challenge is, therefore, to understand how to harness this code switching practice in a systematic way in order to enhance conceptual development in the mother or primary language. We need further research in order to ascertain whether the type of code switching used by teachers truly builds an academic understanding of a concept, or whether it dilutes scientific meanings through the use of colloquial examples p. 166.

Choice of unfamiliar languages such as English, as LOLT in rural and township schools, has been observed to significantly encourage teaching practices and strategies such as rote learning (Setati & Adler, 2001), Safe talk (Chick, 1996), recall, memorisation and chorus teaching (Alidou & Brock-Utne, 2011). In such classrooms, teachers do most of the talking relying prevalently on traditional teaching strategies.

Some teacher language practises noted by Chikiwa & Schäfer (2014) that exist in secondary school classrooms include transparent code switching (TCS) and borrowing code switching (BCS). Borrowing Code Switching Strategies (BCS) is where a teacher borrows from the English language either by retaining the English spelling or by adapting the phonology of the borrowing language (Baker, 2011). Two forms of BCS are: Transliteration (TLT) and Loan word borrowing (LWB). Teachers' nativisation of existing English language mathematical terms (Farrugia, 2006) that involve giving a home language spelling and pronunciation to English terms (Barton, Fairhall and Trinick, 1995) is referred to as transliteration. Loan word borrowing (LWB) refers to terms borrowed from the English language where the spelling, meaning and pronunciation of the word are retained (Baker, 2011).

Transparent Code Switching Strategies (TCS) refers to all code switching where meaning of terms is not concealed but noticeable, self-evident and transparent to students (Meaney, Trinick & Fairhill, 2012). Four forms were identified by Gauton et al, (2003); Semantic Transfer (SST), Paraphrase (PAR), Compounding (COM) and Ready Translated Equivalent (RTE). These strategies were used to analyse data as is shown in the sections below.

Teacher training

Probyn (2015: p. 220) notes that

> "…in South Africa, although there has been an unofficial drift towards recognizing code switching as a valid classroom strategy, there is little training that guides teachers towards a coherent systematic approach to using both languages in the classroom in ways designed to enhance opportunities to learn."

In other words, while use of indigenous languages through code switching is now a common phenomenon in South Africa's mathematics classrooms (Rose and van Dulm, 2006), teachers are left to make their own day to day decisions as to when, where and how to code switch during teaching.

Wildsmith-Cromarty (2012) noted that most teachers did not acquire their scientific or mathematical knowledge through an African language since such languages were not available as LOLT during their period of training. This then makes it difficult for most teachers to transfer mathematical knowledge from English to an indigenous African language. Pimm (1987) asserts that knowing the mathematics register in one language (for example second language) does not automatically mean that one knows the same mathematics in another language (say one's first language). Essien (2013) also supports the idea that teachers need to be trained to teach in indigenous African languages. Teachers should not be expected to correctly teach or transmit into an African language what they learn in English.

Prediger, Clarkson & Bose (2012) recommend that, because empirical analysis has shown that code switching can enhance teaching and learning, it is important "… to develop and promote teaching strategies that make more purposeful use of code switching and other links between first and second languages." p. 6214. This implies that use of indigenous languages through code switching cannot be left to chance or allowed to occur unchecked. There needs to be deliberate steps towards promotion of systematic and purposeful use of indigenous languages through code switching.

THEORETICAL FRAMEWORK

This study is informed by the socio-cultural theory as envisaged by Vygotsky (1978), particularly the critical role of language and context in classroom communication and cognitive development. The socio-cultural aspects of Vygotsky's theory illuminate the point that learning and development cannot be dissociated from their context. The social environment influences cognition through its "tools", that is, cultural objects, language, and social institutions. According to Vygotsky (1978), people think and perceive things in a way made possible by the vocabulary and phraseology of their language. Concepts that cannot be encoded by learners in their language will not be accessible to these learners, or at best will prove very difficult to understand (Durkin, 1991). Vygotsky's theory enables exploration of use of learners' first languages to teach trigonometry through code switching in the mathematics multilingual classroom.

Concept formation and development, as purported by Vygotsky (1987), should be examined at two levels; the everyday and the scientific levels, as mediated by language. According to Lerman (2014, p. 10) "the process of mediation is through tools and signs, and the signs is the language, and so the language is carrying all the mediation process." In the process of mediating the teaching and learning of mathematics concepts, language is thus very significant. Vygotsky's theoretical and empirical work on scientific and every day concept formation has been used in this study to conceptualise how mathematics teachers in their multilingual classrooms can use language to support students' mental development. The merging of everyday ways of talking and learning, and scientific ways of behaving, results in deep meaningful concept formation (Vygotsky, 1987) since every day concepts are a fundamental foundation to the development and building of scientific concepts. Vygotsky (1987) suggests that the student's home language and his learning of it represents the learning of everyday concepts as it occurs in everyday, non-systematic and unplanned ways.

The framework by Vygotsky assists in understanding how merging the mathematical language, the everyday language and the thoughts that each individual contributes in the learning process presents opportunities for development of the students involved. By using indigenous languages through code switching, teachers are, in a way, providing students with mathematical concepts in the language that they are already familiar with and competent in.

SAMPLE AND RESEARCH PROCESS

This study used a case study approach that enabled the researcher to gain a detailed view of teacher code switching practices manifested during teaching trigonometry in multilingual classrooms. Three grade 11 mathematics teachers from three districts in the Eastern Cape Province of South Africa participated in this study. Each teacher and his/her class constituted a case. Data were obtained through observing five consecutive lessons per each of the three teachers and interviewing these three grade 11 mathematics teachers purposively selected from three schools in three districts of the Eastern Cape Province.

As explained by Denzin and Lincoln (2008), purposive sampling groups participants according to pre-selected criteria relevant to a particular research question. The following criteria was used to select the sample of teachers for this study:

- Mathematics teachers who were fluent in Xhosa and were willing to participate in this study.
- Teachers with at least five years' experience of teaching Mathematics at secondary level and therefore are well experienced. This was to minimize the possibility that their language practices might be due to lack of teaching experience or recognized qualification.
- Teachers who teach at schools where use of indigenous languages is prevalent.

The three teachers were identified as Teacher A, Teacher B and Teacher C. Each teacher was observed for five consecutive lessons in a week teaching trigonometry. Lesson observations were used to identify teacher language practices and frequency of these practices. With the consent of all the Department of Education, school principals of participating schools and the teachers, lessons were video recorded focusing only on the teacher. At the end of each lesson, each teacher was interviewed. The interviews were following up on language practices teachers demonstrated during the lesson. The videos were transcribed and analysed in two stages, first quantitative followed by the qualitative analysis. Trends and patterns that emerged during the quantitative analysis were later followed up during the qualitative data analysis process.

Validity

The degree to which data collected in the research truly measures that which it was intended to measure is a measure of its validity (Creswell, 2009). Multiple sources of evidence that is lesson observations, document analysis and interviews, were used during data collection thereby increasing the validity of the data in this study. Transcriptions were done by an experienced transcriber and were verified by two isiXhosa first language speakers who are English language specialists.

DATA ANALYSIS AND DISCUSSION

Data analysis was also done quantitatively and qualitatively. Themes that emerged during the quantitative analysis were followed up through qualitative analysis process. Discussion focuses on the quantitative analysis first followed by the qualitative. For the purposes of this paper, the following findings will be presented and discussed:

- Quantitative data analysis for indigenous language use strategies
- Qualitative data analysis findings
    - Transliteration
    - Loan word borrowing

Quantitative data analysis for indigenous language use strategies

During quantitative data analysis, two broad patterns emerged and were referred to as borrowing code switching (BCS) (Baker, 2011) and transparent code switching (TCS) (Meaney et al, 2012). In this section each of the teachers' language patterns observed in line with BCS and TCS are presented.

Teacher A

From lesson observations, Teacher A used both BCS and TCS in varying frequencies. BCS was prevalently practiced during her teaching of trigonometry. Teacher A's BCS and TCS strategies were inconsistently practised within and across lessons (see Table 1 and Figure 2).

Table 1: Teacher A's Indigenous Language Use Strategies.

		LESSON					Total	
		1	2	3	4	5		
BCS	TLT	0.8	0	1.3	1.3	1.2	4.5	69.8
	LWB	19.2	3.5	17.7	13.4	11.5	65.3	
TCS	SST	1.9	0.3	2.2	0.5	0.6	5.5	30.2
	PAR	1.2	0	0.3	0	0.1	1.5	
	COM	0.3	0	1.8	0.4	0.4	2.8	
	RTE	7.4	1.4	3.8	4.5	3.2	20.4	
TOTAL		30.7	5.1	27	20.1	17	100	

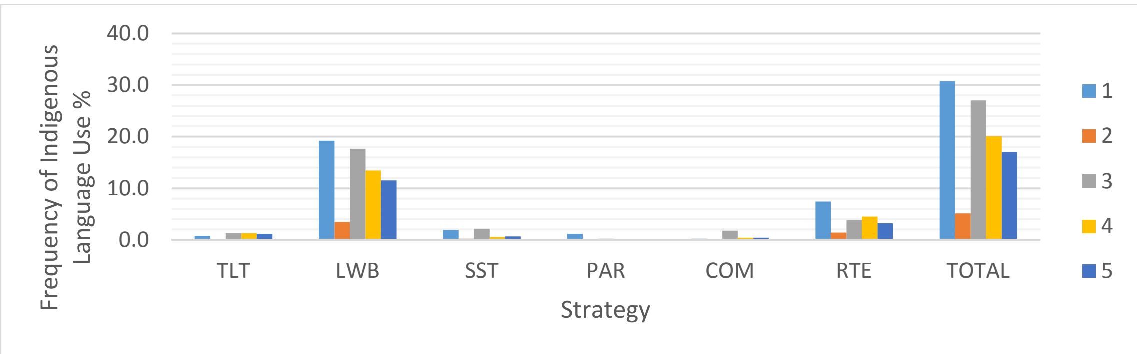

Figure 2: Teacher A's Indigenous Language Use Strategies.

Table 1 shows that in all the five lessons, 65% of the mathematical terms in isiXhosa used by Teacher A in Trigonometry were obtained by loan word borrowing. The use of isiXhosa mathematical terms varied across Lessons 1 to 5 (30.7; 5.1; 27.0; 20.1; 17.0 respectively). In summary, 69.8% of Teacher A's code switched mathematical terms were borrowed and 30.2% were code switched transparently.

Teacher B

Analysis of Teacher B's indigenous language use strategies during lesson observations indicates that borrowing was frequently done during teaching. Table 2 and Figure 3 below illustrate Teacher B's indigenous language use patterns.

Table 2: Teacher B's Indigenous Language Use Strategies.

		LESSON					Total	
		1	2	3	4	5		
BCS	TLT	1.2	0.9	1.6	2.2	2.8	8.7	75.9
	LWB	21.1	13.3	9.9	10.2	12.7	67.2	
TCS	SST	2.2	0.0	0.0	0.0	1.6	3.7	24.1
	PAR	0.6	0.0	0.0	0.0	0.0	0.6	
	COM	0.6	0.0	0.0	0.0	0.3	0.9	
	RTE	5.9	3.7	3.7	2.8	2.8	18.9	
TOTAL		31.6	18	15.2	15.2	20.1	100	

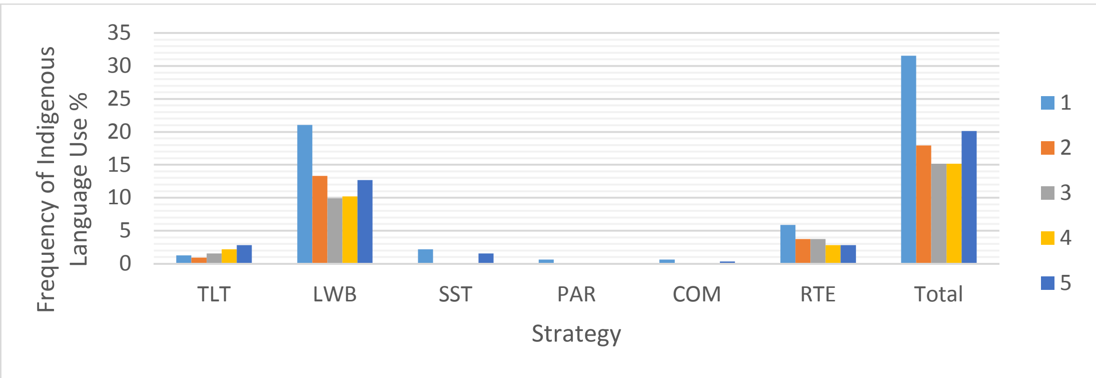

Figure 3: Teacher B's Indigenous Language Use Strategies.

The use of mathematical terms in isiXhosa by Teacher B was inconsistent across lessons as shown in Table 2 and Figure 3. The LWB strategy (67.2%) occurred for most of the time. BCS accounted for 75% of the identified isiXhosa mathematical terms of Teacher B, used during the teaching of trigonometry. 25% of the mathematical terms were through TCS. Teacher B consistently taught using the BCS strategy.

Teacher C

The indigenous language use practices exhibited by Teacher C during teaching trigonometry varied across lessons and strategies. Most of the mathematical talk as shown in Table 3 and Figure 4, was done through borrowing.

Table 3: Teacher C's Indigenous Language Use Strategies.

		LESSON					Total	
		1	2	3	4	5		
BCS	TLT	4.8	1.3	1.1	0.4	3.1	10.8	78.7
	LWB	22.9	7.7	9.5	8.4	19.6	67.9	
TCS	SST	1.8	1.3	1.1	0.9	0	5.1	21.3
	PAR	0	0	0.2	0	0.2	0.4	
	COM	0.9	0	0.9	0.4	0.2	2.4	
	RTE	5.5	2	2.2	1.3	2.4	13.4	
TOTAL		35.8	12.3	14.9	11.4	25.5	100	

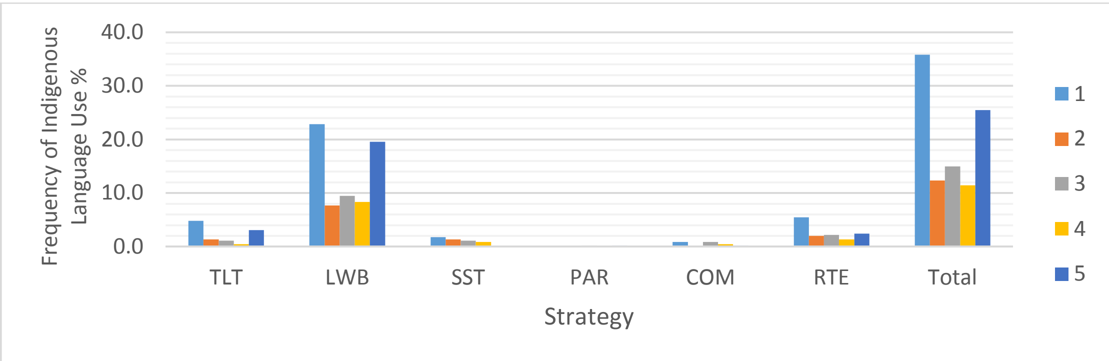

Figure 4: Teacher C's Indigenous Language Use Strategies.

Table 3 shows that 79% of Teacher C's use of mathematical terms in isiXhosa was through borrowing while only 21% was through TCS (see Table 3). For the transparent code switching strategy, RTE was employed 13.4% in the five lessons. Use of isiXhosa mathematical terms varied across the lessons.

Comparison of teacher indigenous language use strategies

Indigenous language use strategies from the observed lessons were compared across teachers to determine any patterns in their language practices. Borrowing was done frequently by all the participating teachers as is shown in Table 4 and Figure 5.

Table 4: Comparison of Teachers' Indigenous Language Use Strategies.

			Teacher A	Teacher B	Teacher C	Total
Strategy	BCS	TLT	4.5	8.7	10.8	78.7
		LWB	65.3	67.2	67.9	
	TCS	SST	5.5	3.7	5.1	21.3
		PAR	1.5	0.6	0.4	
		COM	2.8	0.9	2.4	
		RTE	20.4	18.9	13.4	

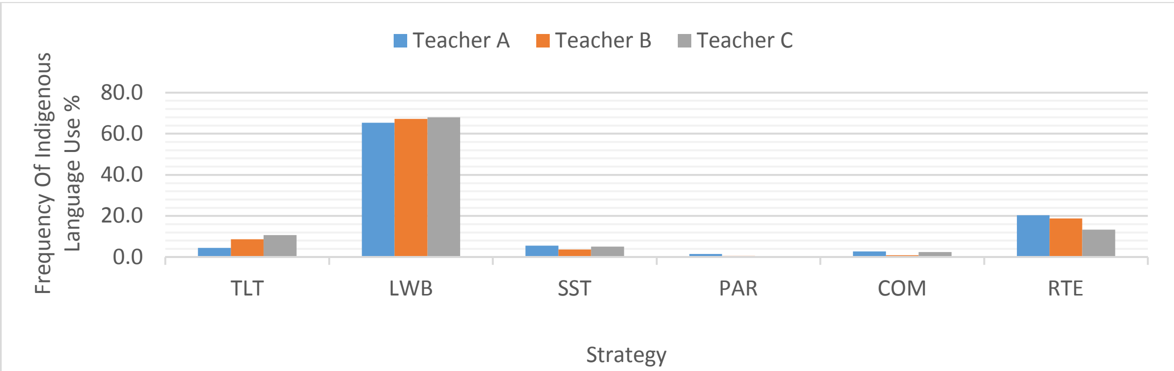

Figure 5: Comparison of Teachers' Indigenous Language Use Strategies.

During the teaching of trigonometry, it is apparent from Figure 5, teachers consistently used the LWB (A-65.3 %; B-67.2 %; C-67.9 %) strategy throughout the five lessons. TLT (A-4.5 %, B-8.7 % and C-10.8 %) was not consistently practiced across these teachers. The greater part of the mathematical talk in isiXhosa was done through borrowing where teachers would attach prefixes to already existing English mathematical terms. All the teachers consistently used the borrowing strategy (A-69.8%; B-75.9%; C-88.7%) throughout the teaching of trigonometry more than their use of the transparent code switching strategy (A-30.5%; B-24.1%; 21.3%). Teacher C borrowed 89% of mathematical terms from English and transparently code switched only 11% of the time. Document analysis revealed that all participating teachers did not have any teaching materials written in isiXhosa. Lack of such materials was attributed to prevalent borrowing by these teachers.

Of the four transparent code switching strategies, RTE had the highest frequencies (A-20.4%, B-18.9 %, C-13.4 %) across teachers. The presence of RTE indicates that while there was high borrowing frequency, there are situations where these teachers used isiXhosa mathematical terms that are readily available and used in everyday language.

Quantitative data from lesson observations presented here shows that use of indigenous languages in observed South African classrooms is dominated by borrowing from English. Very little transparent use of indigenous languages is evident from these lessons. The trends of current teacher strategies when using indigenous languages for mathematical purposes are not beneficial use. Prevalent borrowing, a common feature across all observed teachers presents little or no advantages to the teaching of mathematics. The use of LWB was rampant it just occurred unchecked and unrestrained. In the interviews, Teacher A said "I do not plan or think about how are use isiXhosa because it is my first language, I speak it every day." This uncontrolled and unplanned use resulted in this widespread borrowing through LWB. This is strategy, LWB, which was predominantly used by all participating teachers is nothing more than just adding a prefix to an English term or phrase. This calls for best practices that will assist teachers maximize the use indigenous languages for conceptual teaching.

Transliteration (TLT)

The teachers in this study used this form of borrowing (TLT) significantly during their teaching (see Tables 1 to 4 and Figures 2 to 5). Transliteration was commonly practised by all participating teachers during teaching. Some of the transliterated words observed across all the five lessons of each teacher included verbs, some are processes and some are operations (subtract, add, multiply, square, prove, deduce, identify, simplify, round-off, substitute, relate). Some transliterated terms are taken from purely mathematical domains and are thus esoteric in nature. These include: squaring, deducing, substituting and rounding off to some degree of accuracy.

Teacher A: *Kuba besikade si-calculeta* (we've been calculating the) i-sides, *si*-calculator *i*-sides but now *kengoku sizawu-* (now we are going to) calculator *i*-angles. (Lesson 5).

Teacher A: Ok again so it means *sim-provile* that err c^2 is equal to a^2 plus b^2 minus 2ab Cos C. So *u*-x *wethu sim-provile uba u-* (we have proved that) then *sa*-rearrager (we arranged). (Lesson 3).

Teacher B*: Si-squarishe i-first terms nhe yaba ngubani* (first square the first term, and what is it)? (Lesson 4).

Teacher B*: Uyabona ngoku asikwazi uyo addisha ngapha* (can you see that we cannot go add that side). *Siya-multiplyer kengoku* (we multiply) (Lesson 1).

Teacher C: It means *i*-period *yethu apha ukuze ku-formisheke* (our period then forms) one shape. (Lesson 3).

Teacher C: *Masiyiploteni ngoku* (let's plot it now). (Lesson 5).

While teachers used borrowing to maintain precision in code switching thereby operating in the 'safe code switching mode', their indigenous language use strategies varied. Transliteration was practised even with those words that have ready isiXhosa equivalence. Examples include 'add' (*addisha*), 'multiply' (*multiplaya*), 'calculate' (*masicalculeteni*) and 'draw' (*drawisha*). Teachers would alternate between the English version of the word, the transliterated form and the translated form. This resulted in inconsistent use of these terms. Such a practice, while seemingly risk-free, might cause confusion and impact on the cognitive load of the students.

Transliteration of mathematical expressions and terms into indigenous language was found to be of limited value as this form did not express the mathematical concepts transparently. Its use was regarded as adding excess baggage on to teachers and students since their English equivalents were still required for both continuous and summative assessment purposes. This was deemed so because all formal assessment was done in English only.

Some of the transliterated forms were not consistent across teachers. An example from the above is the transliteration for 'round-off.' Teacher A used *siyi-rounder* while Teacher B used *roundishi***.** In both cases they were referring to rounding off answers to a given degree of accuracy. Such inconsistencies in indigenous language use were considered in this study to be a direct result of lack of planning and absence of best practices. Another example taken across teachers was for the concept of drawing. Teacher C used *masiyiploteni* (lets plot) while Teacher A used *drawisha* (draw) to refer to the same activity of drawing. Transliteration by teachers demonstrated the inadequacy in these teachers' vocabulary to use isiXhosa to teach mathematical concepts. This thus reverberates with arguments by researchers (Chikiwa & Schäfer, 2014; Schäfer, 2010) to develop a mathematics register for all domains of mathematics at secondary school level.

Loan word borrowing (LWB)

In all their teaching, participating teachers in this study presented key mathematical words in LWB form. Some examples in the extracts below include *i-table*, *i-calculation*, *ye-Sine and Cosine*, *kwi-parabola, ne-hyperbola* and *le-function*. Some of the words like 'table' and 'function' are everyday terms being used in a scientific sense. The use of a term like *i-table* was considered to be of less benefit to a student who is struggling to distinguish this 'table' from the everyday one. The same applies to the concept of 'function' which is frequently used in everyday life to refer to non-mathematical activities. Since these terms are key and yet the only isiXhosa terms in some of the teachers' sentences, it thus suggests that teachers do not have appropriate isiXhosa mathematical words for these concepts. This teacher borrowing practice in such instances illustrates the teacher's subconscious quest to make key terms familiar and easier to grasp for his/her students.

Teacher A: *Kuba besikade si-* calculator (we've been calculating the) *i*-sides, *si*-calculator *i*-sides but now *kengoku sizawu-* (now we are going to) calculator *i*-angles. (Lesson 5).

Teacher A: *U*-opposite *no*-adjacent, *yeyiphi lo*-ratio (we are going to use opposite and adjacent, which ratio is that)? (Lesson 1).

Teacher B: So it means *sawufuna i*-ratio *ethini*, *e-involver i*-opposite *nantoni ne* (we will find the ratio that involves the opposite and what else)? There is one ratio e-involver i-opposite ne-hypotenuse *akunjalo* (isn't that so)? There is only one ratio that involves *i*-adjacent ne-hypotenuse. (Lesson 1).

Teacher B: So *masithatheni* (let's take) i-combination where we have one unknown, *yeyiphi* (which) *i*-combination where we have one unknown. *I-unknown yethu iphi ngu*-D *nhe* (what is our unknown, D). (Lesson 3).

Teacher C*: Masithi ke nantsi (*let's say here is the*) i-table yakho entle oyithandayo nhe (*your nice table that you like*). Yi*- (it's a) reflection *kaloku, ubona* (you're looking at the) *i*-reflection. (Lesson 5).

Teacher C: Ndenze *le*-function, tomorrow *sizawu* investigata (we will investigate). (Lesson 4).

With all the three teachers, the words that are strongly mathematical, esoteric and highly technical were the ones mostly code switched through borrowing. Even some everyday terms like 'opposite,' 'adjacent,' 'deduce,' 'combine' which were used for mathematical purposes were used in their borrowed forms. With everyday terms acquiring new and specific mathematical meanings they were thus used in borrowed form. This phenomenon recurred consistently across all the participating teachers and across each teacher's lessons. Trigonometry's technical terms were mainly borrowed implying lack of their isiXhosa equivalence in the teacher vocabulary. In instances where these terms were observed, they were not consistently used across the observed teachers.

During the interviews, Teacher C was asked where got the isiXhosa vocabulary he used and his response was *"I do not have any textbook or dictionary written in isiXhosa. All the words are use are mine. I just speak as they come. More so, this is my mother tongue so I do not have any problem."* The prevalent and rampant use of LWB was to a large extend a direct result of lack of teaching materials written in isiXhosa. Teacher A said *"I do not any teaching materials in my home language hence I rely on my own understanding of these mathematical terms when code switching."* Teachers, as noted by Setati (2008), were making moment by moment decisions as to the when, why, how and what of indigenous language use. Document analysis revealed that all participating teachers' textbooks, lesson plans, charts and others were only in English. This revealed lack of support available to these teachers in their use of isiXhosa during teaching.

In situations where the isiXhosa equivalent was available, this was found not to be used in everyday life of both the teacher and the students. During interviews, all the three teachers agreed that some of the words had isiXhosa equivalents that were more difficult than the English terms. Teacher B said *"Most of the words we use [in mathematics] should have their isiXhosa counterparts. It just needs time. If you ask those people who are doing language they should be able to tell us. Some of these terms are used in rural areas and not familiar to township people."* Teachers also mentioned that most of their students were only familiar with the modern dialects of isiXhosa and were struggling with the 'old' or 'rural' Xhosa.

CONCLUSION

Results show that teachers predominantly use borrowing strategies which, as this study concludes, still dominated by English mathematics register except for the isiXhosa prefixes attached to the mathematical terms. Loan word borrowing (LWB) was commonly used than transliteration (TLT). Transparent code switching (TCS) was not widely practiced by these teachers.

Teachers agree and concur that use of indigenous languages through strategies such as code switching is useful and can be used to enhance conceptual understanding. Such an appreciation motivated teachers to use indigenous languages even in situations where they could not do so transparently as in most of the cases above. This implies that teachers have the right motive to help their students grasp mathematical concepts that are crucial for students' mathematical growth but lack beneficial and meaningful ways of using indigenous African languages. It is argued in this study that if teachers in their classrooms use indigenous languages transparently, mathematics will be presented to students in a familiar language. This will help teachers to potentially place more focus on enhancing conceptual teaching and understanding resulting in restoring and reclaiming our African pride through the meaningful teaching of mathematics.

All the three teachers used borrowing consistently in their lessons. This was achieved because there was not much effort on the teacher's part to find more transparent terms. They used the 'easy way-out method' of adding an isiXhosa prefix to an English term. Any group of people, like teachers in their classrooms, who regularly talk about mathematical concepts and ideas will have specific ways to succinctly convey their meanings (Meaney, Fairhill & Trinick, 2008). While these teachers felt the need for, and importance of, indigenous language use, their way out was mostly through borrowing. Prevalent use of LWB which is not transparent suggest the need for intellectualisation of indigenous African languages for them to meaningfully function as LOLT at secondary school level. The only isiXhosa terms that were transparently used for mathematical purposes were those that teachers and the students use in everyday life, and those mathematical terms that students were exposed to in the early years of their learning, the Foundation Phase of the South African curriculum.

This paper recommends for well thought out, judicious and systematic use of indigenous languages in the teaching of secondary school mathematics. Teachers are encouraged to plan for the use of any indigenous languages instead of the spontaneous and ad hoc use which is the current phenomenon in secondary schools. Planning should address when, what, how and why questions of indigenous language use. Teachers will need to be trained and supported in this endeavour if such use of indigenous languages is to be successful and appreciated. Development of the mathematics register in indigenous languages specifically targeting secondary school mathematics is urgently required. Such a development of the mathematics register, coupled with the development of teaching materials in indigenous languages, will enhance the efficacy of isiXhosa to function as the language of teaching and learning (LOLT) in rural and township secondary school mathematics classrooms.

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'WE LINK IT': A CONVERSATION WITH TEACHERS FROM HISTORICALLY DISADVANTAGED SCHOOLS ABOUT THEIR LOCAL PRACTICES

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There is much that mathematics teacher educators can learn in terms of conversation with teachers from historically disadvantaged schools. Professional development in this case is focusing more and more on these teachers' local practices. Also, conversations between teachers and mathematics teacher educators present rich opportunities for learning what directions teacher collaboration could take. In this paper, selected excerpts are presented from a conversation that the author and colleague had with a group of teachers with whom we intend to collaborate around the use and development of lesson plans for pre-service teachers who were placed with teachers. Drawing mainly on ethnomethodology, I discuss ways the teachers talk about the different types of boundary objects that are present and that determine their local practices. Boundary objects—as reifications—are ideal types, whereas others are more repositories of the mathematics content. The report concludes with the importance of considering the different boundary objects that characterize the teachers' local practices.

Keywords: mathematics, practice, teachers

INTRODUCTION

Lecturers in mathematics teacher education, who intend to collaborate with teachers, have to establish contact with them starting with conversations around the teachers' local practices. This research report focuses on selected excerpts from a first time conversation with two intermediate and senior phase teachers and one high school teacher who teach at the grade 9 and 10 levels. These teachers work at historically disadvantaged schools located in the greater Cape Town area. These teachers acted as mentors to a group of pre-service teachers (PSTs) who were in their third and final year of the Bachelor of Education degree course as well as those in the Postgraduate Certificate in Education programmes enrolled in a teacher education programme. For their teaching practice, the PSTs were placed with teachers at seven different schools, some being historically advantaged and some historically disadvantaged. A meeting was arranged with the teachers at Stellenbosch University towards the end of the academic year, with the goal of finding out how the PSTs were doing, and what particular issues the teachers as well as the PSTs were facing in terms of interaction during the teaching practice period which runs for a period of nine weeks in the third quarter of the school year. Conversations with the teachers gave a sense of the work they were doing at their own schools and, in particular, ways they were aligning their work with respect to demands from different actors such as the principal, the curriculum advisors, the learners, donors of educational technology (e.g. computer software), the public at large, fellow teachers and different types of boundary objects. Boundary objects are lesson plans, pace setters, textbooks, workbooks, classwork books, and software such as computer programs.

This research report is driven by the question of what can be learned from a conversation with these teachers about their responsibilities with regard to their teaching, and to find out how these teachers go about aligning their work with respect to the various boundary objects and actors that they encounter. Another matter addressed in this report is the contact and conversation between two professional modes of practice: mathematics teacher education based at Stellenbosch University, a historically advantaged higher education institution; and mathematics teaching particular to township schools. Each of these practices has very different demands and yet has to be in conversation with each other.

DISCUSSION OF RELATED LITERATURE

The question that drives this report requires a review of literature on the particular teacher population referred to earlier, the notions of local practices, ethnomethodology and boundary objects in relation to school mathematics teaching, and interviews in relation to conversational analysis. The literature was reviewed and integrated for purposes of coming up with a theoretical framework, which will be used to analyse selected excerpts of the conversation with the teachers.

The teachers reported on are portrayed as not being as dedicated as those who work in the more affluent and historically advantaged schools found in the leafy suburbs of South African cities (Spaull, 2013). They are viewed in deficit ways (Keitel, 2005). The 'targeting' (Labaree, 2011) of these teachers can easily be picked up in different newspapers (Die Burger, 2011). This targeting takes the form of holding these teachers, especially, accountable for learner achievement in high-stakes assessments such as the Annual National Assessment and the National Senior Certificate examinations and teacher evaluation, i.e., 'teacher testing' (Spaull, 2013, p. 24). In the United States the Network for Public Education released a major teacher evaluation study with recommendations. This study details the harmful effects of teacher evaluation. An important recommendation of this study is that "more time can be spent on reflection and improvement of instruction."[5] In the South African case many of these teachers are considered as "just too badly educated themselves" (Business Day, May 13, 2016).Together with Spaull's (2013) views, this targeting does not take into account the fact that 'teaching is an extraordinarily complex and demanding form of

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https://dianeravitch.net/2016/04/17/npe-releases-major-teacher-evaluation-study-with-recommendations/

professional practice' (Labaree, 2011, p. 9), especially when the context is considered. The 'daily grind' (Lortie, 1975) is one where these (and other teachers) find themselves having to respond to their learners, their peers, the principal, textbook publishers, curriculum advisors, policy makers, educational technology producers, and the public at large. The author has a particular interest in working with this teacher population.

The different types of boundary objects mentioned earlier on can be considered as a set of working arrangements (Star, 2010) that the teachers have to deal with. The beginning conversation with the teachers that will be reported on, represents a 'nexus of perspectives' (Wenger, 1998) between the teachers and mathematics teacher educators and is of interest to the mathematics teacher education community in light of current issues and debates in professional development, where it becomes imperative to consider the teachers' working arrangements.

The reference to the teachers' local practices is taken from literature on ethnomethodology (Garfinkel, 1967; Lynch, 1993; 1997) and, in particular, the LEDIMTALI (Local Evidence-Driven Improvement of Mathematics Teaching and Learning Initiative) based at the University of the Western Cape. In this initiative there is an emphasis on 'local' as in teachers' local concerns, dilemmas and practices. For this report ethnomethodology is defined as a science of the procedures that constitute what Garfinkel (1967, p. 1) has called "practical sociological action and reasoning". This means that the teachers, in this case, do their work in practical ways where they take into account and interact with the schooling system, the learners, curriculum advisors and donors of software programs, for example. Drawing on Lynch (1993), we see that, in ethno methodological parlance, the adjective 'local' has little to do with subjectivity, perspectival viewpoints, particular interests, or small acts in restricted places. Instead the use of local refers to the heterogeneous grammars of activity. What this means is that people—teachers in this case—go about their work, in ways based on what and whom they need to respond to, as mentioned earlier.

Ethno methodologists are not out to overcome heterogeneity by theoretically postulating a homogeneous domain, for example, the teachers' cognitive structures, such as ways they think 'in their minds' and speak about the mathematics teaching, compared to mathematics teacher educators. For example, mathematics teacher educators may regard, based on their work demands and learning opportunities, actions related to problem solving approaches to mathematics teaching as quite 'orderly' and a norm. Such actions cannot always be guaranteed in the case of mathematics teachers. In the case of teachers these actions may happen in irregular ways. Thus ethno methodologists attempt to investigate a patchwork of 'orderlinesses' without assuming that any single orderly arrangement reflects or exemplifies a determinate set of norms. Mathematics teacher educators may find problem-solving approaches to mathematics teaching as 'the norm' and powerful. This is not to deny the historical and social "contexts" in which social action, such as mathematics teaching, take place; rather, ethno methodologists insist that specifications of such contexts are invariably bound to a local contexture of relevancies (Lynch, 1993). What the above means, is that there has to be an acute awareness on the part of the mathematics teacher educator of what the teachers find relevant in terms of their situation, and what they have to do as they do their work.

In the post-1994 South African education context, we read of an interesting series of interviews revealing the effects of the ‘new’ education system on the consciousness and practices of teachers from historically disadvantaged township schools (Modiba, 1996). This study points out how the teachers were conscious of the constraints such as the lack of access to resources in their work, and how their accounts of their practice should read meaning beyond their simple consciousness (p. 132). What this quote tells us is that the teachers’ comments contained in interview excerpts about their local practices, cannot be interpreted in naïve or simple ways. These comments reveal how the teachers are working towards some kind of order in meeting the demands of their work.

The notion of a boundary object is helpful in terms of analysing excerpts from conversational exchanges between teachers and lecturers, as well as how teachers do their work. Boundary objects can be thought of as “artefacts or abstract constructs that facilitate knowledge sharing and knowledge generation between different communities of practice, for example between teachers and learners, subject advisors and teachers, the education department and academic institutions” (Raymond Smith, personal communication). They determine the work arrangement of teachers (Cobb, et.al. 2003; Star, 2010). This notion was first used by Star and Griesemer (1989) and Star (1989) to describe the kind of communication that had happened between different actors involved in the development of the Museum of Vertebrate Zoology at the University of California, Berkeley. For this project amateur collectors of animal specimens had to speak to professional biologists, for example. Although both groups were interested in the conservation movement, it is clear that they thought very differently about specimens.

Boundary objects in the context of this report are objects such as lesson plans, pace setters or ‘pacing guidelines’ (Stein & Coburn, 2007), classwork books, workbooks, textbooks, Annual National Assessments (ANAs) and software programs. Wenger (1998, p. 58) considers boundary objects as ‘reifications’ that are concrete objects that embody a set of ideas or processes because they hold steady a set of ideas and processes across time and space, which can easily be inferred from the above. Curriculum advisors, for example, use pace setters as a way to communicate requirements and expectations in terms of school mathematics teaching. There is the very real possibility that boundary objects will be used differently and come to have different meanings when they are incorporated by different teachers. Despite these situations, boundary objects can play a significant role in potentially enabling teachers, pre-service teachers, curriculum advisors and teachers to link and to align their work. For this report the following types of boundary objects are useful: (a) repositories and (b) ideal types (Star, 1989, p. 48). Examples of the first are a museum and a library, because they 'store' knowledge and can be used differently by different people. Mathematics textbooks, the CAPS documents and software programs, depending on their respective designs and use, also fall into this category. The same can be said about the ANAs. On the other hand, lesson plans, work books, classwork books and pace setters are ideal types, because one has to adapt them to a local site, as they can be fairly vague.

The meeting with the teachers at the university took the form of an informal 'interview' (conversation), during which views were shared with respect to practices (Kvale, 1996). Although there were prompts for this interview such as asking the teachers how the PSTs were doing during their teaching practice period, what their expectations were from the PSTs, the actual interview had flexible and iterative prompts. These caused the teachers' responses to reflect a better sense of the phenomenon of 'local practices,' i.e. how they were doing their work. In line with the Latin meaning of conversation as 'wandering together with' there is the possibility of gaining a sense of the teachers' lived world. At the most basic level, interviews are conversations wherein there are 'attempts to understand the world from the subjects' points of view, to unfold the meaning of peoples' experiences, to uncover their lived world prior to scientific explanations' (Kvale, 1996).

THEORETICAL FRAMEWORK

The analytic framework for this research report, to be used for data analysis, is framed by key ideas found in the literature on ethnomethodology, boundary objects, teacher practices (Modiba, 1996), conversational analysis (Schegloff, Jefferson & Sacks, 1977) and collective 'qualitative interviewing' (Kvale, 1996).

METHODOLOGY

Literature on conversational analysis (Schegloff, Jefferson & Sacks, 1977) and collective qualitative interviewing (Kvale, 1996) was helpful in guiding the data analysis. From several viewings of the 90 minutes long videotaped meeting with the teachers it was clear that it has been a conversation across two professional modes of practices. In some segments of this videotape the teachers spoke in detail about what they were doing in terms of their daily grind in the form of their daily and weekly lesson planning, and how they use and think about the different types of boundary objects.

As much as the teachers were mentor teachers to the PSTs, they were in fact sharing with us, in direct ways, their local practices. The latter came about because of flexible and iterative prompts during the 90 minute conversation. At times the teachers directly responded to our prompts in consecutive utterances. Then there were times when the conversation 'drifted' (Erickson, 1982), as is well-known in the case of conversations constructed by temporal proximity of each utterance, i.e. an utterance or comment that follows closely in terms of a time span. The use of 'anchoring' (Erickson, 1982) was an attempt to focus on the issue under discussion in the conversational exchanges. At the beginning of the conversation the teachers were prompted in a more general way to talk about how they interacted with the PSTs. They soon started speaking about how they handle the relationships between pace setting, classwork and work books and a software program. The teachers' responses thus presented a better sense of their 'local practices'. In line with the meaning of conversation, it was possible to gain a sense of the teachers' lived world (Kvale, 1996). Lived world, for example, refers to the teachers' classrooms and the school as a whole.

"Teachers' local practices" serve as the unit of analysis. A particular theme that emerged from a transcription of selected excerpts of the videotaped conversation, is around linking and links, which teachers were doing with respect to the different boundary objects — the pace setter, the CAPS document (DBE, 2011; 2012), workbooks, classwork books, software programs and textbooks—as well as what they expected from the PSTs. In line with ethnomethodology literature, there is a noticeable patchwork of 'orderlinesses' (Lynch, 1993) in terms of this linking.

Excerpts showing their local practices are presented in ways where the time (in minutes) since the beginning of the conversation are indicated and where the sentences or utterances in the form of transcribed lines of conversation, are numbered. These excerpts have been grouped, based on the presence of iterative prompts that are aimed at focusing or anchoring the conversation with respect to the above boundary objects. Other excerpts that were analysed are instances of 'drifting' (Erickson, 1982), i.e., where the boundary object is not necessarily the CAPS document or a pace setter but mathematical terminology that the teachers signal to us as being new to learners who were previously taught in the foundation phase.

Due to space constraints, only two data categories on the teachers' local practices with respect to linking and links will be presented: (a) lesson plans, the CAPS document and the pace setter, (b) a computer software program and the ANAs. When a single teacher refers to 'we,' it was assumed, methodologically, that he/she spoke on behalf of other teachers in the school. In the transcribed excerpts given below T1 and T2 refer to different teachers and R1[i] and R2 refer to the two interviewers. The second column from the left gives the time in minutes and seconds since the beginning of the conversation.

DATA AND ANALYSIS

On linking the lesson plan, the CAPS document and the pacesetter, the following data was obtained:

T1	3:24	1	Your lesson plan is different from ours.
R1		2 3	Say how? We also have difficulty with our students and lesson plans, so help me understand…
T1	3:40	4 5	Ours is based on the CAPS document or sometimes the CA gives us a framework of the lesson plan.
R2		6	Give an example?
T1		7	Number and operations, three weeks… Problem solving, one week
T2	5:35	8	We have the weekly lesson plan… then the daily lesson plan
R1		9	That you take from the weekly?
T2		10	Yes, and that weekly lesson plan comes from the pace setter that comes from the CAs. And the CAs are looking through the CAPS document.

In lines 1 through 10 we notice two types of boundary objects, viz. repositories (CAPS document) and ideal types (weekly & daily lesson plan & pace setter) that are linked, mainly by the curriculum advisor (CA), and how they influence the teachers' local practices. Evident, in line 1, is the fact that the teachers regard the PSTs' lesson plan ('your lesson') as coming from us as mathematics teacher educators and as different from theirs. It is the case that the PSTs received guidance from us on lesson plans. The iterative prompts in lines 2-3 and 6-9 caused the teachers to say more about how their lesson plans are structured. In lines 4 and 5 we see the CA's attempts at making the daily and weekly lesson plans tightly bound in the school district, the school and the classroom, by linking them to the pace setter, especially (see line 10). This move on the part of the CA is about linking the lesson plan, which is an ideal type boundary object, to the CAPS document which in this sense is more of a repository, because it provides details on the mathematics curriculum for the whole school year.

The pace setter as an ideal type boundary object has the goal of keeping every teacher 'on the same page' in an 'ideal way.' It has details on 'pacing' represented in columns labelled 'content description,' 'assessment activities,' 'week' and 'completed.' These pace setter details can be found on the Education Department's website[i] . Ideally the list of pace setter mathematics content has to be viewed in relation to a 'framework of a lesson plan' (line 5). The pace setter is thus not a rich repository of information on the mathematics curriculum.

In this regard the CAPS document is more of a repository or 'resource' because it has far more details on the mathematics that has to be 'covered.' On the other hand, a mathematics textbook as a boundary object, is more of a repository compared to the CAPS document. Depending on the quality and representation of the mathematics content in a particular textbook, it usually has introductions, illustrations, examples and exercises. Thus there exists an opportunity, through participation, for users/teachers to get an idea of the intention of the textbook writer as a curriculum developer when studying the representation or framework of content in the particular textbook.

It is thus not surprising that curriculum advisors also provide a 'framework' (line 5), the CAPs document and the pace setter, as ways to structure the mathematics curriculum with the goal of keeping 'every' classroom and school 'on the same page.' Curriculum advisors align these different types of boundary objects, as reifications, in particular ways so that the school functions as a bureaucracy that can enable learning to happen. What comes to the fore thus far, not surprisingly, is a patchwork of orderlinesses in what the teachers do, i.e. their local practices. So far, the challenge is for mathematics teacher educators to work towards participation involving the teachers and the curriculum advisors, at least. In turn the challenge is to devise a type of lesson plan that is more of a repository, and yet aligned with the bureaucratic requirements of the school.

A more complex scenario becomes apparent in the case of the presence and use of computer software and the ANAs in the teachers' work arrangements, which will be considered next.

On linking the pace setter, classwork books and a computer software program and the ANAs, the following data is obtained.

R1	21:54	Have you had any input from government's side on how to work with pace setters and technology?
T3		We did not have any input yet. They have not visited us since we had the projectors. We have 35 computers that have been donated to us. We bring them into the classroom. There is a program called I Can Compute (not real name), where the learners will do the maths, answer questions in those boxes and then they get scores. And if they have high scores, they can choose an avatar that they would like to have when they use the computer. They don't look at it as old boring maths but as a game, but instead they are doing maths. At the same time. All the learners now use this program.
R2	24:51	If I look at the way you talk about the program in relation to the mathematics, I ask…Here are the ANAs which have to be done…Talk about the ANAs, the software… that relationship
T2	25:00	We link it, but linking it is a lot of work, because we have workbooks that we have to cover, and we have the I Can Compute program that we do. We do not want to spoil the relationship with the company that has donated it, and we have classwork books that we need to fill in for the CA. Sure we must make sure that the CA is happy, and the workbook person is happy, and we must also make sure that the company that has donated the program is happy. It's a lot of work. We find ourselves staying at school until 4 o'clock, and sometimes we have to be, we have to come in on a Saturday to make sure that all these things are done. We are using all our resources that we have. But there is a link between all these things because when we do the classwork books, we can do some of the things, and at the same time we are doing our workbooks. Some of the topics that we would open on the I Can Compute program are the same topics that we have.

Evident in this lengthy excerpt are the teachers' efforts to 'link' (lines 14 and 25) the different types of boundary objects, e.g., pace setter, the particular software program, classwork books and the ANAs. As mentioned earlier, the pace setter is an example of a reification that has the goal of monitoring and 'pacing' instruction, i.e. teaching. In line 6 and onwards the teacher talks about how they (his/her colleagues) use a particular software program, and its relationship to the learners. The iterative prompt in line 12 aims at getting to know how the teachers try to connect this program, as a boundary object, with the ANAs.

This prompt is more focused compared to the one in line 1. The response that follows in line 14 onwards gives a picture of a constellation of different types of boundary objects and what the teachers do as they try to 'link' (line 25) them. From line 15 onwards we see that they work at keeping the CA, the software 'company' and 'workbook person' 'happy.' They are aware that there is a 'link between all these things' (line 25) and try to keep track of them all. What emerges in terms of local practices, although a patchwork, is again a sense of orderlinesses.

A useful intervention would be to conceptualise ways of linking lesson plans, a software program, the workbook, class work book, pace setters and ANAs. Depending on its design features, the software program can be a repository type of boundary object where the user, a teacher in this case, has to be aware of the designer's intentions. Through appropriate long term participation on the part of teachers, curriculum advisors and mathematics teacher educators, there is the possibility of linking lesson plans as ideal types and reifications, to ones that are potentially more strongly structured as repositories with respect to individual use of the teacher in an actual classroom. This calls for long term participation (Wenger, 1998) with respect to the reifications evident in all these types of boundary objects. Boundary objects per se are objects that embody and that 'hold steady' a set of ideas or processes (Wenger, 1998). On the ground, it implies taking the teachers' existing local practices into account. The fact that the teachers are interested in and are 'linking' (line 14) is a necessary condition.

CONCLUDING REMARKS

The findings reported have implications for mathematics teacher education. It is tempting for mathematics teacher educators to treat the minutiae of the teachers' local practices as reported in their conversation as a problem to be solved. This is the well-known 'summons' to theory (Lynch, 1997, p. 19). In addition, it is reductionist to assume that there can be a single orderly work arrangement that the teachers can follow as they interact with the different boundary objects and their actors. More importantly, it necessary not to presuppose our epistemological primacy as mathematics teacher educators, with respect to what the teachers do, per se. When the teachers say 'we link it', it implies that it is upon mathematics teacher educators to look for ways to mediate between what the teachers are already doing and what can be added, mediated and shared, so that there is an enhancement of learning on the part of mathematics teacher educators as well as the teachers.

There is benefit in studying the teachers' local practices and their detail per se. Mathematics teacher educators have to be aware of the boundary objects that determine teachers' work. This report informs us that there are 'gaps' in terms of what the teachers do and are doing. The teachers are aware of these 'gaps' and thus try to 'link' what they have to do and need to do. Here these specific excerpts reveal the teachers' consciousness (Modiba, 1996).

The issue about lesson plans in particular, whether coming from teachers or mathematics teacher educators, is that they are ideal type boundary objects and reifications. To make them meaningful requires participation involving conversation in the context of on-going practice.

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A LOGICAL DISCOVERY BUILT AND THEN VERIFIED AND GENERALIZED FURTHER THROUGH EXPERIMENTATION

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This qualitative descriptive study firstly focuses on how a pre-service mathematics teacher (PMT) discovered on logical grounds that the perimeter of a parallelogram, which was formed by connecting the midpoints of the sides of a convex quadrilateral, is equal to the sum of the diagonals of the convex quadrilateral, and only thereafter verified the logical discovery through experimentation in a dynamic geometric context. This in itself, brings to the fore the notion that results in mathematics are not necessarily discovered (or conjectured) firstly through experimentation and then subsequently explained to be true for all cases via logical reasoning (or proof) but rather the reverse trajectory path can also permeate mathematical discoveries. Secondly, this study illustrates how a logical discovery, namely the perimeter of a parallelogram is equal to the sum of the diagonals of the quadrilateral, which was initially confined to convex quadrilaterals was further conjectured to also permeate concave and crossed quadrilaterals through the process of experimentation within a dynamic geometry context.

Keywords: Deductive reasoning experimentation, inductive reasoning, logical discovery

INTRODUCTION

Across the geometry curriculum in South African schools, teachers do to some extent engage their students in mathematical investigations wherein learners are provided opportunities to experiment, observe regularities, make conjectures, and then justify them through using deductive arguments (logical proof) (DBE, 2011). However, there is not much evidence of items or activities that provide opportunities for learners to discover a new geometric results through logical reasoning (i.e. through using deductive arguments). Furthermore, many people working in the field of mathematics education and many mathematics teachers appear to have a general misconception that new geometrical results are only first discovered though the process of experimentation (empirical methods) prior to being verified by construction of logical proofs. However, there are many examples in mathematics where new results are discovered or invented just via deductive arguments (De Villiers, 2003). For example, suppose one was examining the properties of isosceles trapezia. Assume that we already knew that in general, as shown as shown in Figure 1, that an isosceles trapezium has the following properties namely: PQ//SR; PS = QR; $\hat{P} = \hat{Q}$; $\hat{S} = \hat{R}$ and PR = QS.

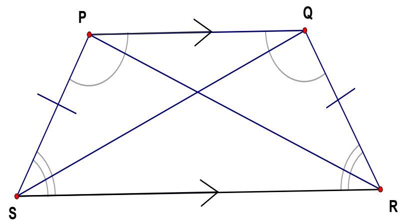


Figure 1: An isosceles trapezium

By considering the known properties, one could easily develop the following kind of argument by logically analysing its properties:

$\hat{P} + \hat{S} = 180^0$ (co-interior angles, PQ//SR)

and $\hat{S} = \hat{R}$

$\therefore \quad \hat{P} + \hat{R} = 180^0$

$\Rightarrow$ the isosceles trapezium *PQRS* is a cyclic quadrilateral.

Thus, in this particular case, we have discovered new knowledge in a deductive manner and not in the conventional inductive manner by first using construction and measurement.

If future teachers have to go into mathematics classrooms and provide opportunities for their mathematical learners to discover new geometric results through deductive arguments, it seems advisable that they ought to have experienced similar teaching in at least one university module. In this paper I provide a descriptive account of the moves pursued by one of the PMTs from a class of 8 prospective secondary teachers, who worked on a set of task-based geometry activities, and which I believe provided an opportunity for prospective teachers to experience and see how a new geometric result can be discovered through logical reasoning.

THEORETICAL PERSPECTIVES

> [Generalization is the] passing from the consideration of one object to the consideration of a set containing the object; or passing from the consideration of a restricted set to that of a more comprehensive set containing the restricted one (Polya, 1957, p.108).

Across most mathematical curricula both at school and higher education levels, inductive and deductive reasoning and generalizing permeates mathematical activities in varying degrees and extent.

Inductive reasoning, which commonly underpins the process of experimentation, is a process of examining and working through a set of examples, which we often refer to as specific instances, and recognizing what stays the same even as parts of the situation varies, and then articulating the identified prevailing commonality as a conjecture or conclusion or general statement, which is often true but are not always or necessarily true (De Villiers, 1992; Govender, 2013; Polya, 1954).

Even though inductive reasoning is useful in making discoveries, it can lead to false conclusions if the examples considered are not representative or are misinterpreted. A common way of demonstrating that conjecture is false (or partially wrong), is through constructing or producing a counter-example, which shows that the assertion(s) in the conjecture is false (Stewart, Redlin, & Watson, 2012; Larson, 2016).

On the other hand, a common way of explaining why an established conjecture is always true (or wrong) is the development of a valid mathematical argument (or proof) through the use of deductive reasoning, which is a logical process of *reasoning* from one or more statements (premises) that are generally assumed to be true to reach a logically certain conclusion (De Villiers, 1992; Govender, 2013). The true premises, which augments the validity of the argument, are often statements that are substantiated or justified by the appropriate invocation of previously established axioms, definitions, theorems and corollaries. A common form of deductive reasoning is the syllogism, in which two statements, a major premise and a minor premise, reach a logical conclusion. For example, the premise "Every A is B" could be followed by another premise, "Every C is A." Those statements would lead to the conclusion "Every C is B." (Johnson-Laird, Oakhill, & Bull, 1986).

Syllogisms are considered a good way to test deductive reasoning to make sure the argument is valid. With that in mind, it is imperative to observe that deduction will produce correct conclusions, but only if you start with correct assumptions.

For the purposes of this study, experimentation is considered as the employment of inductive reasoning within the dynamic geometry context, to test the generality of established mathematical conjectures (generalizations) via a series of special cases through using construction and measurement, and also to develop conjectures (generalizations) through dragging a given figure and analyzing which attributes of a figure stay the same and which changes when the figure is transformed in some way via dragging (Govender, 2013; Driscoll, Egan, DiMatteo, & Nikula, 2010). According to De Villiers (2004, p.93), the functions of experimentation include conjecturing (looking for an inductive pattern, generalization), verification (obtaining certainty about the truth or validity of a statement or conjecture), global refutation, heuristic refutation, understanding.

Just as deductive reasoning is invaluable in developing valid mathematical arguments (or proofs) that explain or justify why conjectures formulated through the processes of experimentation are always true (or false or partially false) it can be used in reverse to also establish conjectures or make discoveries on logical grounds without necessarily following the path of experimentation (compare De Villiers, 2003).

METHODOLOGY

The study reported here followed a qualitative methods design since it involved pre-service teachers as human beings, and aimed to understand pre-service teachers and their practices in a specific context or setting (Marilyn, 2013). According to Maxwell (2003), a qualitative approach which emphasizes the perspective of teachers and the understanding of particular settings has great potential in informing educational practices.

The research took on a descriptive and interpretive framework. The study is descriptive in the sense it seeks to find out how a preservice teacher discovered a new geometrical result through using deductive arguments in terms of potentially observable behavior or events (Maxwell, 2003; Marilyn, 2013). In other words it seeks to find out if the pre-service teacher employed deductive argument(s) to produce a new result, and if any challenges were observed, or any additional and helpful, observable phenomena. The study is interpretative in nature in the sense it seeks to reveal, understand and interpret a pre-service teachers' mathematical moves in a context where she had to work through a scaffolded task-based activity to discover a new geometric result.

Qualitative data were gathered via a task based worksheet with the aim of exploring how the PMT made use of deductive arguments to discover a new geometric result. An inductive-analytical frame-work was adopted and used for the purposes of data analysis in this study. The study reported here sought to answer the following questions:

1. To what extent can pre-service teachers use deductive arguments to discover new geometrical results?

Participants

The purposive sample of the study consisted of a group of 8 Post Graduate Certificate (PGCE) from one of the Universities in Western Cape, South Africa, who were doing a mathematics methodology module linked to the teaching and learning of grades 10-12 mathematics. All 8 students had at least passed Mathematics 2 in their undergraduate degree or 360 credit diploma. All 8 students were conversant with English, which was the medium of instruction at the University.

Research Instrument

As mentioned earlier, the data for the study was obtained through a task-based activity worksheet, wherein items were assimilated from De Villiers (2003). The following questions were presented to the participants in the task-based worksheet:

You have discovered previously that if you connect the midpoints of the sides of any quadrilateral, you get …..

Without using construction and measurement, work through the following questions using the diagram as shown in Figure 2:

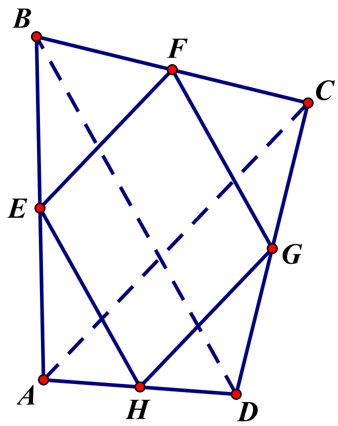


Figure 2: Quadrilateral ABCD

1. Write an equation relating the lengths of EF and
2. HG to the length of AC.
3. Write an equation relating the lengths of EF and HG to the length of AC.
4. Write an equation relating EH and FG to BD.
5. Explain how you found your equations in Questions 1 and 2.
6. Use Questions 1 and 2 to describe the relationship between the perimeter of the inscribed parallelogram EFGH and the diagonals of quadrilateral ABCD.

Thereafter, students were encouraged to check out the validity of their discoveries by construction and measurement. The task read as follows:

1. Make constructions with appropriate measurements in *Sketchpad* to confirm your conclusions from Question 4. Be sure to check the concave and crossed cases for quadrilateral ABCD. Summarize your results. Your summary may be in paper form and may include a presentation sketch in Sketchpad. You may want to discuss the summary with your partner or group.

Procedure

The task based worksheet, which was assimilated from curriculum material produced by De Villiers (2003) was issued to all students at the same time after they discovered and justified that if you connect the midpoints of the sides of any quadrilateral, you get a parallelogram.

The completion of the task-based worksheet by the students was facilitated by the researcher. Initially students worked through Questions 1-4. During the time the facilitator moved around the class and observed what students were doing and made notes. After all students completed questions 1-4 after about 20 minutes, a general class discussion was held and it was found that most students did establish a relationship between the perimeter of the inscribed parallelogram EFGH and the diagonals of quadrilateral ABCD.

On those grounds the facilitator, requested the students to proceed to answer Question 5. In doing Question 5, it was realized that many students did not know what are concave and crossed quadrilaterals. At an opportune moment the facilitator intervened and had a class discussion about convex, concave and crossed quadrilaterals. Through cross-questioning and dragging of a vertex of a convex quadrilateral in a dynamic geometric environment, the facilitator illustrated the variations across convex, concave and crossed quadrilaterals, with the focus on the diagonal(s) of the specific quadrilateral.

DATA ANALYSIS, RESULTS AND DISCUSSION

For the purposes of this study, the data analysis, results and discussion will focus on the responses of a pre-service teacher (Jenny by pseudonym) to the items presented in the task-based worksheet. In doing so, two vignettes will be constructed – Logical Discovery and Experimentation.

Vignette 1: Logical Discovery

At the beginning of the worksheet, PMTs were asked to complete the following statement:

> You have discovered previously that if you connect the midpoints of the sides of any quadrilateral, you get …..

Jenny was able to provide the expected response, namely a parallelogram as indicate in her response in Figure 3

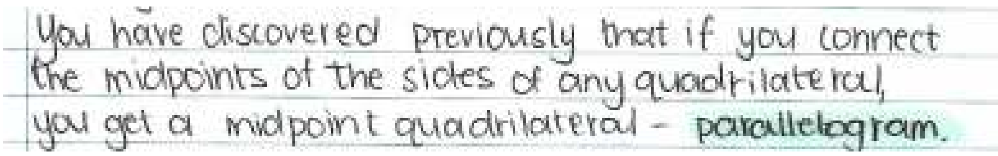

You have discovered previously that if you connect the midpoints of the sides of any quadrilateral, you get a midpoint quadrilateral - parallelogram.

Figure 3: Jenny's response: Parallelogram

The expected responses to Questions 1, 2 and 3 depended to a large extent on PMTs prior knowledge and experience with respect to the following previously established results:

a. The Midpoint Theorem, which states that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and is equal to half the third side (DBE, 2011)

b. Varignon's Theorem, which states that if you connect the midpoints of any quadrilateral, you get a parallelogram (De Villiers, 2003, p.63).

As illustrated in Figure 4, Jenny responded to Questions 1 and 2 by expressing each diagonal of the original quadrilateral ABCD respectively in terms of the sum of the sides of the parallelogram which are parallel to the specified diagonal. In doing so, Jenny managed to invoke the appropriate theorem (namely, the midpoint theorem) to explain how he established her equations in questions 1 and 2.

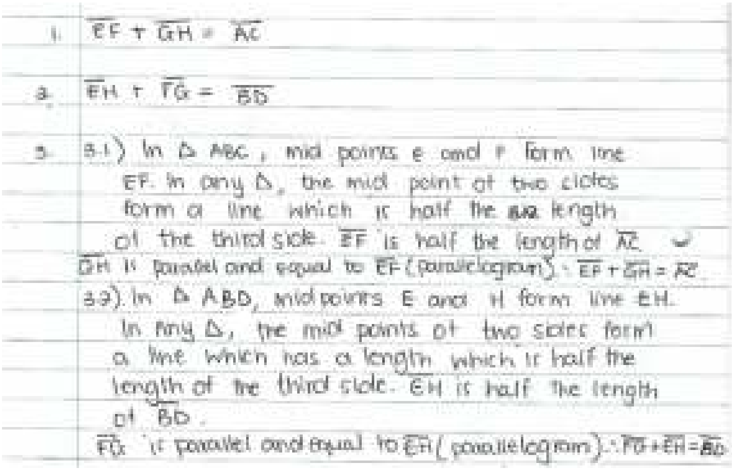

1. $\overline{EF} + \overline{GH} = \overline{AC}$
2. $\overline{EH} + \overline{FG} = \overline{BD}$
3. 3.1) In Δ ABC, mid points E and F form line EF. In any Δ, the mid point of two sides form a line which is half the length of the third side. $\overline{EF}$ is half the length of $\overline{AC}$. $\overline{GH}$ is parallel and equal to $\overline{EF}$ (parallelogram) ∴ $\overline{EF} + \overline{GH} = \overline{AC}$
3.2) In Δ ABD, mid points E and H form line EH. In any Δ, the mid points of two sides form a line which has a length which is half the length of the third side. $\overline{EH}$ is half the length of $\overline{BD}$.
$\overline{FG}$ is parallel and equal to $\overline{EH}$ (parallelogram) ∴ $\overline{FG} + \overline{EH} = \overline{BD}$

Figure 4: Jenny's responses to Questions 1-3

The purpose of Question 4 in the task based worksheet, was to provide an opportunity for the pre-service teachers to use logical reasoning to discover a relationship between the inscribed parallelogram EFGH and the diagonals of quadrilateral ABCD. It appears as if this has been realized in the case of Jenny as can be seen in her response illustrated in Figure 5.

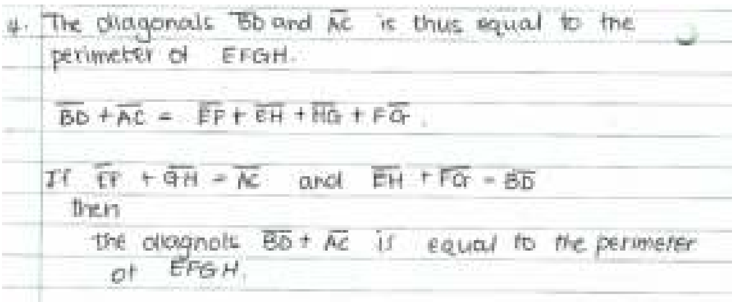

4. The diagonals $\overline{BD}$ and $\overline{AC}$ is thus equal to the perimeter of EFGH.
$\overline{BD} + \overline{AC} = \overline{EF} + \overline{EH} + \overline{HG} + \overline{FG}$
If $\overline{EF} + \overline{GH} = \overline{AC}$ and $\overline{EH} + \overline{FG} = \overline{BD}$
then
the diagnols $\overline{BD} + \overline{AC}$ is equal to the perimeter of EFGH.

Figure 5: Jenny's responses to Question 4

Although the discovery that the perimeter of the inscribed parallelogram EFGH (which is formed by joining the midpoints of a quadrilateral ABCD) is equal to the sum of the diagonals of quadrilateral ABCD is not original (for example, see De Villiers, 2003, p.63), it was certainly experienced as such by the pre-service teachers like Jenny.

The pedagogical value of the discovery lies in the fact that it was accomplished through development of a valid argument, by first establishing a set of true premises (namely: $EF + HG = AC$ and $EH + FG = BD$) that were justified by the application of an established accepted mathematical theorem (namely, the Midpoint theorem) in relation to a specific geometrical representation, and finally connecting them through to deductive logic. As demonstrated in Vignette 1, the pre-service teacher, Jenny, has discovered for herself a new geometric result purely by logical reasoning and not following the route of first discovering through experimentation and then using logical reasoning to explain why the discovery is true.

Although the didactical moves that Jenny has made, may look simple and trivial to mathematical readers, the power in the example lies in the fact that we generally do not engage our learners at schools or tertiary institutions in such kind of meaningful activity, wherein they first discover a mathematical property through using deductive reasoning, and only thereafter engage in experimentation within a dynamic geometry context to verify whether the invariance still prevails across broader set off geometrical figures bearing the same set initial generic conditions, and concurrently give concrete meaning to their results. To illustrate the latter traits of behavior, Vigenette 2 is constructed to capture the didactical and dynamic moves made by the pre-service teacher Jenny.

Vigenette 2: Experimentation

In the first part of Question 5, PMTs were requested to make constructions with appropriate measurements in a dynamic geometry environment to conform their deductive conclusion produced in Question 4 (which relates specifically to a convex quadrilateral). Figure 6, is a direct copy of a dynamic Sketch which Jenny captured amongst her series of drags to verify and confirm her already established conclusion.

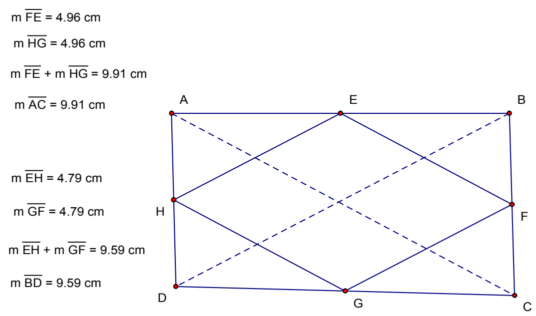
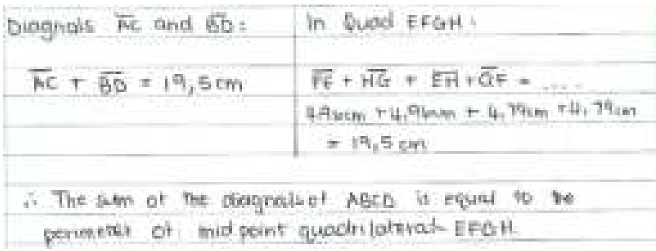


Figure 6: Jenny's convex case calculations

The following is what Jenny wrote in her workbook:



Diagnals $\overline{AC}$ and $\overline{BD}$:	In Quad EFGH:
$\overline{AC} + \overline{BD} = 19,5$cm	$\overline{FE} + \overline{HG} + \overline{EH} + \overline{GF} = \ldots$ 4,9cm + 4,9cm + 4,79cm + 4,79cm = 19,5 cm

∴ The sum of the diagonals of ABCD is equal to the perimeter of mid point quadrilateral EFGH.

Figure 7: Jenny's convex case conclusion

The pre-service mathematics teachers were encouraged to check via experimentation if their result also holds true for concave and crossed quadrilaterals.

Figure 8, is a direct copy of a dynamic Sketch which Jenny captured amongst her series of drags to verify and confirm whether her result established in (4) can be extended to concave quadrilaterals.

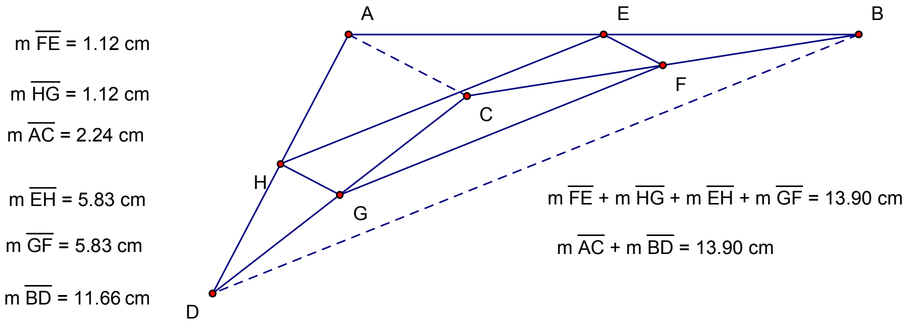
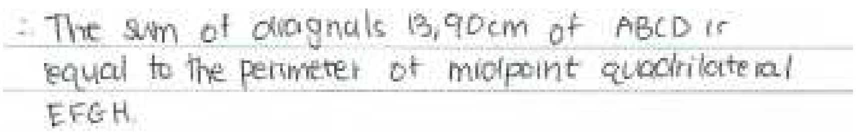


Figure 8: Jenny's concave case calculations

The following is what Jenny wrote in her workbook:



∴ The sum of diagnals 13,90cm of ABCD is equal to the perimeter of midpoint quadrilateral EFGH.

Figure 9: Jenny's concave case conclusion

Thereafter, Jenny moved onto dragging one of the vertices of her constructed quadrilateral ABCD to produce a series of crossed quadrilateral, which was also dragged around where maintaining its crossed quadrilateral identity. Figure 10 represents one such captured instance.

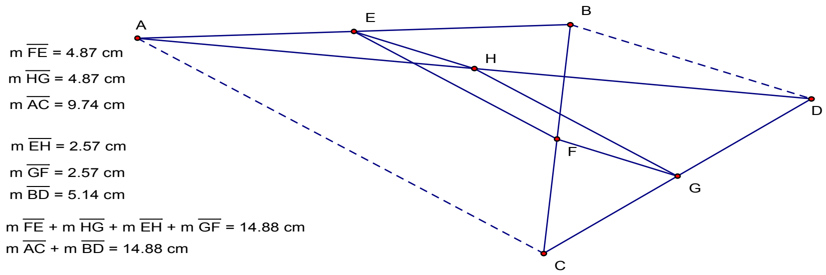


Figure 10: Jenny's crossed case calculations

The following is what Jenny wrote in her workbook:

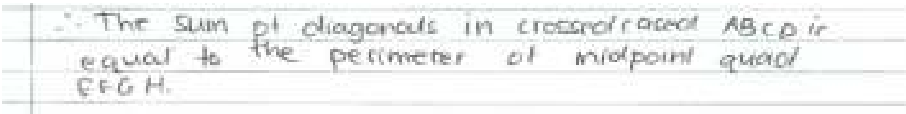


Figure 11: Jenny's crossed case conclusion

As a result of experimenting with a series of convex, concave and crossed quadrilaterals and seeing that the invariant property prevailed across a range of these kinds of quadrilaterals, it seems that Jenny's degree of confidence in her initial deductive conclusion for concave quadrilaterals not only grew for convex quadrilateral cases but was also extended with a high degree of confidence to concave and crossed quadrilateral cases, which ultimately made her write the following in her workbook:

Figure 12: Jenny's generalization to ALL quadrilaterals (convex, concave and crossed)

The aforementioned assertion, wherein Jenny emphasizes that the result holds for 'ANY" case articulates the generality that she propounds over her initial result for convex quadrilaterals and to a wider range of quadrilaterals such as concave and crossed quadrilaterals.

CONCLUDING REMARKS

Without using construction and measurement, a pre-service teacher like some of her peers in a her class managed to successfully build a set of true premises through appropriate justifications, which then enabled her to establish a true conclusion, that is, the perimeter of the inscribed parallelogram (which is formed by joining the midpoints of the original quadrilateral) is equal to the sum of the diagonals of the original quadrilateral. Even though the discovery of the result through the evolution of a valid argument is not original, it was certainly a new experience for the pre-service teacher (and her peers) in that she (like her peers) has come to see that the birth of a mathematical discovery is not necessarily preceded by experimentation (or inductive reasoning) but rather it could be established first via deductive reasoning and then later verified by experimentation. More importantly the study demonstrated how a logically discovered result for a convex quadrilateral can be convincingly generalized to the set of concave and crossed quadrilaterals on empirical grounds, even though one is aware that it is necessary to advance a convincing logical argument to explain (or justify) why the result also holds for the concave and crossed cases before accepting that the generalization is always true.

ACKNOWLEDGEMENT

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SOLVING A WORD PROBLEM THROUGH MATHEMATICAL MODELLING WITH EQUATIONS: THE CASE OF A PRE-SERVICE MATHEMATICS TEACHER

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University of Western Cape

There are no hard and fast rules that will ensure success in solving problems. However, it is possible to outline some general steps in the problem solving process and to give some principles that may be useful in the solution of certain problems. Within the latter context, this qualitative study anchored in an interpretivist paradigm attempts to track and examine the critical moves that a pre-service mathematics teacher (PMT) pursued in solving a speed-distance-time problem which is consonant with a modelling approach. The results of the study showed that the PMT developed an equation as a mathematical model which he used to solve the problem, but the development of this mathematical model hinged on a number of connected sequential moves which characterise the generic modelling approach. In particular the study demonstrated the power of pictorial representations in helping the PMT to comprehend and understand the problem and thus enabling him to introduce variable(s), construct mathematical expression(s), use speed-distance- time formulas to form relationships and finally establish a master equation (i.e. mathematical model) to successfully solve the speed-distance- time problem.

Keywords: Equations, mathematics, modelling, problem solving, teacher

INTRODUCTION

Problem solving is a very important aspect of doing mathematics and most mathematics curricula throughout the world emphasize the need for students to be proficient in problem solving. For instance the National Council for Teachers of Mathematics (NCTM, 2000) propose that problem solving should play a central role in the teaching of mathematics. In particular, they define it as follows:

> Problem solving means of engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge and, through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics, but also a major means of doing so. (NCTM, 2000, p.52)

The South African Curriculum for the Further Education and Training Phase (FET), which is embraced in the Curriculum and Assessment Policy Statements (CAPS) for Mathematics also concurs with the afore-mentioned view (DoBE, 2011, p.3)

This paper reports on a mathematical problem solving activity done by a third year pre-service teacher specializing in the teaching of senior phase mathematics, and doing a module focusing specifically on problem solving. The module concentrates on ways of working in mathematics with a particular focus on word problems. In the problem solving activity reflected on in this paper, emphasis is placed on the process of arriving at the solution rather than the solution itself when doing 'word problems'

This implies that it is more important for us to see how students go about doing the problems, the challenges they encounter and how these are resolved. Furthermore the module gives students the opportunity to realize that there are numerous problem solving strategies which can be utilised to provide efficient and elegant solutions to many problems.

The problem solving activity in this study focused on the use of equations as mathematical models to solve a distance-speed-time word problem.

WORD PROBLEMS

We understand word problems in mathematics to refer to a textual description of a real-life or contrived situation with imposed conditions and constraints with superfluous data at times which students have to unravel to distil salient information which will enable them to find an acceptable solution. Hence, to solve word problems, the student must be able to read the text and understand the language in which it is expressed. This implies that mathematical word problem solving involves comprehension and translation of the given information into mathematical language.

According to Chan & Brenner (1998, p.4), three major kinds of knowledge are involved in mathematical word problem solving, namely, semantic, schematic and procedural knowledge. Semantic knowledge is closely related to the comprehension and the translation of the problem. Schematic knowledge involves the understanding of problem types and how they can be solved. Procedural knowledge involves the understanding and proficiency in the performance of mathematical operations or procedures to solve a given problem.

Some mathematics educators like Sangwin (2011) regard mathematical word problem solving as an early entry into mathematical modelling. This could be attributed to the translation of the word problem into a mathematical equation or geometric representation, which can be used a tools to determine the required solution. However, the process of engaging students in the modelling process to solve word problems should not be treated as a linear process but rather as a cyclic process (Burkhardt, 1994; Lesh & lemon, 1993). In this respect, Vershaffel & De Corte (1997) as cited in Ferrucci, Yeap & Carter (2001, p.26) describe the problem-solving process as cycle involving five processes, namely: (1) comprehending and understanding (making sense) of the problem situation; (2) building and developing a mathematical problem that represents and describes the situational elements and relations;

(3) engaging and operating with the mathematical model and rearranging the model if necessary to identify and work with unknown elements; (4) looking back with the aim to interpret and evaluate the outcome of the procedural and thinking processes in terms of the practical situation nested in the constructed mathematical model; and (5) articulating and communicating the solution in meaningful ways.

THE PICTORIAL (MODEL) APPROACH IN SOLVING WORD PROBLEMS

The pictorial (or model) approach has been considered to be an effective strategy to prepare students to engage and respond to unfamiliar situations by thinking flexibly and creatively and solve word problems (Mousoulides, Pittalis, Christou, & Sriraman, 2010). In this respect, Fong (1994) affirms that the pictorial (model) approach helps students understand abstract mathematical concepts by providing a diagrammatic representation of the quantities of a problem and their associated problem structures. After a diagram is drawn, a series of logical steps eventually lead to a solution of the problem. Through this process of drawing diagrams and creating a logical sequence, students are able to develop a deeper comprehension and understanding of the structures of the problem and its known and unknown quantities.

MODELLING WITH EQUATIONS

Many problems in the sciences, economics, finances, medicine, and numerous other fields can be translated into algebraic equations. This is one reason why algebra and the understanding of its structures and rules are so useful. In using equations as mathematical models to solve problems, Stewart, Redlin & Watson (2012, p.57) suggest the following guidelines to help students set up equations which model situations described in words:

1. Identify the variable: Identify the quantity that the problem asks you to find. This quantity can usually be determined by careful reading of the question posed at the end of the problem. Then introduce notation for the variable (call it x or some other letter).
2. Translate from words to algebra: Read each sentence in the problem again, and express all the quantities mentioned in the problem in terms of the variable you defined in Step1. To organize this information, it is sometimes helpful to draw a diagram or make a table.
3. Set up the model: Find the crucial fact in the problem that gives the relationship between the expressions you listed in Step 2. Set up an equation (or model) that expresses this relationship.
4. Solve the equation and check your answer: Solve the equation, check your answer, and express it as a sentence that answers the question posed in the problem.

RESEARCH QUESTION

To what extent does a pre-service mathematics teacher (PMT) pursue the modelling approach to solve a speed-distance-time problem?

RESEARCH DESIGN

This study adopted a qualitative approach anchored in the interpretive paradigm since it allows a researcher to collect textual data which can be thoroughly analysed in order to comprehend human behaviour and understand how people make sense of the contexts and problems confronted (Cohen, Manion and Morrison, 2007, p.21). This study made use of the interpretivist paradigm because we wanted to interpret and track the moves that that a pre-service mathematics teacher pursued in his attempt to develop an equation as a mathematical model, which he ultimately used to solve a speed-distance time problem. Hence, the data for this study was obtained from the said PMT's written solution response, wherein he developed an equation as a mathematical model solve the following distance-speed-time word problem.

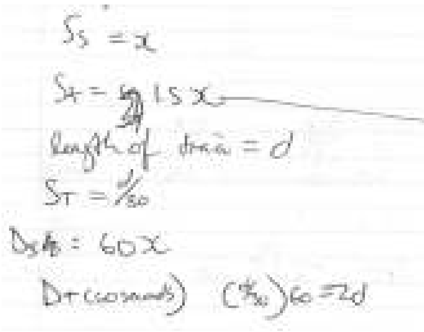
> At a certain place the freeway is directly next to and parallel to a railway line. Two cars on the freeway are travelling in the same direction as the train travelling on the railway line. One car is moving at $1\frac{1}{2}$ times the speed of the other. At a certain instant the drivers of both cars are simultaneously directly next to the rearmost point of the train. The slower car takes 60 seconds to reach the front point of the train. The train itself takes 30 seconds to cover a distance equal to its length. If the train and the cars each travel at a constant speed, how long will it take the faster car to reach the front point of the train? (Reddy, Rambaran, Hansraj, 1987)

DATA ANALYSIS

Data analysis is the process of systematically searching and arranging the raw data with the aim of increasing one's own understanding of the data (Creswell, 2003). The data analysis in this study consisted of examining and categorizing the evidence in meaningful chunks in order to address the research question. In particular, the analysis included identification of data which relates to sections that illuminate the sequential moves that characterize the modelling approach. This approach assisted us in managing the relevant data while disregarding irrelevant information in an attempt to make meaning, and to answer the research question. It is envisaged that the analysis and findings of this study will yield plausible problem solving trajectories and pathways that our pre-service teachers could come to see and understand, and thereby take into the classrooms when they practice as qualified teachers.

DATA ANALYSIS, RESULTS AND DISCUSSION

Ricky **[pseudonym]** initially began by developing a pictorial representation to try and understand the given information in the problem as shown in Figure 1.



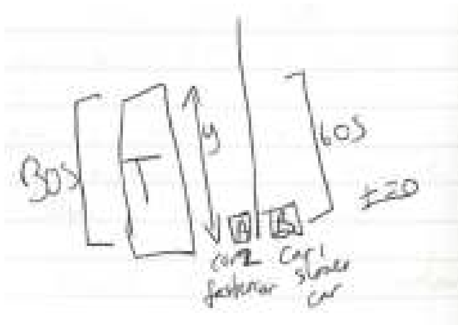
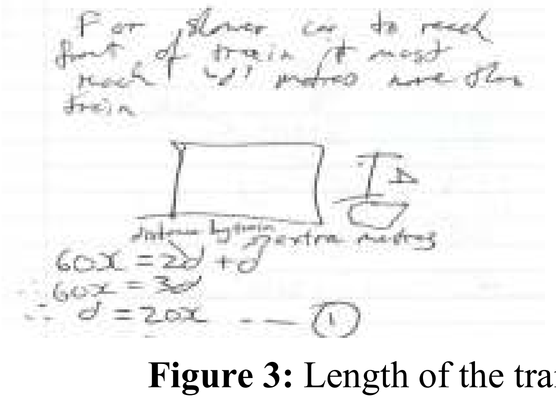


Figure 1: Pictorial Representation **Figure 2:** Identification and use of variables to build expressions

Through making sense of each statement in the problem, Ricky then proceeded to express relationships articulated in the problem using appropriate variables (like x and d) . He initially lets the speed of the slow car (S), which is denoted by S_S, to be x. Then representing the fastest car by the letter f, and using that given information, namely 'one car is moving at $1\frac{1}{2}$ times the speed of the other', he correctly expressed the speed of the fastest car in terms of the variable x as $S_f = 1.5x$. He thereafter assumed that length of the train is d units, and using the fact that 'the train itself takes 30 seconds to cover a distance equal to its length' he moved on to express the speed of the train correctly as $S_T = \frac{d}{30}$.

Having expressed the quantities mentioned in the problem in terms of the variables x and d, Ricky through invoking the formula, which states that $distance = speed \times time,$ established2that the distance travelled by the slow car is given by $D_S = S_s \times t = x \times 60 = 60x$ (see step 5 in Figure 4). Similarly, using the given information that the train travels its own length in 30 seconds, Ricky with the necessary intuition (see step 6 in Figure 4) correctly calculated the distance the train travels in 60 seconds as follows:

$$D_T(60seconds):\ speed\ of\ train \times\ time\ travelled\ by\ train = \left(\frac{d}{30}\right)(60) = 2d.$$



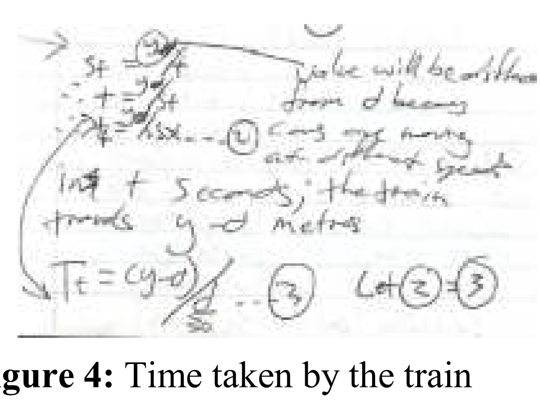


Figure 3: Length of the train **Figure 4:** Time taken by the train

As per Figure 3, Ricky uses a simplified diagram to rationalize that the slow car needs to travel the distance the train travels (represented using capital D) plus an additional distance which is equal to the length of the train as articulated in his assertion: "For the slower car to reach the front of the train it must reach'd' metres more than the train. As evident in Figure 3, Ricky demonstrates understanding of the problem's mathematical concepts and principles, by moving onto expressing the distance of the slow car travelled in terms of 'd' as follows: $60x = 2d + d$, where $2d$ is distance the train travelled and d is the additional distance (which is equal to the length of the train) that the slow car must travel. Establishing this relationship is a crucial step in solving this problem, and it seems that his pictorial representation assisted him to do so and hence proceed to express d in terms of x, namely $d = 20x$, which he defines as his equation (1).

Now reverting to Figure 1, we see Ricky seemed to have indicated on his pictorial representation that the fasted car (f) will travel a distance of y metres. He then assumed that the fastest car will travel for a given time t at the established speed of $S_f = x$ (which he expressed in step 2 in Figure 2) to cover distance y. As seen in Figure 4, he uses these established relationships to determine the amount of time (denoted by $t_f = \frac{y}{1.5x}$), which the fastest car will take to travel distance y, and he calls this his equation 2.

Using his initial assumptions that the fastest car travels 'y' metres and the length of the train is 'd' metres, Ricky accurately computes that the distance the train has to travel in t seconds such that the fastest car meets the front of the train is given by $(y - d)$ metres (see lines 4&5 in Figure 4). Having intuitively reasoned that the time taken for fastest car to meet the front of the train is the same as the amount of time the train travelled, Ricky proceeded to express the total time (T_t) taken by the train in terms of d as follows:$T_t = \frac{(y-d)}{\frac{d}{30}}$, where $(y - d)$ is the distance the train travelled and $\frac{d}{30}$ is the established speed of the train (See line 6 in Figure 4). Up to this stage, we see that Ricky has read each sentence in the problem and expressed all the quantities mentioned in the problems in terms of respective selected variables, and has used pictorial representations to facilitate his development thus far.

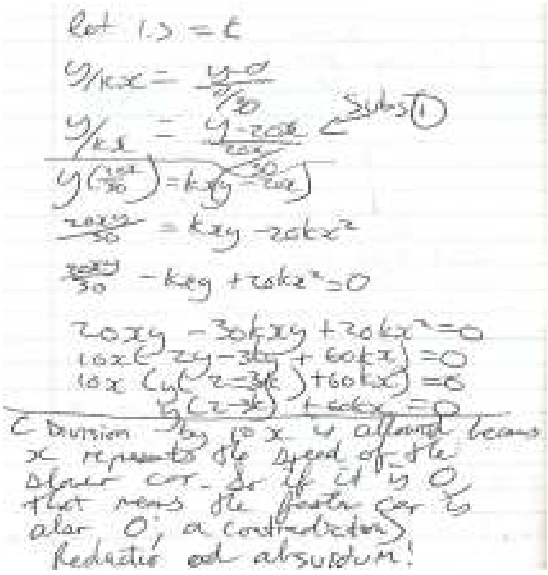


Figure 5: Equation as mathematical model

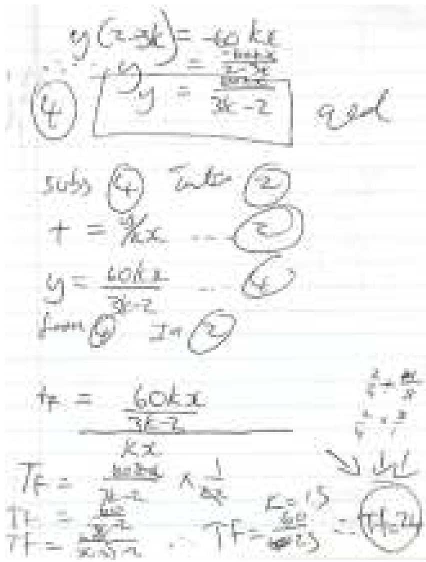


Figure 6: Solving system of equations

Identifying a crucial fact, namely commonality of time that the train and fastest car travelled, Ricky proceeded to uses this critical piece of information to effectively set up an equation (or model) that expresses a relationship between t_f , which represents the time the fastest car travelled, and T_t , which represents the time the train travelled (see Figure 4, where he begins to state let (2) = (3)). In setting (2) = (3), Ricky really meant that $\frac{y}{1.5x} = \frac{(y-d)}{\frac{d}{30}}$. However, for some unknown reason Ricky let the factor 1.5 to be k (see step 1 in Figure 5) but then proceeded to apply all the necessary mathematical procedures, rules and algorithms to systematically establish the following correct equation (as can be seen in step 9 in Figure 5):$10x(y(2-3k)+60kx)=0$.

Interestingly, Ricky seemed to have appropriately reasoned, rationalized and justified for himself as to why he can divide the LHS and RHS of the equation $10x(y(2-3k)+60kx)=0$ by $10x$, through taking cognisance of the fact that division by zero is inadmissible. As explained in Figure 5, he correctly argues that if x, which represents the speed of the slower car is zero, then the speed of the fastest car, which he derived to be 1.5x (see Figure 1), will consequently be zero (as $1.5\times 0=0$), and this will be a contradiction to fact that the faster car travels at 1, 5 times the speed of the slower car. This is undoubtedly a clear and convincing argument, and provides the necessary foundation to support the idea that $10x \neq 0$, and hence the equation $10x(y(2-3k)+60kx)=0$ can be divided by $10x$ on both sides. Consequently, on dividing both sides of the equation by $10x$, he obtained $(y(2-3k)+60kx)=0$, and went on to procedurally solve for y to obtain $y=\frac{60kx}{3k-2}$, which he called equation (4) (see Figure 6).

Even though Ricky interchangeably uses T_f instead of t_f to mean the time of the fastest car (as can be seen in the last 4 steps in Figure 6), he successfully set up a system of equations (as can be seen in Figure 6) and solved them simultaneously to arrive at $T_f = \frac{60}{3k-2}$ (see line 9 in Figure 6). He finally replaces k by 1.5, and simplifies the expression on RHS of the equation to obtain $T_f = 24$(see line 10 in Figure 6), and emphatically ends by writing the following: "the time of the faster car: 24 minutes". Even though Ricky may/may not have looked back to see if his answer was correct or not, he does provide a detailed reflection of his experiences as he pursued to solve the given word problem as follows:

> This was quite a challenge. I might have taken a long approach but it worked out to provide a solution. This problem required a lot of mathematical knowledge on my part because it required me to remember a lot of rules pertaining to functions and basic algebra. My approach was to remember that the train and the cars are all moving at the same time. Initially I could not understand why the speed at which the train was moving was significant but while I was working I realized that the train was in motion while the car was trying to reach the front of the train. Using this idea and the idea ratios and proportions, I was able to generate an equation to represent the distance the fastest car travelled. Once, I had that it was all a matter of substituting the factor by which it was faster than the slower car to get the speed. Finally I was able to determine the time using this knowledge. This problem was quite challenging and required me to think very much out of the box! I enjoyed it and it made realize not to accept everything just at face value but to think deeper about mathematical problems.

Evident in Ricky's reflection is that his engagement with the speed, distance and time problem forced him to not only make direct connections with the concepts of speed, distance and time but rather to engage these concepts in relation to the conditions posed in the problem, and then through deep thinking develop and pursue a solution strategy through making relevant connections with algebraic procedures and rules with the aid of pictorial representations. As evident in presentation of Ricky's solution in this paper, the problem itself involved some critical deep thinking moves, namely: calculating that the train has to travel a distance of $2d$ meters; realizing that the slowest car has to travel d meters more than the train; the train in relation to the distance y meters travelled by the fastest car, would have to travel $(y - d)$ metres; building up a system of equations and relating them via crucial fact in the problem (namely: the time travelled by the train and fastest car such that it reaches the front of the train is the same) and hence constructing a mathematical model (the construction of an equation); solving the mathematical model (equation) in a simultaneous manner to obtain $t = 24$ seconds, which is correctly the required amount of time it would take the fastest car to reach the front of the train. Consonant with the view that deepening-thinking problems should provide an opportunity for students to cognitively stretch their minds to an extent that their procedural and conceptual knowledge are enhanced and their problem solving skills are challenged, Ricky confesses that the speed-distance-time problem was of such a nature and that he enjoyed every bit of it (see last 3 lines of his reflection).

CONCLUDING REMARKS

This article illustrates the use of pictorial representations (i.e. models) to facilitate representations of given information, problem unknowns and constraints in a visual form, which then makes it possible for a student to better understand the underlying data, concepts and conditions governing the problem. In this way as student is able to build mathematical expressions and equations, which can be connected by a crucial fact in the problem that paves the way to find the solution the given problem.

More importantly, the article provides a hands-on exposition of solving a word problem through modelling with equations. The trajectory of modelling with equations in this study began with a pictorial representation of the situation; then the introduction of variables after careful reading and understanding of the problem; then the cyclical expression of all quantities and conditions in the problem in terms of selected and defined variables within the context pictorial representations (i.e. translation from words to algebra); then the identification and justification of a crucial fact in the problem that gives the relationship between established expressions (for example in this instance, it was distilled that the time taken by the fastest car to reach the front of the train can be equated to the amount of time the train travels for this to occur), followed by setting up an equation (or model) that expresses this relationship; and ultimately solving the equation to obtain the required answer. Even though is no concrete evidence to confirm that the PMT looked back to check if answer is correct or makes sense, it is possible that PMT could have looked back and checked his answer. Despite the PMT acknowledging in his reflection that the speed-distance-time problem was challenging it has been encouraging to read in his reflection that he enjoyed every bit of the problem solving process that culminated in the development of a correct solution.

The PMT was able to engage systematically with the various stages of the modelling approach, and this enabled him to solve the speed-distance-time problem with a good degree of success. Finally, the modelling approach articulated in this empirical study, demonstrates how mathematics teacher educators can provide an opportunities for pre-service mathematics teachers to engage with genuine algebraic thinking processes and procedures in authentic problem solving contexts, which they can model to the learners when they begin to teach in schools.

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A REPORT ON A SUPPLEMENTARY TUITION PROGRAMME FOR GRADE 12 MATHEMATICAL LITERARY LEARNERS: IMPLICATIONS FOR TEACHING AND LEARNING

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Mathematical Literacy is a relatively new subject in South Africa. Since its introduction in 2006, Mathematical Literacy has served as a good "alternative" to Mathematics. As a result of increasing learner numbers in Mathematical Literacy, more Mathematics teachers are now teaching Mathematical Literacy. However, declining school enrolments at some schools has resulted in teacher redeployment to other schools. Thus, in smaller schools it is not uncommon to find teachers teaching both Mathematics and Mathematical Literacy. This becomes an impossible task when the teacher is the school principal. This paper examines one such school where the school principal was not able to teach the subject. A successful supplementary tuition programme was arranged for grade 12 Mathematical Literacy learners at this school. This paper interrogates the success of the programme and comes up with suggestions on teaching and learning practices in Mathematical Literacy and Mathematics as well as supplementary tuition programmes in these subjects.

Keywords: learning, mathematics, mathematical literacy, teaching

INTRODUCTION AND BACKGROUND

Mathematical Literacy is a new subject in South Africa, having been introduced in grade 10 in 2006 as an alternative to Mathematics in the Further Education and Training (FET) band. In the CAPS document for Mathematical Literacy, under the heading "What is Mathematical Literacy", the following is stated:

> "The competencies developed through Mathematical Literacy allow individuals to make sense of, participate in and contribute to the twenty-first century world — a world characterised by numbers, numerically based arguments and data represented and misrepresented in a number of different ways. Such competencies include the ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology. Learners must be exposed to both mathematical content and real-life contexts to develop these competencies. Mathematical content is needed to make sense of real-life contexts; on the other hand, contexts determine the content that is needed." (DBE, 2011a, p. 8).

At the same time, in the CAPS document for Mathematics, under the heading "What is Mathematics", the following is stated:

> "Mathematics is a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision-making. Mathematical problem solving enables us to understand the world (physical, social and economic) around us, and, most of all, to teach us to think creatively." (DBE, 2011c, p. 8).

It would seem that from the descriptions for Mathematical Literacy and Mathematics that there are both similarities and differences between the subjects. Both subjects involve the use of numbers, problem solving and decision making. However, they differ in approach. In Mathematics, symbols and notations are used to describe numerical, geometric and graphical relationships. In Mathematical Literacy, learners have to make sense of the numbers which are used to describe numerically based arguments and data represented and misrepresented in a number of ways.

The CAPS document lists five key elements underpinning Mathematical Literacy. These are:

- The use of elementary mathematical content
- Authentic real-life contents
- Solving familiar and unfamiliar problems
- Decision making and communication

The use of integrated content and/or skills in problem solving. (DBE, 2011a, pp. 8 - 10). Four of these key elements may also apply to Mathematics. However, the first one *"The use of elementary mathematical content"* appears to only apply to Mathematical Literacy. When it was announced that Mathematical Literacy would be introduced as a new subject in South Africa, there was a massive teacher upgrade programme where teachers, from subjects where there was an oversupply or subjects which were being discontinued, were selected to do an Advanced Certificate in Education (ACE) in which Mathematical Literacy was to be main focus of study.

Now, more than 10 years later, Mathematical Literacy is an established subject in South Africa with a set of teachers who have been trained to teach the subject. The number of learners doing Mathematical Literacy has also been increasing. Table 1 shows the number of learners who wrote grade 12 Mathematical Literacy or Mathematics in South Africa in the years 2012 to 2015.

Table 1: Grade 12 Mathematical Literacy and Mathematics learners (DBE, 2015, p. 137; 150).

Year	Mathematical Literacy	Mathematics
2012	291341	225874
2013	324097	241509
2014	312054	225458
2015	388845	263903

It is evident from Table 1 that the number of grade 12 learners doing Mathematical Literacy as a school subject has increased quite substantially when compared to Mathematics. This means that there is a need for more Mathematical Literacy teachers. Although some universities are training pre-service Mathematical Literacy teachers, a study by Bansilal, Webb and James recommends that pre-service Mathematical Literacy teacher training be expanded to meet the demand for more Mathematical Literacy teachers (Bansilal, Webb and James, 2015). In the meantime, more schools are using Mathematics teachers (old and new) to teach Mathematical Literacy.

At many schools learners who start with Mathematics in grade 10 may not necessarily end up doing Mathematics in grade 12 two years later. According to the policy, changes of subjects are allowed in grade 10, 11 and 12. In grade 10, a learner may change a maximum of two subjects provided it is done by the beginning of the third term, subject to the approval of the principal of the school where the learner is registered. In grade 11, a learner may change two subjects provided this is done before 28 February, also subject to the approval of the principal. A change of one subject is allowed in grade 12, in exceptional cases. This must be done by 31 January of the grade 12 year and approval for changing the subject must be obtained from the head of the assessment body. Certain documents are also needed for the subject change in grade 12 (DBE, 2011b, p.27).

There appears to be more subject changes in the FET are from Mathematics to Mathematical Literacy, than any other subject changes. There are various reasons for these changes. From personal communication with school principals and Mathematics/Mathematical literacy teachers, the following trends emerge:

- Learners struggle with Mathematics in grades 10 and this continues through to grade 12. Learners have gaps in their mathematical knowledge when starting with grade 10. This is probably due to poor primary school preparation, which impacts on learner performance in grades 8 and 9. It would appear that only the more experienced mathematics teachers, especially at grade 10 level, have the knowledge and experience to support learners and “bridge” the gaps in their mathematical knowledge.

- Mathematical Literacy is seen as a “softer” option. There are perceptions in schools and the wider community that learners have far greater chances of passing Mathematical Literacy than Mathematics and this has an impact on the overall pass rate at the schools. While this is probably true, it is not automatic. Since Mathematical Literacy is a different subject, Mathematical Literacy teachers report that when mathematics learners first arrive in their classrooms (after the subject change), these learners tend to take time to adjust to Mathematical Literacy.

The subject change from Mathematics to Mathematical Literacy also affects the staff allocation at schools. This may result in the redeployment of excess teachers to other schools.

REDEPLOYMENT OF TEACHERS

One of the challenges facing schools in South Africa is the ability to retain and grow learner numbers, thereby maintaining or increasing staff numbers. When learner numbers decrease, it means that the number of teachers should also decrease and that excess teachers should be redeployed to schools where learner enrolment has increased. This is done in an attempt to bring equity in education as far as staff provisioning is concerned (Mona, 1997). The focus of the redeployment process lies in the effective and efficient employment of skills and resources. The term resources is all encompassing and includes human, technological, power, financial and information (DOE, 1998).

Redeployment has been on-going in the various provinces and has affected many schools. One such school (which the writer calls school L) is located in the Eastern Cape. In mid-May 2014, the school principal met with the writer to discuss how redeployment affected his school. His school, with declining school enrolment, had its Mathematical Literacy teacher redeployed at the end of February 2014. The principal, who was also the Mathematics teacher, was the only teacher left on the staff who could teach Mathematical Literacy. Thus, at this school, only the school principal was able to teach Mathematics (grades 8 – 12) and Mathematical Literacy (grades 10 – 12). Clearly, this would have been quite a difficult and virtually impossible task.

Since this was a no- fee school, it was not possible for the school governing body to employ a Mathematical Literacy teacher as it did not have the necessary funds. After some discussions with the school principal, the writer decided to assist the grade 12 learners at the school with supplementary tuition (extra classes) in Mathematical Literacy.

LITERATURE SURVEY

There are various types of outreach or support programmes for school learners. These programmes usually involve supplementary tuition. In South Africa, it would appear, through a variety of reasons, that certain high schools, usually in less affluent areas, are not able to give learners quality instruction that is expected in subjects such as Mathematics and Physical Science. Learners from these and other schools are always looking for extra classes where supplementary tuition is provided.

Mogari, Coetzee and Maritz (2009) report on research by Ireson and others which show there is a strong relationship between learners' opportunity to participate in supplementary tuition and their socio-economic background. Lauziere (2010) speaks about American parents waiting until there is a crisis at school before seeking help in mathematics and science. This is probably true for South African parents as well. However, in the South African context, only the middle and upper classes can afford supplementary tuition by a private tutor. The prohibitive cost of private tuition puts it out of reach for many South African households, hence the need for funded outreach programmes.

A case study by Tshabalala and Khosa in the Hwange district of Zimbabwe reveal that most teachers believe that supplementary tuition assisted a number of learners to perform better in their examinations. The study also shows that almost all the teachers took part in supplementary tuition, including those who had not specialized in teaching the subject, proving that Mathematics is a very popular subject among those learners engaging in supplementary tuition (Tshabalala & Khosa, 2014).

In South Africa, the provincial departments of education and districts usually arrange supplementary tuition (extra classes) in subjects such as Mathematics and Physical Sciences for learners from underperforming schools. These schools are usually located in less affluent areas. Despite the large sums of monies spent, it is the writer's experience that the returns from these classes, in terms of improved learner achievement, are very minimal. When developing a supplementary tuition programme, there are various factors to take into account, two of which could be regarded as key to the success of the programme. These are the quality of the facilitator (tutor) and the resource materials used in the programme (Govender, 2011).

It was these two key elements which the writer considered when planning the supplementary tuition programme in Mathematical Literacy for learners in school L (as explained in the previous section).

RESEARCH QUESTION

The following research question was posed for this study:

"What is the impact of a supplementary tuition programme for Mathematical Literacy on grade 12 learners' performance in the subject?"

The following sub-questions were formulated in the context of the research question:

- What should a supplementary tuition programme for Mathematical Literacy consist of?
- What teaching approaches are suitable in a supplementary tuition programme?
- How do learners respond to teaching in a supplementary tuition programme for Mathematical Literacy?
- What impact, if any, does a supplementary tuition programme for Mathematical Literacy have on learner results?

THEORETICAL FRAMEWORK

This study examined the impact of a supplementary tuition programme on learner performance in Mathematical Literacy and consisted of various parts. As stated earlier, two of the key factors in any supplementary tuition programme are the learner support materials used and the teaching. School L was located in a less affluent area of the district. Although it would have made sense for the writer to go to the school to do the sessions, it was not possible. The writer usually uses his personal computer (with a writing pen) for presentations or teaching. All his resource materials were available electronically (power-point slides and exam papers). Thus, he would be to using a data projector and computer during the sessions. The school did not have electricity so these sessions could not take place at the school. Instead, the learners (all 13 of them) were transported to a central venue (where the writer's offices were located) for these contact sessions. A suitable theoretical framework for this study would be that of socio-culturalism as proposed by Vygotsky. His theory states that the development of a learner's intelligence "results from social interaction in the world and that speech, social interaction, and co-operative activity are all important aspects of this social world" (Sutherland, 1993, p.104).

Prior to this study being undertaken, there was little or no social interaction with respect to Mathematical Literacy in the learner's normal school environment as they did not have a teacher in the subject. The supplementary tuition programme, described in this study, took place in an environment which was different from the learners' normal school environment. They were now placed in a social setting which was, probably, far more conducive for learning, with social interaction and co-operative activity all contributing to this learning. Vygotsky also describes a "Zone of Proximal Development" (ZPD) as the distance between the level of development of a child (when working on problems) and his or her level of potential development when working with an adult. The adult is the one driving the learning until the child is able to internalise the knowledge ((Vygotsky, 1978).

It would appear that, in school L, the children's learning of Mathematical Literacy had progressed well in grades 10 and 11 and they reached a certain level of development in the subject. Unfortunately, this development stalled when their teacher (the adult as described by Vygotsky) was redeployed early in their grade 12 year and they were not taught for three months. Although they may have had the necessary background in the subject, it would have been highly unlikely for them to reach their potential without the necessary support of a qualified Mathematical Literacy teacher. It was imperative for these learners to get support to build on what they already knew and acquire new knowledge.

A key feature of this study would be the teaching approaches used in the supplementary tuition programme. Fleming (2006) describes four learning styles in his VARK model of learning, namely, visual (V), auditory (A), read/write (R) and kinaesthetic (K). The teaching approach used in this supplementary tuition programme took this VARK model of learning into account.

OPERATIONAL STRATEGY AND THE COLLECTION OF DATA

There were 13 learners who did Mathematical Literacy at school L. The learners were brought to a central venue for the contact sessions. There were six contact sessions (each of approximately 2 ½ hours duration), four before the June examination, and a further two in the third term, just before the trial examinations. At the beginning of the third term, the school, which acquired some funds in the meantime, appointed a School Governing Body (SGB) teacher who was in her first year of teaching. Despite having a teacher, learners requested further sessions with the writer. Due to various commitments, the writer could accommodate them for only two more sessions. An interesting dynamic to this study was the issue of language. All the learners were Xhosa speaking and the sessions were conducted in the LOLT (language of learning and teaching) which was English (the language spoken by the writer).

The contact sessions comprised the following:

Session 1: Foundational Knowledge for Mathematical Literacy

Session 2: The mathematics in Mathematical Literacy

Session 3: A discussion of the content examined in Mathematical Literacy Paper 1 using the exemplar as a guide

Session 4: A discussion of the content examined in Mathematical Literacy Paper 2 using the exemplar as a guide

Session 5: Revision of the content examined in Mathematical Literacy Paper 1 using the June 2014 as a guide

Session 6: Revision of the content examined in Mathematical Literacy Paper 2 using the June 2014 as a guide

Details of these contact sessions will be discussed under the results section of this study. After session 4, learners wrote the common Grade 12 June Mathematical Literacy papers. These papers are usually set for underperforming schools in the district. Although school L was not an underperforming school, it was included as there was no Mathematical Literacy teacher at the school. Learners were also given a questionnaire to respond to various issues, including their experiences of the contact sessions. This questionnaire was administered after session 4. In the questionnaire they had to respond to the following questions or issues:

- State why they selected Mathematical Literacy as a school subject and what they thought of the subject
- Indicate how long had they been without a Mathematics teacher
- Describe the classes they attended (including the teaching)
- State what they learnt at these classes
- Comment on the effect of the classes on their ability to work through Mathematical Literacy problems, their confidence as a learner in the subject and on their results/performance in Mathematical Literacy
- Give other comments , not covered in the questionnaire

Sessions 5 and 6 took place prior to the trial examinations. Thereafter, learners wrote the trial examinations and then the final examinations. The six contact sessions, learner responses in the questionnaires and the learner results from the various examinations (June; Trial and Final) provided the data for this study.

RESULTS

The data

This study involved the collection of both qualitative data (description of the contact sessions and learner responses in the questionnaires) and quantitative data (learner results in the examinations). This approach allowed for the triangulation of data.

Contact sessions with learners

As stated earlier, there were six contact sessions with the learners. A description of what transpired in each session is given here:

Session 1: Foundational knowledge for mathematical literacy

In the presentation on foundational knowledge for mathematical literacy, the following key content and skills was discussed, with examples given. To consolidate learning, children were given some questions to work through where appropriate:

- Working with numbers in context
- Rounding off and estimation
- Solving simple "word problems" through estimation and then

- checking the calculations
- Working with different types of numbers and fractions
- Ratio and rate
- Direct and inverse proportion
- Scales and conversions
- Length, perimeter, area and volume
- Measures of central tendency such range, mode, median and mean
- Differences between a histogram and bar graph

Session 2: The mathematics in Mathematical Literacy

This presentation emphasised the importance of basic or elementary mathematics when working with Mathematical Literacy and covered the following mathematical content. The content was integrated into relevant contexts.

- Simple interest
- Reading and interpreting graphs
- Percentage increase and decrease
- VAT calculations
- Interpreting and doing calculations from tax tables
- Inflation and exchange rates
- Distance, surface area and volume
- Data handling – interpreting pie charts and tables
- Scale drawings and maps
- Measures of central tendency, including lower and upper quartiles
- Working with different types of graphs and interpretation thereof

Session 3: Content examined in Mathematical Literacy P1 (the exemplar)

After the first two sessions, the writer believed that learners would have sufficient background knowledge to work through the exemplar papers. Learners were given selected problems to work through and these were then discussed during the session. Problems not covered in class were given as homework. The following content was discussed during the session:

- Reading and interpreting financial documents
- Cell phone costs in word and table forms
- Measurement using baking as a context
- Measurement using vegetable gardens as a context
- Working with data in tables and pie charts
- Working with scales using floor plans

- Using maps to identify roads and determine directions
- Tax tables
- Measures of central tendency within the context of pocket money

Session 4: Content examined in Mathematical Literacy P2 (the exemplar)

After the first three sessions there was a need to give learners exposure to higher order questions in Mathematical Literacy. Thus, the writer decided to use the Mathematical Literacy P2 exemplar for this purpose. The writer asked the learners to work through certain problems in the paper. These were then discussed in class. The writer made learners aware of the background knowledge needed for each problem. The following content was discussed during this session:

- Time, distance and area calculations, venue cost, profit and graphs
- (using Dinner & Dance as a context)
- Conversions of units, surface area calculations, maps and distances
- Measures of central tendency, tables and graphs , box and whisker
- plots (car sales)
- Tuck-shop sales involving percentage increase, profit, interpreting
- tables and the layout of the tuck-shop.

After session 4, learners wrote the common grade 12 June examination papers for Mathematical Literacy. Sessions 5 and 6 were held prior to the trial examinations as requested by the learners of school L.

Session 5: Revision of the content examined in Mathematical Literacy Paper 1 (using the June 2014 as a guide)

The content and contexts discussed during this session were similar to the ones for session 3. The writer went through the June exam paper 1 and highlighted key aspects of the content.

- Measures of central tendency (pocket money)
- Reading and interpreting municipal accounts, percentages and
- VAT calculations
- Salary and tax calculations
- Perimeter and area of circles and rectangles
- Interpretation of graphs involving different rates of working and
- break-even point
- Tables and pie charts
- Floor plans and maps
- Exchange rates, volume, mode and upper quartiles

Session 6: Revision of the content examined in Mathematical Literacy Paper 2 using the June 2014 as a guide

The content and contexts discussed during this session were similar to the ones for session 4. The writer went through the June exam paper and highlighted key aspects of the content, especially questions involving multi-step procedures and reasoning and reflecting.

- Calculating and comparing supermarket purchases
- Income and expenses for social function
- Taking out a loan at simple interest to cover purchases
- Area of rectangular surfaces; tiling and volume
- Ratio, percentages, fractions, time and usage of fuel; prediction of
- future petrol price
- Time and distance calculations involving the "Gautrain"
- Maps and direction
- Interpretation of data in table form, including quartiles

TRENDS EMERGING FROM THE CONTACT SESSIONS WITH LEARNERS

It is evident that the sessions with the learners from school L was comprehensive and covered a wide range of mathematical literacy topics. The first two sessions laid the foundation for this supplementary tuition programme and the follow-up sessions enabled learners to use their knowledge and skills to answer questions of different cognitive levels. This gave learners much needed practice in preparation for the June, Trial and Final examinations. Further, the use of English during the sessions did not prove to be an obstacle to learning. All learners were comfortable with the use of English and expressed themselves very well when the writer asked questions during the classes.

The questionnaire (administered after session 4)

The questionnaire was administered to learners after the fourth session. Once again, language was not an issue with all learners expressing themselves very well in English. The learners' responses were analysed with a view to coming up with trends and patterns of coherence.

On their selection of Mathematical Literacy as a subject

Learners responded very positively to this statement. Most of them stated they *"loved"* the subject and were pleased by its utilitarian nature and use in everyday life. In this regard, they mentioned calculations involved in measurement, and the analysis of financial documents such as utility bills, income and expenditure statements and salary slips. They found the subject *"interesting"* and *"less difficult"* than Mathematics.

The thoughts of learners on Mathematical Literacy as a school subject

For most learners it was their third year of doing Mathematical Literacy so they were accustomed to the subject. Once again, their responses were favourable. Their actual thoughts are indicated here with some editing to ensure correct spelling and grammar:

> "It is easy to calculate and I enjoy it"; "I get high marks in Mathematical Literacy"; "It gives us more knowledge and what is happening in our daily lives"; "Some of my friends struggle with the subject but it is my favourite subject"; "It is a good alternative to Mathematics"; "I learn about various things like currencies"; " It is a good alternative to Mathematics"; "It is a useful subject: we can calculate water and electricity consumption according to the given tariffs and learn not to waste water and electricity)"; "It is a good subject and sharpens the brain on decision making in our daily life"; "It is a good subject and enables us to understand about things in our daily lives and what we will experience in the future"; " It is a good alternative for those who do not do well in Mathematics."

Period for which learners were without a Mathematical Literacy teacher

All learners gave the same response; three months. It was this critical fact which convinced the writer to assist these learners with supplementary tuition.

Description of classes conducted by the writer

The learners responded very favourably to the question. Their actual words are indicated here with some editing to ensure correct spelling and grammar:

> "The classes were excellent and helped me a lot. Even when you study at home you remember what was said"; "It was fascinating"; "It was awesome; Dr Govender is a good teacher and he helped us a lot as we did not have a teacher for quite some time"; "I was very happy with the classes as it made me understand the work". "Dr Govender helped us a lot; we struggled without a teacher so it was good to get help from Dr Govender"; "I enjoyed the teaching which helped me in some things I did not understand"; It made me solve problems which I did not know how to solve"; "It was quite interesting and understandable"; "It was a good experience"; "It was a quiet place where one could study with no disruptions"; "The atmosphere was nice and comfortable for learning"; "We were able to understand since a data projector was used and this gave us a better picture of the calculations."

Learning at these classes

Most learners were very specific about what they learnt at these classes and their responses tended to triangulate with the data from the contact sessions. Included in their responses were:

> "How to calculate percentages"; "Conversions of units"; "How to analyse break- even point"; "Perimeter, area and volume" ; "More knowledge in the subject"; "I learnt a lot about VAT calculations (including/excluding VAT) as we had not done this before"; "Percentage increases and decrease"; "Different calculations"; "Revision of key topics"; "Simplification (ratios and fractions)"; "Reading and interpreting tables and graphs"; "Distance and speed"; "Topics from past year papers".

Description of the teaching at these classes

There was some overlap with the earlier question on the description of the classes. Once again, the learners responded favourably. All learners complimented the writer on his ability to make the classes interesting and facilitate learning. They were very pleased with the classes as everything was *"explained very well"*. They were exposed to *"more difficult"* work and it was *"very interesting"*. They also commented that the writer made things very clear to them using language they could understand. They were impressed with the use of technology (data projector) as this was the first time they were taught in this manner. The work on screen was clear to them. They enjoyed working through the problems given to them and the subsequent discussion of the solutions to show them where they "went wrong". Although the teaching was at a good pace, some mentioned that the pace was "fast at times". However, the learners were very happy that they could stop the writer and ask questions when they did not understand.

The effect of the classes on their ability to solve Mathematical Literacy problems

It would appear that the classes left an indelible mark on the learners. All claimed that they were now better prepared to work through Mathematical Literacy problems. For purposes of impact, the actual words of learners are indicated here:

> "I used to struggle with calculating perimeter and area and conversions and now I know how to work these out"; "I learnt different methods to solve problems"; " I know how to work with numbers and can solve any numerical and financial problems in Mathematical Literacy"; "I can finally solve difficult Mathematical Literacy problems"; " I have improved my speed in calculations"; "I can calculate percentages"; I used to struggle with conversions and VAT calculations but I can do these now"; "I am able to better understand Mathematical Literacy questions"; "The classes gave me good preparation to solve problems"; "I can solve many problems with the knowledge that I now have".

The effect of the classes on their confidence in Mathematical Literacy

One often hears from Mathematical Literacy teachers that their learners tend to lack confidence in the subject, especially in grade 10. Although this confidence tends to grow in grades 11 and 12, one still finds learners in grade 12, lacking in confidence. However, after these classes, it would seem that learners from school L gained more confidence in Mathematical Literacy. Once again for impact, their own words are indicated here:

> "I have become more confident in Mathematical Literacy"; "I have a high confidence"; "I am faster with my calculations and can understand questions better"; "I am better and can work out difficult problems"; I have a high self-esteem and I have developed more confidence"; "I have gained a lot of confidence"; "I am a better learner in the subject and will pass"; " My confidence has been boosted and I will be able to solve problems."

The effect on the classes on their results/performance in Mathematical Literacy:
As described above, learners' confidence in Mathematical Literacy increased after the contact sessions. This increased confidence came to the fore when they had to state how they would do in the subject at the end of the year. All responded that they would do well and their actual responses are captured here:

> "I will perform well and get good results"; "I will be able to pass the subject at a moderate or adequate level of achievement"; "I think my results will improve"; "My results dropped in term 2 but I think my results with improve"; "I am lazy but I know I will do well"; "I will get good performance"; "I will get good results"; "I will be able to pass with a good percentage"; "It will close the gap of not having a teacher and I will get better results"

Other comments
Despite this part of the questionnaire being open-ended, most of the comments given were in praise of the supplementary tuition programme conducted by the writer.

The learners expressed their thanks, appreciation and gratitude for the classes conducted by the writer. One learner expressed a wish for the writer to do the same for others who needed help. When they were without a teacher, they were not taught and were struggling with Mathematical Literacy. The classes came at the right time and helped them. They were grateful for the concern shown by the writer and the support given. All expressed a wish for more classes before the trial examinations classes.

Trends emerging from the responses in the questionnaire

Learners provided rich data in the questionnaires. They were able to explain why they chose Mathematical Literacy as a school subject and give their thoughts on the subject. They were able to describe the classes held by the writer and what they learnt from these classes. In fact, most of them were very specific about what they learnt and this triangulated well with the details given by the writer. They were full of praise for the writer on his ability to get the best out of them. Learners indicated that they were better prepared to solve problems in Mathematical Literacy and their confidence was high. All indicated that they would do well in their final examination. This was very heartening to note, especially since they were without a teacher for three months.

Learner results
As stated earlier in the paper, learners from school L wrote the Grade 12 June common district papers, the provincial trial examination papers and the final national examination papers for Mathematical Literacy. Their results (including the final grade 11 mark) are shown in table 2. All results are recorded as percentages for easy comparison.

Table 2: Mathematical Literacy exam results for learners at school L

Learner	Grade 11 Final mark Nov 2013	Grade 12 June Final 2014	Difference June – Nov	Trial Sept 2014	Final Nov 2014	Difference Final - trial
Max mark	100	100	+/-	100	100	+/-
1	59	47	-12	28	43	+15
2	40	46	+6	26	38	+12
3	79	77	-2	73	75	+2
4	44	54	+10	33	51	+18
5	46	57	+11	38	51	+13
6	60	59	-6	38	48	+10
7	54	42	-12	37	39	+2
8	61	55	-6	33	50	+17
9	32	45	+13	28	42	+14
10	32	42	+10	41	48	+7
11	29	18	-11	20	30	+10
12	30	45	+15	27	50	+23
13	30	48	+18	43	47	+4
Class Ave%	45.8	48.8	+2.6	35.8	47.1	+11.3
Below 30%	1	1		5	0	

The learners' results are self-explanatory. The class average increased, by 3% from November 2013 to June 2014, despite them not having a teacher for three months. Unfortunately, there was a big drop in the class average for the trial examinations with five learners obtaining less than 30%. Of these, three were close to pass mark of 30%. Despite the set-back in the trial examinations, the class average increased by 11.3% in the final examinations. In fact, all marks went up in the final grade 12 examinations. The increases range from a small 2% to a very big 23%.

The class average for the grade 12 final examinations were marginally up by 1,3% when compared to their grade 11 results, a year earlier.

FINDINGS

The findings of this study are listed in the context of the research sub-questions and then the research question. Firstly, the research sub-questions are discussed:

- A supplementary tuition programme for Mathematical Literacy should be well-organised and structured to enable learning. It should consist of mathematical content relevant to Mathematical Literacy and give learners the opportunity of working through questions of different cognitive levels. These questions could come from text-books, exam papers and other sources. When learners work through questions after being taught certain topics it helps them to internalise the knowledge.
- One must take into account that learners who normally do Mathematical Literacy are not necessarily the top learners at school. However, as indicated in this study, they are willing to learn provided the teaching approaches used are conducive for learning. This means that the teaching approaches should take into account that learners have different learning styles. As described earlier, the teaching approaches used during the sessions took into account Fleming's VARK model of learning (Fleming, 2006). In this study, the writer projected all material onto a screen using a data projector. Learners were able to see clear formulae, calculations, pictures, diagrams and graphs. This catered for the "visual" aspect of their learning. It would appear from the responses in the questionnaire that the writer facilitated the contact sessions in simple language which learners were able to understand. This enabled the learners to learn by "listening", thus, ensuring that "auditory" learning also featured during these sessions. Learners were given the opportunity of reading the information and writing down key tips when working through calculations. This catered for the read/write aspect of their learning. During the sessions learners were taken from the basics to more complex material. This was done in a developmental manner with learners given a number of questions to work through. This meant that learners were involved in "hands-on" or "kinaesthetic" learning. Thus, the writer provided for a combination of all four learning styles of the VARK model during the sessions with learners.

- As mentioned earlier in the paper, many outreach programmes for learners do not have the desired outcomes because of various factors, two of which are the materials used and teaching in the programme. The learners at school L were without a teacher for three months and were, probably, without hope. This supplementary tuition programme came at the right time for them. They responded positively to the programme because someone cared about them and gave them much-needed support. This support was given in a new environment by an experienced teacher who used materials and teaching approaches which encouraged their learning.
- This programme helped to build on what learners knew from grades 10 and 11 and attempted to consolidate their learning. It probably made the task of the SGB teacher, who was appointed in the third term, easier as she just needed to focus on revision rather than teaching learners with knowledge gaps. Usually, the success of any supplementary tuition programme is dependent on learner performance. As is shown in table 2, learners did extremely well in the final examinations. All 13 learners obtained the minimum 30%, with 10 of them getting more than 40%. Despite not have a Mathematical Literacy teacher for three months, all learners passed the subject. It would be reasonable to claim that the supplementary tuition programme impacted positively on learner results in Mathematical Literacy. In fact, school L was one of 24 schools in district with a 100% pass in the subject.

After an analysing the responses to the research sub-questions in this study, it is now possible to answer the research question, "*What is the impact of a supplementary tuition programme for Mathematical Literacy on grade 12 learners' performances in the subject?*".

The supplementary tuition programme was well-structured and organised. It had an experienced teacher who used teaching approaches which catered for different learning styles. The materials covered all what learners needed to know in Mathematical Literacy. Learners responded positively and were active in the learning process. They gained confidence in the subject and were able to work out all types of questions, from the easy to the more difficult.

A learner's performance in a subject is more than just the result that the learner obtains in the subject. It is how the learner views the subject, responds in class and works thorough problems or questions in class (or at home). If the answers to all of these questions are positive, then the learner should perform well in examinations. From the learners' qualitative feedback in the questionnaire and the results in the various grade 12 examinations, especially the final examinations, one can conclude that the supplementary tuition programme was fairly successful and impacted positively on the learners' performances in Mathematical Literacy.

CONCLUSION

Although the sample in this study was small with 13 learners participating, the study yielded some rich data which the writer could work with. This study has major implications for the teaching and learning not only of Mathematical Literacy but other subjects such as Mathematics and Physical Sciences. These include:

- Teachers should be well prepared and take the curriculum demands and their learners' needs into account when preparing their lessons. It is very likely that well-prepared lessons can lead to better learning outcomes.
- Children learn using different learning styles. It is important for teachers to prepare for a combination of these learning styles (visual; auditory; read/write and kinaesthetic).
- Teachers should ensure that the materials used during their lessons are appropriate and should give learners the opportunity of working through problems which are easy and then moving onto to more complex ones. In this way, learners will become better and more confident in the subject matter.
- Technology can help enhance learning. However, most teachers may not have access to technology at their schools. The lack of technology should not inhibit learning. Teachers can use coloured chalk and write clearly on the board so learners can see formulae, calculations, simple diagrams and graphs. Where possible, pictures and complex graphs and diagrams may be copied for use by learners.
- Language need not necessarily be a barrier to learning. Most South Africans learn through a second language, usually English. If a teacher uses simple, clear and understandable language in class, then learners do not feel alienated and are willing to participate. At FET level, they should also be encouraged to express themselves in the language of teaching and learning (LOLT). This practice will count in their favour when they answer questions in class exercises, tests or examinations.
- An important factor is the environment in which teaching and learning takes place. A well-prepared teacher and a conducive learning environment is likely to impact positively on learner performance and results in their subjects.

Poverty, inequality and location continue to impact on education in South Africa. It is common knowledge that learners from the "advantaged" communities in South Africa go to some of the top schools and probably have some of the best teachers. The consequence of this is that these learners usually get good results.

However, as this small-scale study has shown, learners from “disadvantaged” communities, where poverty is rife, can also do well, provided they work in conditions which are conducive to learning. In this regard, various role players, including district officials and parents should do more to promote quality teaching practices at schools in “disadvantaged” areas.

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THE ASSESSMENT OF EUCLIDEAN GEOMETRY IN GRADE 12 MATHEMATICS PAPERS: A COMPARISON OF TWO EXTERNAL EXAMINATION PAPERS

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Euclidean Geometry has always been an integral part of the Mathematics curriculum in South Africa. However, poor learner performance in Euclidean Geometry resulted in it becoming optional at South African schools from 2008 till 2013. This meant that a large percentage of mathematics learners left school without having done Euclidean Geometry in these years. As a result of submissions made by various stakeholders such as AMESA and the universities, Euclidean Geometry was made compulsory and assessed as part of Grade 12 Mathematics paper 2 with effect from November 2014. This paper examines the assessment of Euclidean Geometry in the grade 12 examinations in 2014 and 2015. There are notable differences with regard to the way Euclidean Geometry was assessed in both these years and these are highlighted in this paper. The paper argues for a more innovative way of assessing Euclidean Geometry to make it more accessible to learners rather than the "old" style of setting such papers which made the papers more difficult and beyond the scope of most learners.

Keywords: Assessment, Euclidean geometry, mathematics

INTRODUCTION AND BACKGROUND

Euclidean Geometry has always been an integral part of the South African mathematics curriculum. When the NCS was introduced in grade 10 in 2006, Euclidean Geometry in the FET was optional. In 2008 when the first group of NCS learners wrote the grade 12 examinations, Euclidean Geometry was included in Mathematics Paper 3, an optional paper. One of the main reasons for making Euclidean Geometry optional in South Africa at the time is that the teachers were not familiar with the content (Bowie, 2009). It was not taught well at schools and learner performance was poor. Thus, during the period in which Euclidean Geometry was optional, there would be an intensive teacher training programme in Euclidean Geometry and other optional topics. Teacher training would be conducted by the Department of Education and other stakeholders such as universities and NGOs.

However, this did not appear to be the case and that this training only commenced in 2011 when the when it became known that Euclidean Geometry would once again be compulsory (and assessed as part of Mathematics paper 2) with the introduction of the Curriculum and Assessment Policy Statement (CAPS) as the revised curriculum in South Africa. Thus, from 2008 till 2013 only a few learners in South Africa completed Grade 12 Mathematics with Euclidean Geometry.

Not having done Euclidean Geometry and other paper 3 topics would have some serious implications for learners going on to study mathematics related programmes at universities. A newspaper report in the Johannesburg Star (1 April 2010), suggested that the then new NCS mathematics was “not up to university standards”. The report stated that “first year university students are struggling in maths subjects far more than in previous years”. Lecturers had claimed that “first year students were not coping with maths at university – meaning that the standard had dropped significantly with the new curriculum”. This had resulted in universities devising bridging programmes in mathematics to assist students. Universities had also encouraged learners to do Mathematics P3 as it prepared learners better for the conceptual skills needed for mathematics at university level (Serrao, 2010).

However, not many learners did Mathematics P3 (the optional paper). Table 1 shows the number of learners writing the Grade 12 Mathematics exams and the numbers who did Mathematics P3, which included Euclidean Geometry for 2010 to 2013. This data was gleaned from the DBE diagnostic reports for 2013. The fourth column has been added to show the percentage of learners who did paper 3 (DBE, 2013). [NB: Paper 3 was discontinued in 2014].

Table 1: Number of learners writing Mathematics and Mathematics P3.

Year	Number of Mathematics learners	Number of learners doing Mathematics P3 (optional)	Percentage of learners doing Mathematics P3
2010	263034	9454	3,59%
2011	224635	8871	3,94%
2012	225874	8878	3,93%
2013	241509	9302	3,85%

The figures show that the percentage of learners who did Euclidean Geometry at Grade 12 level ranged from 3, 59% to 3, 94%, which is very low. Govender (2010) outlines reasons for the low percentage of learners who did Mathematics paper 3. Some of these were:

Teachers at the schools did not have the required capacity to teach paper 3 topics such as Probability and Euclidean Geometry. Learners did not take up Mathematics P3 because of its optional status and added workload. Learners did not want to spend time on content they thought would have no benefit to them.

When the newly formed Department of Basic Education (DBE) came into existence in 2009-2010, it came up with a proposal to include most of the Mathematics P3 content into two mathematics papers. Euclidean Geometry would be included in paper 2 and Probability in paper 1, thus, circumventing the need for an optional third paper. Other proposals by the DBE included the following:

- Technical High Schools should have a choice of offering either Engineering Mathematics or Mathematics in place of their current offering of Mathematical Literacy or Mathematics.
- Engineering Mathematics then would be designed to suit learners who would pursue trade industries and Mathematics would be for learners who would like to go to universities and universities of technology (formerly Technikons). Mathematical Literacy did not provide learners with the necessary skills they would need in trade and technical industries.
- The training of teachers on the Euclidean Geometry and Probability would be done during school holidays for not less than two weeks in 2011, so that this could be implemented in 2012 in grade 10. (Govender, 2010)

(NB: Technical Mathematics and Technical Science are being offered for the first time in Technical Schools with effect from January 2016).

The Association for Mathematics Education of South Africa (AMESA) was in agreement with the DBE proposals and came up with some recommendations. Only the ones relevant to this paper are stated here:

- There should be thorough continuous in-depth training for teachers in geometry, probability and all aspects related to Engineering Mathematics. The training should be well organized and compulsory for all educators and no one should be left behind.
- The DBE should get experts to develop the training materials in Geometry and Probability, so that training and learning could be effectively accomplished at all levels with much understanding and confidence.

- Since the core mathematics curriculum will become overloaded, with the inclusion of both Probability and Euclidean Geometry as compulsory content, the writing team should take cognizance of the available teaching time, and prepare a detailed teaching work schedule for each affected grade. The work schedule and pace setters should clearly indicate amongst other aspects, at least the topics/content and assessment standards that will be dealt with during each week during a given academic year, grade-wise. The developed work schedules should be work-shopped with all affected educators.
- There should be continuous supervision and monitoring at regular intervals at all levels of curriculum implementation. Unless, this is done diligently, the educators will always cite “time constraint” as an excuse if they do not complete the curriculum in a given grade in time for the final examinations or if their learners perform poorly in any of the examinations, particularly the grade 12 NSC examinations.
- The writing team should decrease the transformation geometry because it is currently over-weighted. This could be one possible way to create some space for the weighting of Euclidean Geometry in Paper 2.
- More space could be created for Euclidean geometry with the leaving out of “reciprocal ratios” and the “tan compound angles” as these skills could easily be acquired later in tertiary study. (Govender, 2010).

Most of the AMESA’s recommendations were implemented when the NCS Grades R-12 (commonly known as CAPS) was introduced in grade 10 in 2012. There was also intensive nationwide training in Euclidean Geometry, Statistics and Probability for teachers in 2011 (grade 10), 2012 (grade 11) and 2013 (grade 12). However, the true impact of the changes in the FET Mathematics curriculum and the training of teachers would be seen when the first grade 12 Mathematics examinations (set according to CAPS) was written at the end of 2014. One of the key issues would be the types of questions set for Euclidean Geometry which would be compulsory for the first time in grade 12 examinations since 2007.

LITERATURE SURVEY

Euclidean Geometry is an important branch of mathematics across all school grades. It allows for school learners to understand the world by comparing shapes, objects and their connections. This literature examines some research into the teaching and learning of Euclidean Geometry. Kutama (2002) did research into process-based instruction in grade 8 and 9 Euclidean Geometry and came up with the following recommendations:

- Geometry worksheets should be developed to take into account each learner's van Hiele level of thinking. The language should be simple and learners should be encouraged to communicate their ideas in their mother tongue; talking to each other about their experiences will make them confident in communicating thought; may facilitate progress to higher van Hiele levels of thinking.
- Knowledge of concepts and theorems is not sufficient to equip them with skills to solve problems; the more they practice the more skills they acquire; start with simple problems in order to boost their confidence; then attempt more difficult problems.
- A variety of resource materials should be used to teach communication, concepts and theorems.
- Teachers should find out what and how children learn and think when they communicate thought, form concepts and master theorems. This will enable them to gain insight into learning difficulties and immediately find a way to guide them. If learners are given the opportunity to explain what they have discovered, then teachers will be able to identify their problems.
- Assessment and teaching of Geometry should be integrated. Teachers should not only assess learners' solutions, they should assess the processes involved in arriving at the solution.

(NB: The van Hiele's levels of thinking is explained further in the theoretical framework of this paper)

A case study on Euclidean Geometry by Kotze (2007) for grade 10 teachers and their learners listed the following recommendations:

- A 'feel' for geometric aspects may be developed by exposing aspiring mathematicians to a wide spectrum of experiences in geometry.
- Practical experiences with space and shape can develop spatial sense. Although an understanding of measurement and proportional reasoning is an advantage in developing mastery in geometry, experiences need not only be in formal Euclidean geometry. However, a firm Euclidean structure should be built up, eventually.
- Apart from a formal approach, other approaches to space and shape may be implemented to develop learners' thinking from informal to formal.

Dhlamini (2012) refers to challenges with teachers' reasoning skills when working through Euclidean Geometry questions.

Some of the findings from his research, which investigated grade 12 teachers' understanding of Euclidean Geometry, are shown here:

- Teachers performed poorly in Bloom's taxonomy category 4 - 5
- Teachers also performed poorly in van Hiele levels 3 - 4
- They were very poor when it came to visually estimate angle size.
- The majority of teachers exposed their learners to the standard representation of a particular theorem.
- Those teachers who did not teach Mathematics Paper 3 topics could not answer non-routine problems.

Gunhan's (2014) study evaluated the reasoning skills in geometry-related subjects of six grade 8 learners. Some of the key findings of the study are shown below:

- The school curriculum should place more emphasis on reasoning skills. With regard to geometrical concepts, learners should be presented with problems that allow them to use different reasoning skills to reflect their geometrical knowledge, visual perception, and logical arguments.
- Teachers can better understand the way in which their learners think when they are solving geometry problems, and can help them see their students' reasoning skills. It encourages teachers to reconsider the techniques used in learning processes.
- Teachers should have the necessary skills to facilitate the acquisition of reasoning skills by their learners.

In the aforementioned literature survey both learning and teaching issues are discussed. Learners proceed through various levels of understanding (van Hiele's levels) in Euclidean Geometry, eventually reaching a stage where they are able use their "reasoning" and "proof" skills. This means that learners should experience a wide range of activities to ease their transition from informal to formal Euclidean Geometry. Teachers should be aware of how children learn and think when solving geometry problems. This will enable them to integrate assessment into their teaching (assessment for learning) where the teachers could assess learners' solutions and the processes involved in arriving at the solutions.

However, as indicated at the beginning of this paper and in the study by Dhlamini (2012), Euclidean Geometry has not been well taught in South African schools. Teachers tend to teach at a basic level, focussing only on standard or routine problems with their learners getting little or no exposure to higher order questions. This is bound to impact on learner performance in the Euclidean Geometry part of Grade 12 Mathematics paper 2 and Mathematics as a whole.

All papers are set according to cognitive levels which should cover knowledge, routine, complex and problem solving type questions.

The last two question types are regarded as higher order questions. Learners who have not been exposed to higher order questions would not be able to score well in such questions.

2014 and 2015 marked the first two years in which all grade 12 learners wrote Euclidean Geometry as part of Mathematics paper 2 for the first time since 2007. This paper focuses on the questions set for Euclidean Geometry for both these years.

RESEARCH QUESTION

The following research question was designed for this study:

How does the assessment of Euclidean Geometry in the 2015 Grade 12 Final Examinations compare to that of 2014?

The following sub-questions were formulated to answer the research question:

What are the views of various stakeholders about the inclusion of Euclidean Geometry and its assessment as a compulsory part of the Grade 12 Mathematics curriculum?

What are the key elements in Euclidean Geometry question setting?

Were there any differences in the assessment of Euclidean Geometry in 2015 when compared to 2014?

THEORETICAL FRAMEWORK

This study compares the Euclidean Geometry set in Grade 12 examinations in South Africa for two years, 2014 and 2015. These years mark the first two years of the implementation of Curriculum and Assessment Policy Statement (CAPS) and the first time since 2007 that Euclidean Geometry formed part of Mathematics paper 2. The CAPS document for Mathematics (grades 10 – 12) defines assessment as follows:

> Assessment is a continuous planned process of identifying, gathering and interpreting information about the performance of learners, using various forms of assessment. It involves four steps: generating and collecting evidence of achievement; evaluating this evidence; recording the findings and using this information to understand and assist in the learner's development to improve the process of learning and teaching. (DBE, 2011, p. 51).

The CAPS document also distinguishes between informal assessment (Assessment for Learning) and formal assessment (Assessment of Learning). Since this study is about the assessment of Euclidean Geometry in the grade 12 examinations, it falls in line with formal assessment or assessment of learning.

In both 2014 and 2015, Euclidean Geometry comprised 50 marks out of the 150 allocated for Mathematics P2 (DBE, 2014b; 2015c). There are four cognitive levels for the assessment of Mathematics and these are based on the TIMSS study of 1999. Table 2 shows the cognitive levels for FET Mathematics.

The middle column of the table gives descriptions which are relevant to Euclidean Geometry (DBE, 2011, p. 53).

Table 2: Cognitive levels for Mathematics.

Cognitive level	**Description (relevance to Euclidean Geometry)**	**Percentage allocated**
Knowledge (L1)	Straight recall; appropriate use of mathematical vocabulary	20%
Routine procedures (L2)	Proofs of prescribed theorems; perform well known procedures; generally similar to those encountered in class	35%
Complex procedures (L3)	Problems involving complex calculations and/or higher order reasoning; there is no obvious route to the solution; could involve making significant connections between different representations; requires conceptual understanding	30%
Problem solving (L4)	Non-routine problems which are not necessarily difficult; higher order reasoning and processes involved; might require the ability to break the problem down into its constituent parts	15%

Another important way in assessing Geometry is using van Hiele levels. The Van Hiele theory of geometric thought describes the different levels of understanding through which children progress when learning Geometry (Usiskin, 1982). The basis of the theory is the idea that a child's growth in Geometry takes place in terms of distinguishable levels of thinking. Table 3 shows the van Hiele levels with descriptions, the ability of the learner and the possible school grades applicable to the levels. The fourth column on the possible grades of learners is a guideline and depends on quality teaching and learning. Also, the boundaries between levels are not absolute.

Table 3: The van Hiele model (adapted from Howse and Howse, 2014).

Level	Description	Ability of learner	Possible grades of learners
1	Visual	The learner identifies, names, compares shapes (by appearance)	Grades R - 2
2	Analysis	The learner analyses the shapes and discovers properties through observation	Grades 3 – 6
3	Abstraction/ informal deduction	The learner discovers and formulates generalisations about previously learned properties and rules and develops informal arguments to prove these generalisations	Grades 7 - 9
4	Deduction	The learner attains logical reasoning and proves theorems deductively	Grades 10 - 12
5	Rigour	The learner establishes theorems in different systems of postulates and compares and analyses deductive systems/	Advanced school learners and students at university

If one examines the above table, then it means that grade 12 learners should be operating at a "deduction" level. However, some top learners in grades 10 -12, by virtue of their ability, from class results and participation in Mathematics Olympiads, could be operating at the van Hiele level of "rigour". At the same time, learners in grade 10 and possibly grade 11 may be operating at an "informal deduction" level.

RESEARCH METHODOLOGY

As mentioned earlier, the Euclidean Geometry questions of both the 2014 and 2015 papers are analysed. These analyses are based on the AMESA submission to the DBE for both years. The van Hiele model is also used to give another perspective on the questions set. The diagnostic reports of 2014 (DBE, 2014) and 2015 (DBE, 2015) are used as possible indicators of learner performance in Euclidean Geometry in both these years.

Mathematics Paper 2 2014

Since 2009, AMESA (The Association for Mathematics Education of South Africa) has been making submissions to the Department of Basic Education (DBE) on the grade 12 Mathematics and Mathematical Literacy papers. The curriculum committee of AMESA gets input from the various AMESA regions on the papers and consolidates all the input as one AMESA submission. In 2014, questions 8 to 10 comprised the Euclidean Geometry part of the paper (DBE, 2014b). A summary of the question-by-question analyses for the Euclidean Geometry part of the paper is shown here (AMESA, 2014):

Table 4: AMESA analyses of question 8 of 2014.

Question 8 Euclidean Geometry								
Quest.	**Content**	**Levels**				**Marks**	**Topic Code**	**Comment**
		1	**2**	**3**	**4**			
8.1	Size of an angle	4	0			4	4	Direct application of theorem
8.2	Size of an angle	2	2			4		Direct application of theorem
8.3.1	Reason for statements	2				2		Direct application of theorem
8.3.2	Calculate length		4			4		Use Pythagoras
	TOTAL	**8**	**6**	**0**	**0**	**14**		

Table 5: AMESA analyses of question 9 of 2014.

Question 9 Euclidean Geometry								
Quest.	**Content**	**Levels**				**Marks**	**Topic Code**	**Comment**
	Ratio & Prop.	**1**	**2**	**3**	**4**			
9.1.1	Areas of Δs between same parallel lines	1				1	4	Knowledge of axiom
9.1.2	Completion of theorem	5				5		Completing parts of a proof by writing missing statements or reasons
9.2.1	Ratio and proportion		3			3		Proportion, line // to a side of Δ
9.2.2	Ratio and proportion			3		3		Proportion, line // to a side of Δ
9.2.3	Ratio of areas of Δs			4		4		Areas of Δs, express in ratio form
	TOTAL	**6**	**3**	**7**	**0**	**16**		

Table 6: AMESA analyses of question 10 of 2014.

Question 10 Euclidean Geometry								
Quest.	**Content**	**Levels**				**Marks**	**Topic Code**	**Comment**
		1	**2**	**3**	**4**			
10.1	Reasons for various statements	5				5	4	Applying various theorems
10.2	Rewriting in terms of RT			2		2		Using proportionality (parallel lines)
10.3	Identify angles		4			4		Using exterior angle of cyclic quadrilaterals and parallel lines
10.4	Proving two angles equal			3		3		Making use of other relationships
10.5	Proving two triangles similar		3			3		Make use of equal angles
10.6	Application of similarity				3	3		Make use of previous results
	TOTAL	**5**	**7**	**5**	**3**	**20**		

The preceding analyses of questions 8 to 10 are now combined in Table 7.

Table 7: AMESA analyses of the Euclidean Geometry of 2014.

	L1	L2	L3	L4	Total
Question 8	**8**	**6**	**0**	**0**	**14**
Question 9	**6**	**3**	**7**	**0**	**16**
Question 10	**5**	**7**	**5**	**3**	**20**
TOTAL	19	16	12	3	50

According to the AMESA submission, of the 50 marks allocated to Euclidean Geometry, 35 marks were allocated to level 1 (knowledge) and level 2 (routine procedures) questions. Interestingly, only three marks were allocated to level 4 (problem solving). This data analysis focuses on the level 1 and level 4 questions in the paper.

In question 8 learners were given a drawing which involved the use of "the angle at centre" theorem and the isosceles triangles. The angle at the circumference was given. In question 8.1.1 they had to calculate the angle at the centre and in question 8.1.2 they had to calculate one of the base angles of the isosceles triangle. These questions could be classified as fairly easy knowledge type questions and probably at an "informal deduction" level using the van Hiele model.

From the DBE diagnostic report for 2014, in question 8.1.1 some candidates made the mistake of dividing by 2 instead of multiplying by 2. Many did not give the reason or gave an incomplete reason such as 'angle at centre'. Some gave either 'angle at circumference is twice angle at centre' or 'centre of circle theorem' as the reason. All of these reasons were incorrect.

In 8.3.1 a diagram with two tangents, a secant and a diameter was given. Learners were given two statements relevant to the diagram. They had to provide reasons for such statements. This is shown below:

8.3.1 Give reasons for the statements below.
Complete the table on DIAGRAM SHEET 3.

	Statement	**Reason**
(a)	$A\hat{B}C = 90°$	
(b)	$AB = x$	

(2)

This question was fairly straight-forward and the majority of learners should have had no problem with this question and would be regarded as a level 1 (knowledge question). In the van Hiele model, this question would be classified at the deduction level. According to the DBE diagnostic report of 2014, candidates gave incorrect reasons for question 8.3.1. The most common incorrect reasons were "tan-chord theorem" and "AB = AE". In 9.1.2, the question on the proportional intercept theorem was set in a very interesting way.

9.1.2 Given below is the partially completed proof of the theorem that states that if in any ΔABC the line $DE \mid\mid BC$ then $\frac{AD}{DB} = \frac{AE}{EC}$.

Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 4.

Construction: Construct the altitudes (heights) h and k in ΔADE.

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(BD)(h)} = \ldots\ldots\ldots$$

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \frac{AE}{EC}$$

But area ΔDEB = ... (reason: ....)

$$\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

(5)

Learners were given statements relevant to the diagram which they had to complete and provide one reason. Thus, the theorem was set in a “scaffolding” manner. This was a departure from the normal way of examining this theorem and could be regarded as an innovative item. Using the van Hiele model, this question involved the proof of a theorem in which deductive reasoning is involved and would be rated at the deduction level. It would appear that this question went well with candidates as the only comment from the DBE diagnostic report for question 9.1.2 was that candidates used h for the height instead of k. In question 10 learners were given statements based on the diagram given. They had to provide reasons for such statements.

10.1 Give reasons for the statements below.
Complete the table on DIAGRAM SHEET 6.

Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$		
	Statement	**Reason**
10.1.1	$\hat{T}_3 = x$	
10.1.2	$\hat{P}_1 = x$	
10.1.3	WT | | SP	
10.1.4	$\hat{S}_1 = y$	
10.1.5	$\hat{T}_2 = y$	

(5)

Question 10 would be regarded as fairly easy and is classified as level 1 (knowledge). Since this question involves linking statements with reasons, it would be rated as a deduction type question using the van Hiele model.

In the DBE diagnostic report for 2014 it was reported that many candidates had a good understanding of the tan-chord theorem and used it correctly in questions 10.1.1 and 10.1.2. However, some candidates gave irrelevant reasons as their answers. In question 10.1.3, many candidates simply gave the answer as "corresponding angles" rather than "corresponding angles are equal". Some candidates did not realise that the two angles were subtended by the same chord. In question 10.1.5, candidates gave the reason "alternate angles" instead of "alternate angles WT || SP".

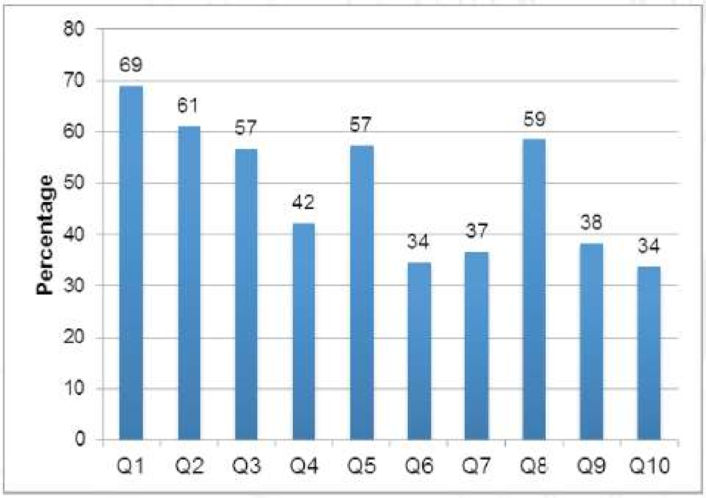
According to the AMESA submission, the problem-solving type question (level 4) comprised only three marks (question 10.6). Learners had to use the results and information from 10.1 to 10.5 to work out question 10.6. This question is shown below:

10.6 Hence, prove that $\frac{WR}{RQ} = \frac{RS^2}{RP^2}$. (3)

There is no obvious route to solving question 6. Although this question involves some deduction, it tends to lean more towards rigour on the van Hiele model. The DBE diagnostic report stated that, in question 10.6, candidates created incorrect ratios and/or proportions and used these to solve the question. Many candidates were able to deduce the proportion from the similar triangles in question 10.5 but could not go any further as they did not see the link with question 10.2. Some candidates attempted to work backwards. This proved to be fruitless.

Learner performance (from the Diagnostic reports for 2014)

The DBE analyses learner performance using 100 scripts from each province. The 100 scripts cover top, middle and bottom learner performances. The bar graph below shows the average learner performance for the Mathematics paper 2 from 2014.



Q1	Data Handling
Q2	Data Handling
Q3	Analytical Geometry
Q4	Analytical Geometry
Q5	Trigonometry
Q6	Trigonometry
Q7	Trigonometry
Q8	Euclidean Geometry
Q9	Euclidean Geometry
Q10	Euclidean Geometry

The performance in Euclidean Geometry from the above graph is isolated and shown in table 8.

Table 8: Learner performance in Euclidean Geometry in 2014.

Question	**Total mark**	**Average learner %**	**Average learner marks**
8	14	59%	8
9	16	38%	6
10	20	34%	7
Total	50	42%	21

Table 8 indicates that the average learner mark for Euclidean Geometry was 21marks out of 50 or 42%. This would appear very reasonable as it was the first time that Euclidean Geometry was assessed as part of Mathematics paper 2, since 2007.

AMESA verdict on the 2014 Mathematics paper 2

Despite issues listed in the DBE diagnostic reports, it would appear that the type of questions set in the 2014 paper 2 had the approval of most teachers. The following extract on the standard of the paper comes from the AMESA submission to the DBE on the grade 12 papers for 2014.

> "Teachers were very impressed with the ways by which the examining panel went to some lengths to make the paper manageable and user-friendly (especially in view of all the "new" work). Candidates were often led through, or pointed in the right direction, and (by virtue of the way the question was phrased) were not necessarily penalised by getting the first part wrong. Questions were generally neatly scaffold. However, the number of separate pieces of paper to be handed in would be a logistical concern. Teachers wanted to know whether this new "style" of setting was just for 2014 or can one expect this for 2015 as well. They also inquired whether this type of setting should be introduced at school level". (AMESA, 2014, pp. 8 - 9).

In the same submission, the following was stated regarding an overall verdict of the paper:

> "We would like to pay tribute to the examining panel. They came up with a very good paper, which was fair to all and very "cleverly" constructed. There would be enough questions to enable learners to pass; a number of learners would be able get 60% but to obtain 80% and higher would be regarded as a significant achievement. However, the Department of Basic Education should keep the paper at this level for the foreseeable future. Teachers reminded us of the debacle in 2009 when the standard of both Mathematics papers was raised so drastically and took all teachers by surprise". (AMESA, 2014, p. 9)

The Euclidean Geometry for 2014 found favour on two fronts. Firstly, the teachers, through their participation in the AMESA review for 2014, were pleased with the standard of the questions and the innovative way in which some questions were set. Secondly, the learners appeared to do well with an average of 42% in Euclidean Geometry.

Mathematics Paper 2 2015

In 2015, questions 8 to 11 comprised the Euclidean Geometry part of the paper (DBE, 2015c). A summary of the question-by-question analyses for the Euclidean Geometry part of the paper is shown here (AMESA, 2015):

Table 9: AMESA analyses of question 8 of 2015.

Question 8 Euclidean Geometry								
Quest.	**Content**	**Levels**				**Marks**	**Topic Code**	**Comment**
		1	**2**	**3**	**4**			
8.1.1	Completion of theorem statement	1				1	4	Straightforward
8.1.2	Proof of theorem		3			3		Very easy as diagram is given with markings at O
8.2	Proving two lines parallel, using information from cyclic quads		3	2		5		Prove that $\hat{A}+\hat{E}=180^\circ$ using properties of cyclic quads
	TOTAL	**1**	**6**	**2**		**9**		

Table 10: AMESA analyses of question 9 of 2015.

Question 9 Euclidean Geometry								
Quest.	**Content**	**Levels**				**Marks**	**Topic Code**	**Comment**
		1	**2**	**3**	**4**			
9.1	Tan-chord theorem and parallel lines		2	2		4	4	$\hat{C} = x$ and $\hat{K}_3 = \hat{C}$
9.2	Cyclic quad (converse of angles in the same segment)		2			2		$\hat{K}_3 = x$
9.3	Proving that TK bisects $A\hat{K}B$			2	2	4		Show that $\hat{K}_3 = \hat{K}_2 = x$
9.4	Prove that TA is a tangent to circle AKH			1	1	2		$\hat{K}_2 = x$
9.5	Explain why A, S, K and T are con-cyclic				2	2		$A\hat{S}B = A\hat{K}B = 2x$ $A\hat{T}B = 180^\circ - 2x$
	TOTAL		**4**	**5**	**5**	**14**		

Table 11: AMESA analyses of question 10 of 2015.

Question 10 Euclidean Geometry								
Quest.	**Content**	**Levels**				**Marks**	**Topic Code**	**Comment**
		1	**2**	**3**	**4**			
10.1	Length of DC	2	1			3	4	Theorem of Pythagoras
10.2.1	Length of CF		3			3		Use given ratio
10.2.2	Proving two triangles similar			5		5		Common angle C and 90° angle
10.2.3	Length of AC			2	2	4		Use results of similarity
10.2.4	Radius of circle through A, B and C				2	2		AC will be the diameter of this circle
	TOTAL	**2**	**4**	**7**	**4**	**17**		

Table 12: AMESA analyses of question 11of 2015.

Question 11 Euclidean Geometry								
Quest.	**Content**	**Levels**				**Marks**	**Topic Code**	**Comment**
		1	**2**	**3**	**4**			
11.1	Complete statement of theorem	1				1	4	Straightforward
11.2.1	Prove two triangles similar		2	1		3		Use corresponding sides
11.2.2	Length of NQ				6	6		$\hat{P} = 90^{\circ}$ and ΔKPM///ΔRNM///ΔRPQ
	TOTAL	**1**	**2**	**1**	**6**	**10**		

The preceding analyses in Tables 9 to 12 are now combined in Table 13.

Table 13: AMESA analyses of the Euclidean Geometry of 2015.

	L1	L2	L3	L4	Total
Question 8	1	6	2		9
Question 9		4	5	5	14
Question 10	2	4	7	4	17
Question 11	1	2	1	6	10
TOTAL	**4**	**16**	**15**	**15**	**50**

According to the AMESA submission in 2015, of the 50 marks allocated to Euclidean Geometry, only 20 marks were allocated to level 1 (knowledge) and level 2 (routine procedures) questions. At the same time 15 marks were allocated to both level 3 (complex procedures) and level 4 (problem solving) questions. It would appear, that based on the AMESA submission to the DBE in 2015, that the Euclidean Geometry in the 2015 paper, was cognitively far more demanding when compared to the 2014 paper. As for the 2014 paper, these analyses also looks at level 1 (knowledge) and level 4 (problem solving) questions, as classified by AMESA. In question 8.1.1, learners had to fill in a "word" to complete the statement of the theorem so this was a "knowledge" question. Using the van Hiele model, it would appear that this question would be at the informal deduction level.

According to the DBE diagnostic report for 2015, some candidates stated that the angle at the centre was "half" the angle at the circumference. This was the only possible mistake attributed to question 8.1.1. In question 9, the first two questions were of a routine nature. Questions 9.3 to 9.5 were of a higher cognitive demand. In question 9.3, they had to prove that "TK bisects $A\hat{K}B$" .This question required learners to make use of the results in 9.1 and 9.2 and other theorems. Two marks were allocated at level 3 (complex procedures) and two marks at level 4 (problem solving). In terms of the van Hiele model, this question would be regarded as a question involving deduction.

According to the DBE diagnostic report for question 9.3, candidates who made incorrect assumptions in questions 9.1and 9.2, provided incorrect reasoning in question 9.3 and this led to a breakdown. In 9.4, they had to prove that "TA is a tangent to the circle passing through points A, K and H". This question also depended on previous results and the use of the converse of the "angle between tangent and chord theorem". Thus, one mark was allocated at level 3(complex procedures) and one mark at level 4 (problem solving). Since this question involved deduction it would be rated as a deduction type question on the van Hiele model. Further, it was stated in the DBE diagnostic report that many candidates were unable to link $\hat{A}_3$ and $\hat{K}_2$. Thus, they could not prove that AT was a tangent. Question 9.5 required learners to prove that the points A, S, B and T were con-cyclic, if it was known that A, S, K and B were con-cyclic. The solution to this question is not obvious and requires learners to do quite a bit of work. In this regard this question is set at level 4 (problem solving). Using the van Hiele model, it would appear that this question would lean towards rigour, rather than deduction. The DBE diagnostic report stated that many candidates were not familiar with the word “con-cyclic”.

In question 10.1, they had to use the theorem of Pythagoras to calculate the length of DC. This was so because ΔBDC was a right-angled triangle. Two of the three sides of the triangle were known so it was a straight-forward application of the theorem of Pythagoras and is classified at level 1 (knowledge). Using the van Hiele model, the question would be regarded as an abstraction or informal deduction question.

According to the DBE diagnostic report, many candidates were able to calculate the length of DC correctly but failed to give a reason why ΔBDC was a right-angled triangle. Others failed to realise that DC was not the hypotenuse and wrote down an incorrect application of the Theorem of Pythagoras

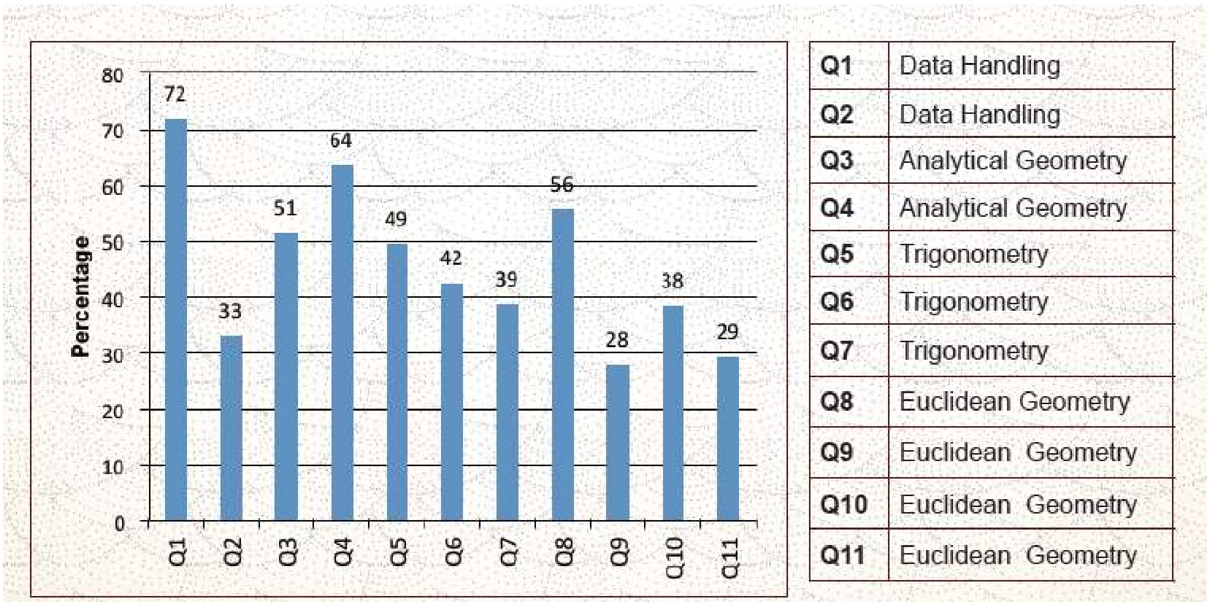
In question 10.2.3 two marks were allocated to level 3 (complex procedures) and two marks to level 4 (problem solving). These questions depend on previous results and being able to manipulate ratios arising out of similarity and proportional intercepts. It was stated in the DBE diagnostic report that many candidates did not link questions 10.2.2 and 10.2.3 and, therefore, found it difficult to answer question 10.2.3

Question 10.2.4 is set at level 4 (problem solving). This question requires learners to deduce that AC is the diameter of the circle. Thus, the radius will be half the diameter. Using the van Hiele model, the question would be at the deduction level. According to the DBE diagnostic report for question 10.2.4, candidates did not realise that AC was the diameter of the circle in question. Many used BC as the diameter.

The DBE diagnostic report stated that, question 11, on the whole, was answered poorly. In question 11.1, learners had to complete the statement of the theorem. This was a “knowledge” type question. However, question 11.2.2 involved problem solving. The route to the proof is not obvious. Learners would have to prove the two small triangles ΔKPM and ΔRNM similar to each other and also similar to ΔPQR. Using the van Hiele model, it would appear that the question involved more rigour than deduction. The DBE diagnostic report indicated that candidates assumed that MN || PQ and used the proportionality intercept theorem to calculate the length of NQ.

Learner performance (from the Diagnostic reports for 2015)

The bar graph below shows the average learner performance for the Mathematics paper 2 from 2015.



The performance in Euclidean Geometry from the above graph is isolated and shown in table 14.

Table 14: Learner performance in Euclidean Geometry in 2015.

Question	**Total mark**	**Average learner %**	**Average learner marks**
8	9	56%	5
9	14	28%	4
10	17	38%	6
11	10	29%	3
Total	50	36%	18

Table 14 indicates that the average learner mark for Euclidean Geometry was 18 marks out of 50 or 36%. This represents a drop of 6% when compared to 2014.

AMESA verdict on the 2015 Mathematics paper 2

From an interrogation of the Euclidean Geometry questions for both 2014 and 2015, there seems to be a definite tilt towards higher order questions. This was stated in the

2015 AMESA submission to the DBE, which featured under the heading, "comparison with the 2014 paper".

> "We note that there is a significant shift towards level 3 and level 4 questions and a drop in level 1 and level 2 questions, making the paper more difficult than the 2014 paper. We are very concerned that the Euclidean Geometry was largely of the pre-2008 style of presentation and there were quite a few tricky stumbling blocks along the way. We urge the examining panel to re-examine this approach as we believe that assessing Euclidean Geometry the "old" way is largely counterproductive and disadvantages learners. The 2014 approach to Euclidean Geometry was innovative and new and we recommend that the panel should go back to that approach. We also note that the Analytical Geometry of the paper required quite a bit of Euclidean Geometry knowledge, thus, disadvantaging learners even further"

It would appear that teachers who participated in the AMESA review of the 2015 Mathematics papers were not pleased with the Euclidean Geometry questions. They were probably expecting questions along the style of the ones set for 2014 and this change to the "old" way of setting questions caught them by surprise. Further, it would seem that learners were not prepared for the "old" way of setting and this impacted negatively on their results.

FINDINGS

The findings of the study are given within the context of the research questions.

The inclusion of Euclidean Geometry as a compulsory part of the curriculum

As the statistics in table 1 show, on average only 3, 5% of the grade 12 mathematics learners did Mathematics Paper 3 and so were assessed in Euclidean Geometry. The vast majority of learners went into higher education without having done Euclidean Geometry. This was a cause for concern by many stakeholders, including the professional body AMESA and the universities. The inclusion of Euclidean Geometry was welcomed by all stakeholders. In fact, most of the proposals made by AMESA in this regard were accepted.

In response to the inclusion of Euclidean Geometry as a compulsory part of Mathematics paper 2, the DBE and other stakeholders ensured that there was a nationwide intensive training of Mathematics teachers in Euclidean Geometry and other paper 3 topics in the years 2011 to 2013.

The key elements in Euclidean Geometry question setting

The examiners of Mathematics question papers have to ensure that papers are set according to the cognitive levels as stated in table of this paper. This means that of the 50 marks allocated for Euclidean Geometry in Mathematics paper 2, 10 marks should be "knowledge" questions, 17 or 18 marks should be questions assessing "routine" procedures, 15 marks should be questions assessing "complex procedures" and the remaining 7 or 8 marks should questions assessing "problem solving".

This means that approximately 28 marks (comprising "knowledge" and "routine procedures" questions) should be within reach of most learners. If learners wanted to excel in Euclidean Geometry and get higher marks, then they would have to perform well in questions involving "complex procedures" and "problem solving".

Another key element that should be considered in the setting of Euclidean Geometry questions is the van Hiele model. The van Hiele suggests that children should be able to demonstrate certain geometric competences at certain age levels. Those setting these questions should also take into account that a number of learners may have not reached level 4 (deduction) on the van Hiele model. While one may be expecting learners to be operating at certain levels, learners without the necessary exposure or experiences in their classes are likely to fall short in such questions.

Comparing the 2014 and 2015 Euclidean Geometry questions

As stated earlier in this paper, the AMESA submissions to the DBE on grade 12 papers in 2014 and 2015, showed marked differences in the cognitive levels of the questions set for Euclidean Geometry for both years.

Table 15: Euclidean Geometry cognitive levels and marks for 2014 and 2015 with exam requirement.

Cognitive level	**2014**	**2015**	**Exam requirement**
Knowledge (L1)	19	4	10
Routine procedures (L2)	16	16	18
Complex procedures (L3)	12	15	15
Problem solving (L4)	3	15	7
Total	**50**	**50**	**50**

While the above calculations were compiled by AMESA and is open to debate, it would appear that the 2015 Euclidean Geometry questions were more difficult than the ones set in 2014. The difficult nature of the 2015 Euclidean Geometry questions could also be seen in the average learner performance for 2015 (36%) when compared to the average learner performance foe 2014 (42%).

The reasons for changing the style of questioning in 2015 when compared to some innovative items in 2014 are not yet known. However, the 2015 questions took us back to the pre-2008 era of higher grade Euclidean Geometry setting. It is widely known that such questions were beyond the scope of most learners. Such poor learner performance at the time resulted in Euclidean Geometry becoming optional with the introduction of the National Curriculum Statement (NCS) in 2006. One would expect that the re-introduction of Euclidean Geometry as part of Mathematics paper 2 would usher in a new way of setting Euclidean Geometry questions. There was an attempt to do so in 2014 but not in 2015.

There needs to be a re-think on the way Euclidean Geometry is set. A good start would be for examiners to look at the innovative way Euclidean Geometry was set in 2014 and attempt to replicate such innovation in future examination papers.

CONCLUSION

South Africa is one of a number of countries where there is a great emphasis on Euclidean Geometry in grades 10 to 12. This is very different from the USA where Euclidean Geometry is done as a once-off subject in either grade 9 for top learners or grade 10 (Sherman, 2010). So most learners in the American school system do not do Euclidean Geometry in grade 11 or grade 12. Yet, there appears to be no complaints in America about such a system and learners do not seem to be disadvantaged by such a system.

In contrast, Euclidean Geometry is a compulsory part of the South African Mathematics curriculum in grades 10 to 12 and appears to have strengthened the curriculum. However, if Euclidean Geometry is to serve its purpose of developing "reasoning" and "proof" skills in learners, then there should be innovative ways of assessing such skills, especially in grades 10 to 12.

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MATHEMATICS TEACHER'S REFLECTIONS ABOUT USING INSTRUCTIONAL DESIGN IN THE TEACHING OF GEOMETRY

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This paper reports on reflections of a novice teacher on experiences in using the GeoGebra as a teaching tool after an intervention programme, carried out in a high school to equip mathematics teachers with technology skills that could enhance improvement on their instructional design in the teaching and learning of Further Education and Training (FET) mathematics. The classes taught ranged from grades 8-12. The intervention strategy followed participatory action research where the teachers were involved in researching their own practice whilst they and the researcher evaluated what worked best. This paper only reports on a case study capturing one teacher's reflections on his experiences. The results revealed that using the GeoGebra in the teaching of various topics in FET mathematics helped the teacher to (i) cover the content in time (ii) close the gaps that existed in their learners' mathematical knowledge in the various grades (iii) ensure conceptual understanding an increased performance in the subject and (iv) improved the understanding of mathematical concepts through discovery of the truths about some theorems in Geometry.

Keywords: Geometry, instructional design, mathematics, reflections, teacher

INTRODUCTION

Go (2012, p. 514) proposes that teaching be conceived as 'goal-less and reflective design' while practiced as 'assessment learning' may assist teachers to address some of the problems they encounter in their practice currently. Schon (1987) understood the teaching profession as design work. Simon (1997) defined design as rational problem-solving, a search process for prospective solutions. Currently there is always greater pressure and emphasis put on Further Education and Training (FET) teachers in South Africa to ensure that their learners' performance exceed 80% at the end of the year. This is not always easily achievable for mathematics teachers as their results in this subject always reflect low scores. These professionals always have to account for a failure rate that keeps on increasing despite their effort to make things work. Most mathematics teachers therefore suffer frustration and low self-esteem as teachers for other subjects look down upon them as they compare performance with the same learners in the same level in other subjects.

This gives rise to debates from government level, institutions of higher learning and amongst mathematics teachers themselves on what pedagogical methods best facilitate learner understanding and retention of mathematical knowledge. These debates are guided by teachers' professional designs which Dewey (1933) addressed as there to convert the indeterminate situations to determinate ones. Determinate situations refer to situations of limited or definite scope or nature while the indeterminate ones include those that are not fixed and vague. The question is how mathematics teachers close the knowledge gap at their matric level whilst equipping learners with higher order mathematics.

According to Schon (1987), teaching might require mathematics teachers to carry out their instructional design process as reflection-in-action involving practical operations in an indeterminate situation for the purpose of discovering meaning and coherence in it. This would greatly enhance conceptual understanding of concepts since for each instructional design used, the teacher would reflect on the outcomes of his practice for a particular lesson. This reflection would be underpinned by the distinction between what teachers make out of pedagogical learning and practical learning. Pedagogical learning incorporates general pedagogical knowledge, which includes knowledge of learners, learning, curriculum and general instructional and assessment strategies. In mathematics, teachers need specialized pedagogical content knowledge (Blomeke, 2012) which includes knowing how to represent the concepts, methods and rules of a discipline in order to create appropriate learning opportunities for diverse learners, as well as how to evaluate their progress in learning those concepts. In the design and planning of each mathematics lesson, the afore-mentioned attributes must be taken into consideration. In reflection-in-action there is practical learning which involves learning in and from practice (Schon, 1987). Blomeke (2012) suggests that learning from practice includes the study of practice, using discursive resources to analyze different practices across a variety of contexts, drawing from case studies, video records, lesson observations, etc., in order to theorize practice and form a basis for learning in practice. Blomeke (2012) further distinguishes learning in practice as involving teaching in authentic and simulated classroom environments, preparing, teaching and reflecting on lessons presented by ones' self as opposed to observing and reflecting on lessons taught by others which would be learning from practice.

Dewey (1933) avers that during reflection teachers examine their teaching performance by critically analyzing what, how and why they do what they do in their teaching with an ultimate goal of improving learner understanding. This article reports on reflection practices of a novice mathematics teacher whose reflective actions were effective in his teaching of geometry. These reflections enhanced the teacher to consciously think about pedagogical actions, instructional design to use for each of his lessons, interpret learners' actions during the lessons, and were constantly using pedagogical approaches enhanced by technology in order to promote learner understanding (Schon, 1987).

BACKGROUND

One of the twenty-seven goals of the Action Plan to 2014: Towards the realization of Schooling 2025, published by the Department of Basic Education (DoBE) in 2011, focused on the need to improve learner performance in languages and mathematics at the Grades 3, 6 and 9 levels. The goal was to move away from the strong tradition in South Africa of focusing on the Grade 12 examination results and their improvement. The education system has in the last decade placed a growing emphasis on monitoring learner performance at the lower grades particularly the Foundation and Intermediate Phases. Table1gives the pass percentages of South Africa National Mathematics Matric Results from 2011-2014.

Table 1: South Africa National Mathematics Matric Results (2011-2014)

Year	Number Wrote	No. achieved at 30% and above	% achieved at 30% and above	No. achieved at 40% and above	No. achieved at 40% and above
2011	224635	104033	46,3	61592	30,1
2012	225874	121970	54,0	80716	35,7
2013	241509	142666	59,1	97790	40,5
2014	225458	120523	53,5	79050	35,1

From Table 1 it can be noticed that in the past five years the highest percentage of learners who obtained more that 40% in the matric mathematics final examinations has never been more than 40, 5%. On the contrary, the implications, damage and gap in the mathematics knowledge to the learners in the Further Education and Training (FET) phase are escalating and widening. Do we leave the matric mathematics teachers to endure the degrading confidence and self-esteem whilst facing the consequences of failure and shame on their shoulders? Whilst various projects embark on closing the mathematics knowledge gap in the lower classes, this study set to document effective reflective practice in a period of two years with a novice in-service teacher whilst teaching matric mathematics in a High school in Pretoria West, South Africa.

This started with a journey where the researcher formed a partnership with the teacher in helping with the teaching and learning of matric mathematics on a two hour slot on Saturdays. All matric learners in this school took mathematics as one of their subjects. This was a heterogeneous class composed of 66 grade 11 learners from four different provinces in South Africa, since this was a new school.

The majority, 62% came from Gauteng, 17% from Eastern Cape, 11% from KwaZulu-Natal and 10% from Limpopo. Seventy six percent of these learners were underprepared to study mathematics at this level. Yet immediate future careers were informed by their performance in mathematics.

An instructional design plan was constructed with the teacher. This plan structure allowed the teacher to decide on strategy of lesson presentation, the kind of resources to be used at a certain stage of the lesson and their availability, presentation of content, evaluation and application. Part of it was that these lessons provided supplementary tuition where learners engaged with problem-solving exercises and intense discussions on topics covered during the previous week. The teacher chose problems based on the topic covered and its related topics and presented them to learners as problem-solving tasks. Discussions would then ensue where clarity of critical characteristics and different ways of approach were debated. There was time to accommodate slow learners and ensure maximum understanding of the solutions to problems. Slow learners were those learners who could not present correct responses to classwork tasks and also indicated no clue on attempting to do their homework exercises.

LITERATURE REVIEW

Waks (1999) notes that in teaching mathematics the teacher's task is to prepare and facilitate classroom tasks that will achieve specific learning goals. He further identifies three activities that the teacher is expected to plan and coordinate: (i) Deductive activities which involve pre-selected information and knowledge presentation, (ii) Discursive activities, which have no prior designs or templates, but may result in a design or pattern emerging out of both the teacher and learners' contributions in the lesson and (iii) Heuristic activities which focus on skill development where the skills are not connected to the teacher's construction of new meaning. Go (2012) notes that one of the fundamental non-negotiable features of effective teaching, is to base instruction on clearly defined learning objectives. The instruction acts as means to aid the desired lesson. It should be noted though that teaching is a thinking person's job; it is not simply a matter of following a script or carrying out other people's instructional designs. It is rather an art where suitable instructional designs become effective under particular conditions, with certain resources in some classroom environment with diverse learners. Perhaps the challenge of the teacher is to master which approach to use when and how.

Hill, Ball & Schilling,.(2008) found the existence of powerful relationships between what and how teachers know about mathematics and what occurs during instruction when they investigated the correlation in teachers' mathematical knowledge for teaching and quality of instruction.

In addition to those, Hauk, Toney, Jackson, Nair & Tsay (2014) identified (i) beliefs about mathematics, (ii) perceptions of mathematics learning and teaching, and (iii) decision-making around adoption of teaching material, as significant factors in mediating teachers' instructional performance in a mathematics classroom. These researchers further argue that instructional realizations in the classroom result from a process of teacher's reshaping and melding of knowledge and beliefs about mathematics and pedagogy. Facilitation of such a process occurs through teacher's careful organization of aspects of communication in the classroom. These include use of (i) discourse practices (Moschkovich, 2007; Ryve, 2011), (ii) vocabulary, the mathematical register, (Wells, 1993), (iii) gestures (Alibali et al., 2012), and (iv) setting of norms for talking about mathematics like, socio-mathematical norms, (Yackel, Rasmussen, & King, 2000). Discourse practices refer to the use of language and the values about mathematical appropriateness to the context, clarity, and precision that is integral to thinking, learning, and communicating, especially in FET mathematics. For a novice teacher the context, classroom environment and the content are all distinguishable unfamiliar and thus coaching is necessary to afford the teacher an appropriate shoulder to lean on whilst thriving to teach mathematics effectively. Such was the partnership between the researcher and the teacher in this article.

Schoenfeld (2013) suggests that teachers must make decisions informed by knowledge that effective teaching extends beyond precise and accurate transmission of facts or uptake by students of information but includes taking into account the mathematical background and experiences of all learners in the classroom, to shape opportunities for learning. Students in the classroom are characterized by different cultural backgrounds and are always operating at different levels to the mathematical register as the teacher talks. To this end, Hauk, et al. (2014) note that there are both similarities and differences between teachers' own mathematical acculturations, their own everyday cultures, prior mathematical enculturation of students, everyday culture of students, intended mathematical enculturation of the curriculum or school, and interim classroom cultures that combine all of those attributes. They conclude that the teacher's intercultural orientation assists both the teacher and the learners to navigate through the different discourse(s) in order to establish particular classroom mathematical discourse(s) using appropriate instructional designs. It was of interest in this study to reflect on the teacher's knowledge of circle geometry, geometric reasoning and problem solving together with the teachers' geometry knowledge for teaching. Effective learning depends both on the knowledge and experience already existing within a student's level of development as well as on the student's potential to learn (Vygotsky, 1978). Understanding a mathematical concept includes knowing facts and concepts and how they connect, and this should be related to knowing how and when to use skills and strategies. A mathematical idea, procedure or fact is understood if it is part of an internal network, the mathematics is understood if its mental representation is part of a network of representations (Asiala et. al, 2004).

All these attributes to understanding of circle geometry were explored through the use of GeoGebra software. This is because of the pedagogical shifts away from concentrating on algorithms and computations but rather focussing on student-centred curricula with teacher as facilitator or coach; discovery activities, open-ended investigations; and sense making suggested by Powell (2004). Technology tools like the GeoGebra have made obsolete many of the algorithms taught in previous mathematics classes and helped teachers to save time while putting emphasis on concepts taught. (Straton, 2011) asserts that GeoGebra is mathematics software that is among the best-known free and easy-to-use educational software for mathematics teaching and learning globally. Powell, (2004) notes that, the challenge for prospective and practicing teachers alike, is to determine and incorporate technology sensibly and effectively enough into the learning and teaching of mathematics. A study conducted in Turkey on facilitating the integration of technology into geometry courses in secondary mathematics, showed that Cabri and Geometer's Sketchpad software gave learners an opportunity to create geometric shapes in virtual environment. It is in this notion that the researcher investigated the effects of using the GeoGebra as one of the dynamic software in enhancing the mathematics teaching skills of the novice teacher.

THEORETICAL FRAMEWORK

In order for the teacher to plan his lesson, he was requested to reflect on what he knew about geometry, his understanding of Euclidean Geometric concepts and how he decides on the approach he suggests in teaching those concepts. This study is underpinned by the framework for reflective practice (Plack & Santasier, 2004).

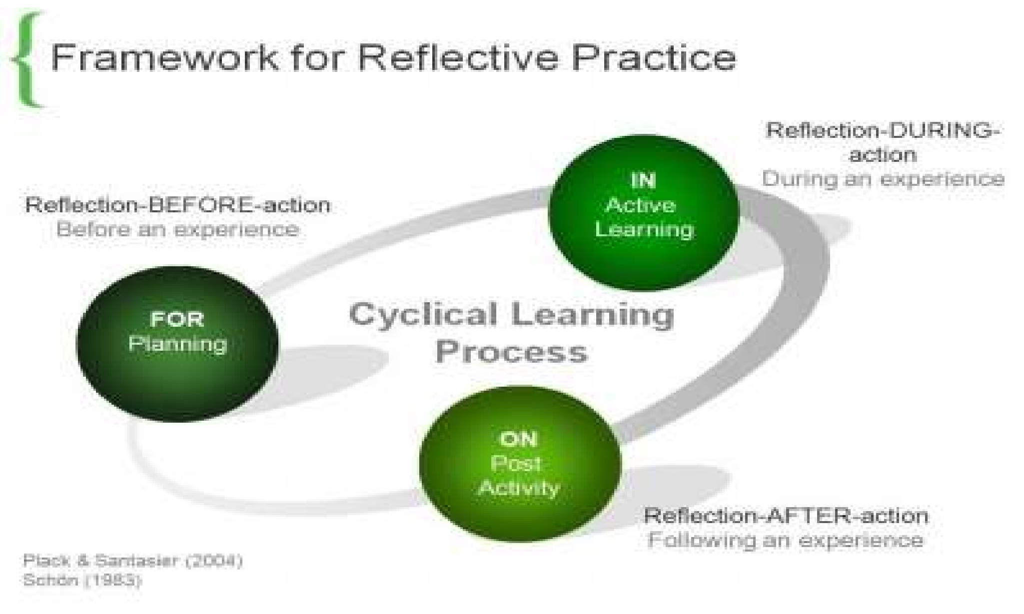


Figure 1: Framework for Reflective Practice

Through the use of the GeoGebra, the teacher was able to afford learners opportunities to discern critical features of geometrical concepts and their related features. They were then able to separate the implications of angles made by the diameter on the circumference and the reason why. The teacher also used the advantage of technology to not only close of geometry knowledge gaps, but also to make learners generalize through a number of examples given and some non-examples that could not work. The question to be answered was: ‘What is the nature of instructional support that can generate in learners the kinds of mental representations that will elicit critical thinking when engaging in learning geometry?’

METHODOLOGY

This was a case study which adopted a participatory action research approach intended to explore how a novice teacher improved on his practice through reflections on his journey on mathematics teaching. The exploration was to determine the current status of the novice teacher’s proficiency in geometric teaching and to design an intervention strategy for improvement of the teacher’s proficiency in mathematics.

An opportunity was given to the teacher to practice personal reflection on teaching; that is, to look back at what has worked and has not worked in the classroom and think about how he could change his teaching strategies to enhance learning. In this case, at the beginning of the intervention, the researcher requested the teacher to keep a reflection journal on his practice. In the journal, the teacher reflected on the challenges and things to improve on his lesson. The 66 learners wrote a pre-test testing their knowledge on prerequisite geometric concepts on the circle geometry. After five consecutive weeks of intensive work with them on geometric concepts, the teacher then administered a post test.

RESULTS

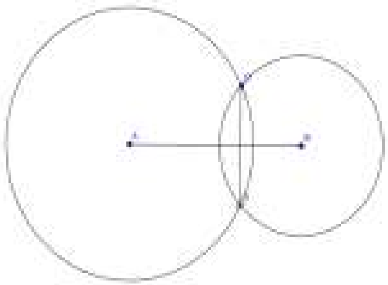
Results in this study were interpreted with regard to experiences of geometry teaching using the GeoGebra as reflected by the teacher.

Reflections before action	Reflection during action	Reflection after action	Intervention
Teacher acquaints himself with the theorem to teach, practise it and chose a few examples on its applications.	Use GeoGebra to draw circles and explore with the learners the properties as displayed in circle geometry	A need to explore other properties of circles using GeoGebra and then probe learners to formulate the theorem	Researcher helps the teacher to discover more on using the GeoGebra for teaching other sections in geometry

One of the problems done with the learners by the teacher was:

CD is the perpendicular bisector of AB where A is the centre of the bigger circle and

B the centre of the smaller one. Predict the quadrilateral that will be formed and identify its characteristics. Draw the resulting figure if centre B lies on centre A. Explain the dynamics.



Using this example, the teacher picked up on geometric concept discussions of meaning of (i) perpendicular bisectors, (ii) quadrilaterals and their properties, and (iii) revisions on geometry dynamics. Data was collected as the teacher became a learner whilst he was introduced to the use of the GeoGebra in the teaching of geometry. The first exercise involved exposure of the teacher to investigations on circle geometry exercises. He was supplied with instructions on how to use GeoGebra to bisect lines, construct triangles and quadrilaterals, reflect on their experiences and findings, what he learnt from each exercise and lastly how he could use what he learnt to ensure the success of students in learning geometry.

DISCUSSION

Some of the comments that were prominent in the conversations included:

Question: What challenges did you experience on researching your own practice as a mathematics teacher?

Teacher: *Since I was researching the impact on my own teaching, I automatically took into account my own teaching strengths and weaknesses. I had to put myself at the level of my learners. Thus I would practice the task before presenting it. The most challenge has been how I make the 'aha' moment for them.*

Researcher: So how did you do this?

Teacher: *You see I have been keeping a journal on which I reflect both what I learnt and what I should teach, Mh….maybe how I should teach it. You know what? Just giving instructions was not helping, but what was fascinating was what they found for themselves whilst playing around with the GeoGebra.*

Researcher: So how did you manage this?

Teacher*: I had to teach myself to be well advanced in knowing my theorems. This I had to do so that I could formalize the excitement brought by learners with reference to particular theorems.*

Researcher: How else did the use of GeoGebra help you in your practice?

Teacher: *Through some of these discoveries, some productive discussions ensued with my class. It was through such discussions that I found a chance to close the mathematical knowledge gap with my learners. Through some of the exercises that I created deliberately so that learners could discern what is and what is not. The discussion would be advanced through the use of tools in the GeoGebra menu to lead the learners to generalize on the meaning of a concept. Also this exercise helped learners to find the critical features of geometrical concepts.*

Researcher*:* Lastly, how would you compare the learners' performance in the pre-test and the post-test?

Teacher*: Oh wow! That one goes without say. I can show you about 74% of the responses were just empty spaces in the pre-test. You won't believe this! With the post-test, of the 94% of the questions attempted, 64% of the learners obtained more than 60% in geometric concepts. I am therefore prepared to explore more on topics that I can teach using the GeoGebra.*

The pre-test covered vocabulary in geometry as well as concepts on circle geometry. It was observed that eighty seven percent of the learners could not correctly label, define or match circle and line geometric concepts to their corresponding names. For example some learners misrepresented the diameter for the chord whilst labelling parts of the circle. The pre- and post-tests needed an identifier so that growth of each student could be tracked; however, the student were assigned a number instead of using their name.

During the intervention when the teacher reflected on his practice, most learners were actively engaged, working together encouraging each other, and using mathematical language to describe the objects and actions they were engaged in. Such an activity was possible because of use of appropriate space provided for learners to use technology. GeoGebra is freely available dynamic mathematics software that can be downloaded and used by both teachers and learners. The implication is that the teachers need to be comfortable with the software and be willing and able to respond flexibly to opportunities, questions, and ideas generated by the learners.

Chun (2011) asserts that action research is done for (i) reflecting more deeply and systematically on teaching practices, and (ii) evaluation of the actions and practices while identifying opportunities to improve teaching and learning. She further notes that it follows a cycle comprised of (i) identification of the problem, (ii) collection and organization of data, (iii) interpretation of data, (iv) action based on data and then (v) reflection.

The teacher's reflection before the experience of using GeoGebra coupled with fear of using the software. He reported that this almost confused him of the geometric facts that he knew and was scared that his computer skills could fail him whilst presenting to the learners. The main problem in this study was that the novice teacher had no previous experience in preparing matric level learners for a successful performance in mathematics. More specifically during the teacher's schooling term and while he trained as a teacher, geometry was optional. This was due to poor mathematics results at school prompted the politicians to influence curriculum designers and policy makers to make Euclidean Geometry optional in the curriculum from 2002 to 2007 during the implementation of the NCS curriculum (Siyepu, 2014). The purpose of optionalizing Geometry was to pretend as if the mathematics results in the country have improved (Siyepu, 2014).The teaching and learning of mathematics, in particular Geometry, presents challenges in these schools like the one under study, due to lack of relevant facilities for its teaching and learning.

This is echoed by Siyepu (2014) who asserts that in the recent past Geometry proved to be a huge challenge to many school mathematics students in South Africa. Its exclusion from the secondary school curriculum presented a problem to both learners and teachers especially as a foundation for studies in architecture, built environment and construction, engineering and many other careers like teaching it later as teachers. Pre-learning became necessary for the teacher to equip himself with correct geometric terms and concept understanding together with learning various ways of using the GeoGebra. The teachers' reflection during his lesson presentation, indicated growth from being a learner on how to use Geogebra, to him taking control of the skill, classroom and content to be covered. He notes a 'fascinating' experience as he experienced in-active learning which allowed him to blend the geometric content he knew together with the discovery of implicative discoveries revealed in action. On post activity, the teacher reported that he realized that he needed to do a lot of practice on using GeoGebra and that that would have given him confidence in presenting his lesson. He also noted that the use of Geogebra could be one of the ways in which meaningful teaching and learning of geometry and other areas of mathematics content can be achieved despite his inexperience in handling the subject and the prevailing poor socio-economic factors.

CONCLUSION

The many teacher preparation programs have fallen short in their technology instruction. Technology is a teaching tool and it can either be used effectively or ineffectively. Mathematics is a key requirement not only for entry to certain careers at university, but also for modern knowledge intensive work. South Africa's development as well as knowledge economy depends partly on the improving the teaching of mathematics. Teachers in well-resourced schools are learning alternative teaching methods using freely available online sources in order to teach mathematics effectively.

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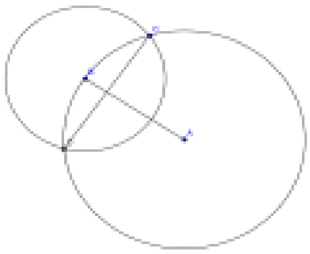
APPENDIX:

1. Explain the relationship between figure 2 and figure 1



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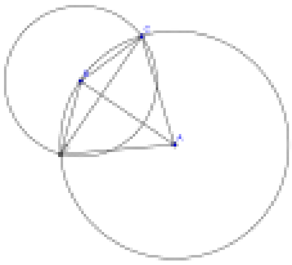

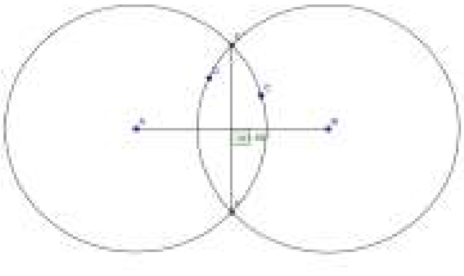

THE TEACHER PROFESSIONAL: ROLES AND RESPONSIBILITIES IN THE AGE OF ASSESSMENT

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In this paper we call on mathematics teachers as professionals to imagine ways of acting with agency in a culture of systemic assessment that is inappropriately used to criticise and blame teachers, rather than the comprehensive system, for weak test results. We debate the issue by considering three questions: What are the purposes of 'good' education, and what mathematics education is espoused (and therefore valued) in the National Curriculum Statement (through the Curriculum and Assessment Policy documents)? What is good mathematics assessment? And, what is the role of mathematics teachers as professionals in ensuring good education and good assessment? With the agency enacted by teachers to boycott the 2015 Annual National Assessment as reference, we propose ways forward for mathematics teachers to recommit to providing good education, and to own their professional development pathways.

Keywords: Professional, teacher, responsibilities, roles

INTRODUCTION

The identity of the teacher as a professional, and teaching as a highly respected occupation is a key feature in countries with the most successful, and generally accepted, high quality education systems, for example, Finland and Alberta, Canada. The requirement to make daily decisions, based on multiple knowledge sources, and to respond timeously to problem situations and effect change makes the teacher a pivotal figure in society. Parents talk directly to teachers about individual problems of learners; teachers spend more time in educational contact with children than parents do; and departments of education are in direct contact with teachers through various channels such as subject advisors and official documents. Why is it then that the teachers' voices are heard so seldom, when educational debates and issues are reported in the media, and even in educational research? Why is it that society's ills and failures are squarely dumped at the feet of teachers (even if we call them collectively "the education system"), without society giving heed to all the contributing educational factors that lie outside the control of the teacher? In this paper we will critically address the role and responsibility of mathematics teachers in relation to increasing systemic assessment. We will debate the issue by considering three questions: What is good mathematics education? What is good mathematics assessment? Who is best able to ensure good mathematics education? Our arguments can be applied to other subjects too, with adaptation for the nature of a subject.

WHAT IS GOOD EDUCATION?

The question, "What is "good education", is not easily answered, especially in diverse communities with diverse aspirations. The answer to what good mathematics education is, depends on the answer to the bigger question of what our society expects of education in general. The goal of the entire education community endorsed by the civil society generally is "quality provision". Though what constitutes quality may be interpreted differently by politicians, departments of education, teachers, parents and learners, the expectation is that quality is to be measured and expressed in numbers. For some a high score by aggregates of learners on the Annual National Assessment signal quality at a national level; for some the total number of distinctions of a cohort in the final school leaving exam signals quality education at school; and it is less often does the quality of the teaching get mentioned when an individual attains excellent results in the school leaving exams. This phenomenon on the part of teachers of owning failure when students do badly, but not claiming achievement when a student excels has been highlighted in the work of Higgins (2011), as limiting the self-cultivation of teachers, self-cultivation being a necessary requirement for good teaching and sadly, for many quality of education is measured by the visible affluence of schools and the figure that reflects the school fees. A small group of parents choose alternative education for their children, some because the problems in state education deter them, others because they value the goals for education of alternative systems like Waldorf schools or Montessori Schools. In general, a view of good education as one that stimulates an insatiable curiosity about the world (and the mathematics that we use to describe it) are seldom pronounced, perhaps because it is difficult to assess and express on numerical scales.

In order to decide on valid indicators of good mathematics education, we need to background numbers for a while and look to a broader vision of our society in relation to mathematics. We might ask questions regarding the holistic view or the needs of individuals and the needs of society, and whether these needs are the same across diverse communities, such as the urban and the rural communities (Mgqwashu, 2016).

Purposes of education

Biesta (2009a) outlines three purposes of education, which in combination have at heart the moral desire to reconcile individual and societal needs. The first, qualification, is the purpose most pronounced in our South African society. The qualification attained at school leaving signals to potential employers and tertiary education institutions that a cohort of young people has attained skills and knowledge that will make them employable, or able to continue with further education. The general aims of the South African curriculum, listed in the Curriculum and Assessment Policy (CAPS) documents (DOE: 2011, p.4) clearly states the qualification purpose of education, both at the exit level, and throughout the school years:

- providing access to higher education;
- facilitating the transition of learners from education institutions to the workplace and providing employers with a sufficient profile of a learner's competences.
- High knowledge and high skills: the minimum standards of knowledge and skills to be achieved at each grade are specified and set high, achievable standards in all subjects;
- Progression: content and context of each grade shows progression from simple to complex and credibility, quality and efficiency: providing an education that is comparable in quality, breadth and depth to those of other countries (DOE, 2011, p. 4-5).
- Individual learners who obtain the National Senior Certificate expect society to take them up in the workforce or in further education programmes. Good education, with the qualification purpose in mind, therefore ensures that the qualification is valid.

Socialisation in a particular social, cultural and political order, is the second purpose outlined by Biesta (2009a). In a culturally diverse country like South Africa the socialisation goal of education also points away from existing cultural practices, and towards influencing ways of being that will bring about morally desirable social change. The CAPS document is unambiguous about socialisation as a purpose of education:

- equipping learners, irrespective of their socio-economic background, race, gender, physical ability or intellectual ability, with the knowledge, skills and values necessary for self-fulfillment, and meaningful participation in society as citizens of a free country;
- Social transformation: ensuring that the educational imbalances of the past are redressed, and that equal educational opportunities are provided for all sections of the population;
- Active and critical learning: encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths;
- Human rights, inclusivity, environmental and social justice: infusing the principles and practices of social and environmental justice and human rights as defined in the Constitution of the Republic of South Africa.
- The National Curriculum Statement Grades R-12 is sensitive to issues of diversity such as poverty, inequality, race, gender, language, age, disability and other factors;
- Valuing indigenous knowledge systems: acknowledging the rich history and heritage of this country as important contributors to nurturing the values contained in the Constitution (DOE: 2011, p.4).

It follows from the above expressed purposes that socialisation is not a simplistic goal.

Individuation or *subjectification* is the third purpose of education identified by Biesta (2009a). This purpose runs counter to socialisation, in some respects, as it is "precisely not about the insertion of 'newcomers' into existing orders, but about ways of being that hint at independence from such orders" (Biesta, 2009a, p. 40). Yet, it is the socialisation function that enables different kinds of individuation – a culture has to enable individuals to strive towards their own goals. Individuation as a purpose of education acknowledges that learners are inescapably individuals with differing life choices ahead. Good education has as goal that individuals develop into independent and autonomous thinkers and responsible actors. The "aims of education" espoused in CAPS provides a vision of individuals who integrate the purposes and principles of education into a way of life: learners that are able to:

- identify and solve problems and make decisions using critical and creative thinking;
- work effectively as individuals and with others as members of a team;
- organise and manage themselves and their activities responsibly and effectively;
- collect, analyse, organise and critically evaluate information;
- communicate effectively using visual, symbolic and/or language skills in various modes;
- use science and technology effectively and critically showing responsibility towards the environment and the health of others; and
- demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation (DOE: 2011, p.5).

As we are building an argument about the role of the teacher professional to provide good education in the age of measurement, we invite the reader at this stage to imagine for a moment what the day to day business of teaching should encompass to answer to a vision of good education in South Africa.

WHAT IS GOOD MATHEMATICS EDUCATION?

The aims of mathematics education have changed fundamentally with the maturation of public schooling that brings many more learners into mathematics classes than was the case a hundred years ago. But the greatest impetus to change the aims of mathematics education and hence curricula is the rapid and ever increasing role of computerised information in developed societies. Even fifty years ago, skill in mathematics calculations were commendable and allowed an individual to excel in the then possible applications of mathematics in science, business and craft. Today computers are routinely used to do even the simplest calculation and the emphasis of education has shifted to deepen the understanding of mathematics and its computation to enable people not merely to do calculations, but to analyse and manage the use of calculation and the application of mathematics.

It is a very real prospect that ever newer mathematics will have to be included in school curricula in the near future, as it filters down into everyday applications. While we do not yet know what mathematics the learners in our classes today will have to learn in their future working lives, good mathematics education endeavours to provide opportunities for all learners to experience personal mathematical thinking, to re-create mathematics and to apply mathematics in problem contexts. Table 1 presents the specific aims and skills for Mathematics education from Grade 1 to Grade 12, as described in CAPS, and relates them to the goals of education as described above.

Table 1: Specific aims for Mathematics Education as in the Curriculum and Assessment Policy Statement

Specific aims of mathematics education		Specific skills in mathematics education	
The teaching and learning of mathematics aims to develop		To develop essential mathematical skills the learner should	
A critical awareness of how mathematical relationships are used in social, environmental, cultural and economic relations	Socialisation	Develop the correct use of the language of Mathematics	Socialisation
Confidence and competence to deal with any mathematical situation without being hindered by a fear of Mathematics	Individuation	Develop number vocabulary, number concept, and calculation and application skills	Individuation
An appreciation for the beauty and elegance of Mathematics	Individuation	Learn to listen, communicate, think, reason logically and apply the knowledge gained	Socialisation
A spirit of curiosity and a love for Mathematics	Individuation	Learn to investigate, analyse, represent and interpret information	Individuation
Recognition that Mathematics is a creative part of human activity	Socialisation and individuation	Learn to pose and solve problems	Individuation
Deep conceptual understandings in order to make sense of Mathematics	Individuation	Build an awareness of the important role that Mathematics plays in real life situations including the personal development of the learner.	Socialisation and individuation
Acquisition of specific knowledge and skills necessary for			
the application of Mathematics to physical, social and mathematical problems	Qualification		
the study of related subject matter	Qualification		
further study in mathematics	Qualification		

There can be no doubt that South Africa has pledged good mathematics education to its citizens. The curriculum goals are aligned to international values of good education. The departments of education are tasked with providing the infrastructure to make good education possible, and also with appointing teacher professionals to facilitate learning in classrooms.

The social pressure on both departments and teachers is significant. Parents and employers place their hopes on good education in order to prosper. In the give and take of blame for the current problems with education in South Africa, the teacher receives direct blows on a daily basis. Individual parents, the press, and the overseeing government officials tend to engage with teachers only when there are problems. When an individual learner fails to progress, parents hold the teacher accountable, when aggregated results are published and departments of education is under pressure, accountability is demanded from individual teachers. We agree that accountability is desirable, but we question the means in which information is gathered and weighed to paint South African teachers as a group, as deficient in some respects. What might an alternative to a deficit model look like?

WHAT IS GOOD (MATHEMATICS) ASSESSMENT?

Good (mathematics) assessment must provide valid information of progress towards the curriculum goals. With regard to the power and influence of assessment, Matters (2009, p. 222) asserts the following;

> The role of assessment, and assessment information in education debate and policy in the early 21st century is an extremely powerful one. [This role] can be justified only if two conditions are met: that the assessment itself is of sufficient strength and quality to support the uses to which it will be put; and that the users of the assessment data – the analysts, the teachers, the administrators, the policy makers – have sufficient expertise and imagination to see beyond the rules of thumb and piece together the true underlying story (whether the story is about one student underperforming in one test, or about a country outperforming Finland in international standardised tests).

The most pervasive use to which assessment results are put is to communicate firmly what is valued in education. Biesta (2009b) warns that the message can be turned on its head, from good intentions to measure what we value, to the unintended message that we value only what we *can* measure. From across the educational field there have been challenges to the view that testing should be used to determine policy changes in education (Messick, 1989; Bennett, 2010; Jennings and Bearak, 2014). Large scale systemic testing with the purpose of international comparison, notably the cyclical Trends in International Mathematics and Science Study (TIMSS) assessments, became the indicators of effective and efficient education across the globe, as nations strive to be ranked in the "top ten" countries. As can be expected the results of the assessment raised heightened curiosity about what teachers do in classrooms to make the leading nations so successful. In the wake of large scale international assessments South Africa adopted a similar multiple choice type systemic assessment, with the good intention to measure, and then improve progress in mathematics learning country wide. The rationale was that by adopting a similar style of assessment the results would improve!

Similarly, the results of the Annual National Assessments in Mathematics (ANA) prompted inquiries into what teachers do in South African classrooms to cause such poor results. The question is whether valid and nuanced enough conclusions about classroom practices can be drawn from results on systemic mathematics tests comprising multiple choice, or short answer, formats. In the early 21st century, the National Research Council of the United States (2001) warned the educational community of the limits of many of the then accepted testing practices. They doubted whether the assessments that are most widely used "effectively capture the kinds of complex knowledge and skills that are emphasized in contemporary standards and deemed essential for success in the information-based economy" (p. 26).

A second issue raised by the NRC (2001) concerns "the usefulness of current assessments for improving teaching and learning" (National Research Council: 2001, p. 27). Because of time constraints and the requirement for such tests to "cover the curriculum" the reach of even the best test is unavoidably shallow. Current systemic type tests provide very little if any information about an individual student's current knowledge about a particular concept, or learning path, that is not already evident to the teacher professional. By the time systemic test results are published the learner has moved on to another grade or teacher, and it is only the teacher's classroom-based assessment that potentially may provide information about changes in the knowledge and skills of individual learners.

A third limitation refers "to the static nature of many current assessments" (NRC: 2001, p. 27). Most assessments provide "snapshots" of achievement at particular points in time, but they do not capture the progression of students' conceptual understanding over time, which is at the heart of learning. This limitation exists largely because most current modes of assessment lack an underlying theoretical framework of how student understanding in a content domain develops over the course of instruction, and the prevailing measurement methods are not designed to capture such growth.

In all fairness to the education authorities, the purpose of systemic testing is not to provide local diagnostic information, but to provide evidence of patterns of strengths and weaknesses in the population measured (DBE, 2011). Hence, a good systemic assessment of the state of mathematics knowledge and skills of learners must reflect such patterns accurately and at a grain level that enables further insight and research into suitable support. Yet, systemic tests like the ANA are set and administered without regard for local influences (we concede that learners' home languages (Afrikaans and other African languages) are taken into account until Grade 3), and claim to present objective information Yet, much is missed when individuation is ruled out in the gathering of information.

An analysis of the mathematics ANA tests over three years, applying the professional knowledge of mathematics educators and teachers, showed that question formulation was often opaque or ambiguous, or without clear meaning from a mathematical perspective (Pournara, Mpofu & Sanders, 2015). This observation is exacerbated when translated into learners' home languages. If test conditions allowed for learners to inquire about the meaning of questions, we propose that more individual learners would have had a better chance to provide a correct answer. Furthermore, we propose that the multiple choice format of the test prohibits productive extended working out in order to reach a solution. Hence the ANA cannot provide information about ways of working that may be true to curriculum goals, but cannot be elicited under such test conditions.

A fourth set of concerns relates to fairness and equity (NRC, 2001, p. 28). Systemic large scale assessment is prone to misalignment with what individual learners in particular schools have been taught. Despite the prescriptive measures like pace setters the matching of test items to what is taught in class cannot possibly be one-to-one. Education is a complex process that cannot be linearly determined, however great the desire is to see all learners and teachers "on the same page" at a specific time. Yet, the pressure on teachers and schools to "perform" through the proxy of systemic learner assessment (Sayed, Kanjee & Rao, 2014) is overwhelming. This scenario points to a severe narrowing of the curriculum in favour of "teaching to the test".

Our contention is with the wrongful and dangerous use of the ANA results to shame and blame schools and teachers publicly (NEEDU: 2013, p.57). Such use is tantamount to the government talking to learners through the test, but then making statements about teachers, or failing to refute statements about teachers in the press, when reckless journalists report research out of context. An alternative, potentially enriching approach would be to gather first-hand information from teachers and classrooms directly, thereby enabling the teacher's voice to be heard.

The ANA is designed according to a transactional curriculum framework, which is controlled through pace setters that prescribe the order and time spent on mathematics topics. Along with the prescription comes a myriad of related paper work for teachers in an attempt to hold teachers accountable. It should already be clear to the reader that such measures must vitiate the goals of mathematics education described in our curriculum. It follows that the ANA can at best assess a snapshot of each topic as prescribed by the DBE for a specific year, and can only assess the efficiency of teaching mathematics for socialisation purposes – what everybody needs to know, and for qualification purposes at certain points in the school career. We accept that the purpose of individuation is at this time not a priority of systemic type assessment; we therefore focus our analysis on the vision of mathematics socialisation and qualification that is espoused by curriculum policy.

THE ROLE OF THE TEACHER PROFESSIONAL IN GOOD MATHEMATICS EDUCATION

While on the one hand the lower pass rate in Mathematics and Science (than in other subjects) is of national concern each year when the results of the National Senior Certificate (NSC) examinations are reported, there is also a concern that the value of this qualification, a pass mark, and even the meaning of a distinction in Mathematics has little meaning (Campbell & Prew, 2014). The underlying issue underlying both the above concerns is that the NSC mathematics examination does not validly assess what employers and tertiary education institutions value as evidence of good mathematics education.

Curiously silent are teachers about whether the NCS mathematics qualification supports and values good mathematics education. But qualification in education is applied throughout the school period. Once a learner receives a report that indicates a pass to the next year of schooling, parents, teachers and learners trust that the qualification is valid and that such learners are qualified to proceed with the learning tasks of the next grade. It is here that the role of the professional teacher as assessor is vitally important, and here too where the risk is real that external assessment uninformed by the voices of teachers may force education off track. It is also at these intermediate levels of assessment for qualification that teacher voices are slowly being heard through their participation in unions. In this section we will reflect on the decision of teacher unions to boycott the administration of ANA in 2015 as an act of agency, and the implications for the role of the teacher professional in ensuring good mathematics education.

Teacher agency

Essentially agency has been defined as the way in which people "critically shape their responses to problematic situations" (Priestley, Biesta & Robinson, 2013). The ecological view (Priestley, et al., 2013) implies that agency does not primarily reside in the teacher but is an outcome of the teacher acting meaningfully on a day to day basis within the educational and social milieu. Far from being problematic or "rocking the boat", teacher agency, or professional agency, can be seen as the condition required for teachers to remain focussed, to hold the line, and to maintain purposiveness and continuity towards curricular goals. A natural consequence of this view of agency is that policy decisions with regard to the day to day role of teachers can either facilitate agency or hinder agency. Teachers as knowledge rich professionals (Gitomer & Zisk, 2015), have to keep in balance the broad purposes of education while maintaining accountability to the children they teach, the education department and the general public. The call to acknowledge the necessary professional agency of teachers has been a theme in previous conference papers (Long & Lampen, 2014; 2015).

In this paper we contemplate the role of agentic teachers facing annual compulsory large scale assessment. For example, should a teacher spend three weeks teaching to the test, under immense pressure to prepare learners for the ANA (or fashion her whole teaching programme to support the systemic assessment), in order to provide a data point on a graph? Or should a teacher who is aware that her learners need more time to develop deeper understanding of the ways of doing mathematics and thinking mathematically, as espoused by the curriculum, refrain from administering the ANA and focus on teaching and learning instead? In 2015 teachers began voicing their concerns.

Agentic teachers call for valid assessment

Teachers through the major teachers' unions voted to boycott the 2015 ANA (NAPTOSA: 2015, Joint media statement). Made out by government as malicious avoiding accountability (NAPTOSA, 2015) teachers' voices were in danger of once again being ignored and their actions forcefully controlled. Yet, the reasons that were provided for the boycott had appeal to the public and to educationists who have a research interest in the provision of quality education. We propose that teachers have been quiet for too long about the conditions of their profession, silenced by fear of being further vilified as unprofessional. We support the agency of teachers and the reasons that teachers have put forward, and hope that these initial steps to formulate problems that compromise their roles as professionals responsible for the education of their learners, will progressively enable them to engage with confidence, the government, the press and parents.

The primary call by teachers through the unions was to suspend the then current ANA process and work with government to design an assessment process that would be valid in varying contexts; be of benefit to learners and teachers in their daily task; and provide information that leads to efficient and effective improvement of the system. In the final part of this paper we discuss ways forward for mathematics teachers to inform valid assessment and improvement of education in South Africa.

THE WAY FORWARD

Mathematics teachers have at least two national forums through which to act as agents of change. One forum is the various teachers' unions, and the other is subject related professional bodies like the Association for Mathematics Education of South Africa (AMESA). We are proposing visions and ways for engagement as AMESA.

Recommit to the curriculum goals

Mathematics teachers should use AMESA conferences as a safe space to deliberate what it would mean to recommit to the goals of mathematics education for socialisation, individuation and qualification, as espoused in our curriculum.

This focus would create opportunities to critically evaluate the various professional development programmes offered by a variety of institutions often with divergent philosophies, beliefs and goals that are imposed on teachers. Not all educators have the same vision for mathematics education, and only deep and critical deliberation about our own commitment to curriculum goals will enable mathematics teacher professionals to choose support and interventions that will bring them closer to the goal of good mathematics education.

Recommit to continuous professional development

Mindset theory (Dweck, 2006) is a powerful call to individuals to return to owning their own learning and development paths. Mindset theory invites us to reflect on our personal views about our own abilities. Supported by cognitive research results about brain plasticity it allows individuals to rethink erroneous notions such as the ability to do mathematics is a gift that only a chosen few have received; or that there is a limit based on age or ability to what we are able to learn. Cognitive research results that underpin mindset theory also challenges us to accept that we only learn when we are challenged, when the work is harder to understand rather than easier. It is a fact, and not a shame, that many experienced teachers have not mastered new curriculum content to a level where they are able to use such knowledge themselves or teach it with confidence. It is a fact and not a shame that newly qualified teachers do not have the same level of content knowledge in all areas of the curriculum, and that they too need to grow through experience and support. It is a fact too that the rapidly changing world we live in will necessitate repeated curriculum changes, and that teachers will always be in a position of inadequate knowledge. If mathematics teacher professionals own their development paths they will be able to request without shame appropriate support from government, unions and professional organisations like AMESA.

Recommit to working together

This paper started by considering what good education is, and what good assessment is. We are aware that we were better able to provide arguments for why assessments are not good. For good assessment, we acknowledge that teachers, teacher educators, government and assessment professionals have to work together until we have assessment systems and means that are valid, and are confident that we value what we teach and teach what we value. AMESA can be the space where we start such work together. We can imagine that AMESA creates a repository for tests and assessments set by teachers, shared for use and comment, and then offering the possibility of further development. Mathematics teacher educators need not be at the receiving end, but can and should lead the quest for curriculum aligned assessment.

Working together as mathematics teacher professionals is also crucial to ensuring that expertise is handed over as our experienced teachers prepare to leave the profession at the end of their careers.

So too is working together crucial to ensure that newly qualified teachers are mentored towards successful teaching, and that the gap between teacher education institutions and schools are closed, in the interests of building on the strengths of both institutions.

Recommit to high levels of agency in individual and collective action

Mathematics teachers can no longer be silent about the complexities of their profession when they are perceived to be solely accountable for an ineffective mathematics education system. When reports are published that do not provide facts in ways that give teachers voice, teachers have to express their views on the reports. If government or teacher unions act in breach of education goals, by corrupt actions that thwart good education, teachers must speak up. It is the teacher that is the clearing house of information about learning to parents, employers and governments. Teachers are role models of educated adulthood, whether they are aware of this role or not. Mathematics teachers must take up their role to provide their learners with opportunities for individuation, and socialisation into a culture where hard work towards deep understanding of mathematics is celebrated and valued.

Enter into scholarly debate about assessment and accountability

This paper aims to stimulate scholarly debate among teachers as well as teacher educators in order to formulate a theoretically sound argument about the role of teachers as professionals within practices of systemic assessment. The argument we present here has to be weighed critically by the professional teacher who then has to work as and with education researchers to find a voice informed by a visions of good education and supported by evidence.

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A TEACHER AND HER GRADE 10 LEARNERS' UNDERSTANDING ABOUT THE TEACHING AND LEARNING OF MATHEMATICAL LITERACY

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This paper is about a teacher and her learners understanding about the teaching and learning of Mathematical Literacy. Using Bernstein codes of recognition and realization rules learners understanding about Mathematical Literacy was analyzed. The study is located within a qualitative approach, adopting a case study two classes which focuses on one secondary school in South Africa. In this school two classes of approximately 30 learners in each, 5 Grade 10 learners in each class (both Mathematics and Mathematical Literacy) and their teacher who teaches both Mathematics and Mathematical Literacy were selected for interviews about their understanding of Mathematics and Mathematical Literacy. The data reveals that the differences between mathematics and mathematical literacy, according to learners and their teacher, is at superficial level but not in terms of content. In this paper, I therefore argue that Mathematics and Mathematical Literacy are inseparable because both have mathematics as their originating frame.

Keywords: Learner, teacher, mathematics literacy, understanding

INTRODUCTION

There has been a debate about the introduction, legitimacy, agendas and the teaching and learning of Mathematical Literacy alongside Mathematics since the inception of Mathematical Literacy in 2006 (Venkatakrishnan & Graven, 2006; Graven & Venkat, 2007; Christiansen 2006; 2007; Venkat, 2007; Mbonani & Bansilal, 2014; Machaba, 2014). These debates have been centered on teachers' views, interpretation of the curriculum (Gaven & Venkat 2007; Machaba, 2014), and teaching of mathematical literacy (Vithal, 2006; Machaba, 2014). Less has been explored with regard to learners and their teachers' understanding on the teaching and learning of Mathematical Literacy alongside Mathematics (Geldenhuys, Kgruger & Moss, 2013; Graven & Venkat, 2006; Venkat & Graven 2008). Thus this study looked at how a teacher and her learners understand about the teaching and learning of Mathematics alongside Mathematical Literacy. The purpose of this paper is to establish if learners and their teacher understands of the teaching and learning of the two learning areas with regard to nature, teaching strategies, and language embedded in them.

The research question which guided this study was: What is Grade 10 learners' understanding of Mathematical Literacy alongside the implementation of Mathematics. How does Mathematical Literacy differ to Mathematics in terms of its nature, teaching strategies and language? To answer this question, firstly Curriculum and Assessment Policy Statement for Mathematics and Mathematical Literacy have been analyzed. Secondly, a teacher and Grade 10 learners' views would be analyses using Bernstein's construct of recognition and realization rules.

MATHEMATICS AND MATHEMATICAL LITERACY

In 2006, Mathematical Literacy and Mathematics were introduced into South African Further Education and Training (FET) curriculum. Their introduction made a mathematically-orientated subject – either Mathematics or Mathematical Literacy compulsory for all FET learners

The Curriculum and Assessment Policy statement (2011: 10) defines Mathematical Literacy as:

> a subject that develop competencies that allow learners to make sense of, participate in and contribute to the twenty-first century world – a world charactered by numbers of different ways. Such competencies include the ability to reason, make decision, solve problems, manage resources, interpret information, schedule events and use and apply technology.

The definition above indicate that Mathematical Literacy develop mathematical competencies such "ability to reason, make decision, solve problems, manage resources, interpret information, schedule events and use and apply technology". These competencies are not different from the ones indicated in the Mathematics CAPS document as indicated below, which are "logical and critical thinking, accuracy and problem solving that will contribute in decision-making". Mathematics has been defined within the Curriculum and Assessment Policy Statement (CAPS) of the FET phase in the following terms:

> Mathematics is a language that makes use of symbols and notations for describing numerical, geometrical and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision-making. Mathematical problem solving enables us to understand the world (physical, social, and economic) around us, and most of all, to teach us to think creatively (DBE, 2011, p. 8).

Internationally Mathematical Literacy is referred to as Quantitative literacy which is aimed at developing students with a flexible range of quantitative skills to be applied in a diverse range of contexts (Steen, 1990; 2001). For example, Steen (1990; 2001) differentiates between quantitative literacy and traditional mathematics as typical school mathematics problems which involve simplified numbers and straightforward procedures but require sophisticated abstract concepts, while quantitative literacy involves mathematics that learners would use in their daily life experiences.

It is clear from the preceding definition that Mathematics and Quantitative Literacy, which is referred as Mathematical Literacy, differ in context and purpose. Mathematics aims at developing mathematical knowledge within mathematics itself at an abstract level while quantitative literacy or Mathematical Literacy is aimed at analysing and interpreting the world using mathematics. In South Africa, Mathematical Literacy was introduced as an alternative to mathematics to address the aims as outlined in the definition of quantitative literacy as defined above. I now discus how Mathematical Literacy was developed as a school subject in South Africa.

INFLUENCES OF THE CONCEPTUALIZATION OF MATHEMATICAL LITERACY IN SOUTH AFRICA

The introduction of Mathematical Literacy alongside Mathematics was to make a mathematical–oriented course available and compulsory for all learners in South Africa. Brombacher (2006) states that Mathematical Literacy in South Africa in particular and across the world in general was developed out of two pressures: “democratisation of mathematics” and “mathematics for democracy”.

Democratisation of Mathematics

Democratisation of mathematics refers to the provision of greater access to mathematics for more people. One of the reasons Mathematical Literacy was introduced in South Africa was to make sure that many learners do mathematics-oriented courses. Vithal and Bishop (2006) state that “the focus on Mathematical Literacy in South African context needs to be understood both historically in how access to mathematics was denied to the vast majority of black people” (p.3).

In the same vein, Christiansen (2006) suggests that one of the main reasons Mathematical Literacy was introduced was to reach the 200,000 learners leaving grade 12 every year without mathematics and the 200,000 additional learners who fail mathematics yearly. Christiansen (2006) further argues the failure of South African learners in international comparison tests, for example TIMMS-R, also added to this.

The “birth” of Mathematical Literacy was to remedy the divide between Mathematics Higher grade (HG) and Mathematics Standard Grade (SG). Over 40% of learners at the end of FET phase senior certificate candidates nationally did not take mathematics courses at all in the FET phase. 50% of learners entered for the standard Grade (SG) mathematics examination and fewer than 9% entered for the higher grade examination – subject and examination pass needed for entry to higher educational courses with a significant mathematical component (Venkatakrishnan & Graven, 2006). By entering many more candidates for the SG band in order to secure better pass rates, the Department of Education appeared to be encouraging schools to abuse the HG/SG curriculum differentiation model.

Similarly, the body representing the higher education sector was particularly concerned that this abuse further reduced the pool of learners studying higher level mathematics, and consequently, restricted many learners access to mathematical disciplines at graduate level (SAUVCA/CTP, 2003). So, the structuring of Mathematics and Mathematical Literacy is often interpreted as a replacement of the previous Higher Grade (HG) and Standard Grade (SG) distinction that used to exist in Mathematics (Venkatakrishnan & Graven, 2008). This misinterpretation is despite the strong statement from those involved in the development of the Mathematical Literacy curriculum that Mathematical Literacy is not equivalent to standard grade mathematics (Brombacher, 2006; Laridon, 2004).

Mathematics for Democracy

The second influence to the introduction of Mathematical Literacy emanated from a push by government to enable more people to use mathematics to facilitate effective participation and contribution to the twenty-first century world (Brombacher, 2006). Many of the intentions in the critical and developmental outcomes highlighted in the new Mathematical Literacy curriculum policy document (Department of Education [DoE], 2003) resembles those embodied in the broad emerging literature that explores the social, cultural political, historical and economic dimensions of mathematics education (Vithal & Bishop, 2006). This can be observed in the critical and development outcomes which are as follows:

- use mathematical process skills to identify, pose and solve problems creatively and critically.
- work collaboratively in teams or groups to enhance mathematical understanding.
- organise, interpret and manage authentic activities in substantial mathematics ways that demonstrate responsibility and sensitivity to personal and broader societal concerns.
- collect, analyse and organise quantitative data to evaluate and critique conclusions.
- communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams.
- use mathematical literacy in critical and effective manner to ensure that science and Technology are applied responsibly to the environment and to the health of others.
- demonstrate that a knowledge of mathematics assists in understanding the interrelatedness of systems and how they affect each other.
- be prepared to use a variety of individual and co-operative strategies in learning mathematics.
- engage responsibly with quantitative arguments relating to local, national and global issues.
- be sensitive to the aesthetic value of mathematics.

- explore the importance of mathematical literacy for career opportunities
- realise that mathematical literacy contributes to entrepreneurial success (DoE, 2003, p. 10)

The above critical and developmental outcomes show that mathematical literacy should enable a learner to use mathematics to critique society.

Christiansen (2006, p. 6) observed that the National Curriculum for Mathematical Literacy's main agenda is for the improvement of living conditions, social justice and democracy. For example the National Curriculum for Mathematical Literacy states that "to be a participating citizen in a developing democracy, it is essential that the adolescent and adult have acquired a critical stance with regard to the mathematical argument presented in the media and other platforms". This quote shows the powerful role that can be played by mathematics. It shows that Mathematics, and Mathematical Literacy, can provide powerful modelling tools. It can help learners to develop insight into complex contexts such as ecology which they had never encountered. However, Christiansen (2006) argues that it is assumed the role played by mathematics will be known to both teachers and learners.

> Mathematical Literacy [is] defined as part of a progressive agenda for increased democracy and social justice, however the superficial engagement with complex applications of mathematics implied by the MLNCS is not likely to live up to its claim" (p. 6).

In support, Venkatakrishnan and Graven (2006) observed that the National Curriculum for Mathematical Literacy also keeps in place the basic structure of the Mathematics curriculum. While the definition and the opening section of the document describes the purpose of introducing mathematics as developing the "self-managing person", the "contributing worker" and the "participating citizen" (DoE, 2003, pp. 9-10), it also emphasizes the subject as useful in a broad, future–life oriented sense.

AMESA (2003) report that the reason for the use of a content-led approach based on these areas was for the sake of "portability and mobility" between the two subjects – Mathematics and Mathematical Literacy. However, AMESA cautioned that "the two subjects are so dissimilar in philosophy and purpose that such portability and mobility should not be a consideration" (p. 4). AMESA's view seems to be consistent with the argument made by Steen (2001) and other local researchers in relation to quantitative literacy. The claim is that the curricula which is structured in terms of mathematical skills has a tendency of being interpreted by teachers and learners as focusing on acquiring procedures as opposed to developing mathematical attitudes. AMESA appeared to be calling for a behaviour-defined curriculum, more focussed on the kinds of actions and attitudes that are helpful when faced with a mathematical problem that can be mathematised (Christiansen, 2006; Skovesmose, 1998; Venkatakrishnan & Graven, 2006; Vithal, 2003).

Venkatakrishnan and Graven (2008) citing Hallendorf (2003) and Brombacher (2006), in relation to the development of the Mathematical Literacy unit standards note that given the time constraints for publication this curriculum used the existing FET mathematics curriculum as a starting point, and attempted to extract the fundamental mathematics while removing the pure mathematics. Hence, AMESA (2001) expressed concerns that at the time this approach would tend to work against the aims that Mathematical Literacy was intended for. AMESA was concerned that mathematical literacy should not be a "watered down" academic mathematics but rather mathematics with different emphasis, the mathematics that develop thinking skills – habits of mind – to be able to apply that learning in various contexts (p. 1).

The concern by AMESA is based on viewing mathematical Literacy using Steen's (2001), Christiansen's (2006) and Vithal's (2003) lenses that consider Mathematics Literacy as not developing new mathematical knowledge but developing mathematics thinking to interpret and analyse the world.

Graven and Venkat (2008) found that learners were very much worried about the uncertainties over whether Mathematical Literacy would be accepted in the Higher Education sector for access to degree courses in finance-related disciplines. This is the group of learners who were achieving highly in Mathematical Literacy and strongly aspired to study degree level courses in Economics/Commerce or related areas. Graven and Venkat (2008) noted that this group of learners exhibits the kinds of flexible, applied problem solving capabilities that have often been seen as lacking in students coming through with appropriate qualifications in school mathematics.

One of the things which were observed during the implementation of Mathematical Literacy was how ML is perceived by parents, teachers and learners. Some learners, parents and teachers perceive mathematical literacy classes to be for "mathematics for stupid learners" (Graven & Venkat, 2006; Geldenhuys, Kruger & Moss, 2013). Many students interviewed by Graven and Venkat (2006) responded by saying: "When you choose Maths Literacy, it is like you know you are a stupid kind of person". This kind of statement affects the morale of Mathematical Literacy learners. Christiansen (2007, p. 10) notes the separation between Mathematics and Mathematical Literacy "serves to maintain a class distinction, which of course is related. It is a well-known sorting tool of learners into those who master the decotextualised, self-referential discourses, and those who do not". Therefore the introduction of Mathematical Literacy as a school subject in South Africa was driven by a vision of a non-esoteric mathematics with real use value, which could still provide reasonable access to further education.

The fact that Mathematical literacy could still provide reasonable access to further education raises a very important question related to whether it is possible to combine epistemological access (access to mathematical knowledge) and social empowerment of learners.

THEORETICAL CONSTRUCTS

According to Bernstein (2000), the fact that Mathematics is a content-oriented subject makes it to be strongly classified and framed, which results in its recognition and realisation rules being clearer.

Bernstein (1982, p. 59) refers to Classification as:

> the nature of differentiation between contents. Where classification is strong, contents are well insulated from each other by strong boundaries. Where classification is weak, there is a reduced insulation between contents, for the boundaries between contents are weak and blurred.

Framing refers to the 'form of the context in which knowledge is transmitted and received and refers to the specific pedagogical relationship between the teacher and the taught' (Bernstein, 1982, p. 59). The concepts of classification and framing, according to Bernstein, yield to concepts of recognition and realisation rules. Recognition rules, according to Bernstein (2000), are criteria (special relationships) for making distinctions, for distinguishing the speciality of a thing or a practice or a specialisation or a context, what makes it what it is. Recognition rules are principles for recognising the 'legitimate text' (p. 50), the voice to be acquired, and are determined by the classification principle at work (relations between different knowledge discourses and practices). Realisation rules are the 'means for creating and producing the special relationship internal to what is recognised as the "legitimate text", i.e. the means for reproducing/creating the speciality in practice' (Bernstein, p.50).

According to Bernstein (2000), this implies the mathematics context can easily be recognised due to its strong classification of a knowledge structure, unique identity, unique voice and internal rules. The acquirer (the learner or the teacher) is able to recognise mathematics pedagogical text (e.g. textbooks) in the classroom. This means that teachers or learners can identify themselves in the context of Mathematics. They can ably recognise 'the speciality of the context they are in' (Bernstein 1996, p. 31). However; this may not necessarily happen in the case of Mathematical Literacy.

RESEARCH DESIGN AND METHODOLOGY

The research design for this study is best characterized as an explorative, multiple case study design (Leedy, 1997). Macmillan and Schumacher (2001); and Opie (2004) argue that qualitative research uses a case study design in which data analysis focuses on one phenomenon, which the researcher selects to understand in-depth, regardless of the number of sites or the participants of the study. The selection of participants in this qualitative case study followed "purposive sampling" (Newman, 1994).

The participants were chosen to illuminate the key theoretical constructs concerning their views in the implementation of Mathematics and Mathematical Literacy. According to Mcmillan and Schumcher (2001) purposive sampling, in contrast to probabilistic sampling, is "selecting information–rich for study in–depth" when one wants to understand something about cases without desiring to generalize to all such cases. Purposive sampling is done to increase the utility of the information obtained from small samples. Thus in this study, a school with two classes of approximately 30 learners, 5 Grade 10 learners in each class (both Mathematics and Mathematical Literacy) and their teacher who teaches both Mathematics and Mathematical Literacy were selected for interviews about their understanding of Mathematics and Mathematical Literacy.

Grounded theory (Inductive) and deductive approaches were used in this data analysis. Although I was open to see the emerging codes in the process, those codes could be related to existing theoretical orientations. The recognition and realisation rules which were used to analyse the National Curriculum Statements for Mathematics and Mathematical Literacy to examine the kinds of knowledge privileged in terms of Graven and Venkat's (2007) spectrum of agendas, were also used as evaluative criteria to teacher and learners' views and perceptions about Mathematics and Mathematical Literacy.

The interview schedule was divided into three parts, namely teaching and learning approaches, and the nature of Mathematics and Mathematical Literacy because researcher wanted to find out what are learners and their teacher understanding with regard to the nature of mathematics and mathematical literacy, teaching approaches and languages in embedded in these two subjects. The data used here involves secondary school learners for both Mathematics and Mathematical Literacy and their teacher on their understanding of the learning and teaching of Mathematics and Mathematical Literacy. I analyzed individual interviews of 10 learners from Mathematics and Mathematical Literacy class and one teacher who teach both two classes. In the interview below the following codes were used: R for Researcher, T for the teacher, ML1 for mathematics learner 1, ML2 for mathematics learner 2, MLL1 for Mathematical Literacy learner 1 and MLL2 for mathematical Literacy learner 2.

FINDINGS AND DISCUSSION

In presenting the findings, I have drawn from Graven's (2000b) orientations to mathematical knowledge and Graven and Venkat's (2007) "spectrum of pedagogical agendas". I have further drawn on Bernstein's (1996; 2000) constructs of recontextualization, in particular the notion of pedagogical identity, which is linked to the notions of classification and framing, recognition and realisation rules. The findings that emerged from this paper are: Nature of Mathematics and Mathematical Literacy, teaching and learning strategies associated to Mathematics and Mathematical Literacy, the issue of Language. These issues will be discussed in detail.

The Nature of Mathematics and Mathematical Literacy

From the interview it is evident that mathematics learners enjoy the subject due to its relevancy with respect to mathematics and it is the subject which is linked to their everyday life situation. Even if there is some little contradiction with the latter statement in their responses that ML is very challenging due to its form of case study and scenarios. It is also evident from maths learners that they do not like ML because it demands a lot of reading and also, it is considered as watered down or standard mathematics which therefore is considered as maths for "small kids". The same sentiment has been slightly echoed by their teacher when she responded by saying

> "It is because ML is the basic of pure math"

When asked about the differences between Mathematics and Mathematical Literacy, ML2, MLL5 and ML1 said the following:

> ML2:I love maths than ML because maths is straight forward and simple, if it solve for x is solve for x and proof is proof, but ML is of more of paragraph, words and I don't like reading. And ML is for small kids.
>
> MLL5: And ML is not as easy as perceived because it deals with things that are related to our real life, like cell phones which need one to read, and understand.
>
> ML1: Maths lit makes us to understand what is happening around us using maths.
>
> T: Both subjects have "solve for x", surds, ratio, decimal places, percentages and proportion, but in ML are in worded form.

From the extract above, ML2 describe mathematics in terms of being "straight forward and has "solve for x" element, while MLL1 and MLL5 describe mathematical literacy in terms of its applicability to real life and a subject that makes sense because it makes them to understand mathematics. Although, ML2 indicated that Mathematics could be identified by "solve for x", her teacher – however – indicated that in Mathematical Literacy there is also solve for x, but in words. Based on these learners description of mathematics and mathematical literacy, both two subjects do not seem to be incompatible to each other because the issue of real life context is also predominant in mathematics. Again, solve for x could be a representation of what is happening in real life context. This is confirmed by what the teacher said, when asked about the differences between Mathematics and Mathematical Literacy, she said:

> Basically… there are no differences because if you look, the textbooks that we are using are the same especially in the first chapters in terms of the content but ML has more words problems than in Mathematics textbook, that could contribute on using different teaching approaches like in ML.

What mathematical Literacy learners and a teacher explained in describing mathematical literacy can, in my view, can also be applicable in mathematics. The notions of regarding Mathematical Literacy as "low standard Mathematics" or standard grade Mathematics as indicated ML2 below is a wrong perception which ML2 confirmed by saying, "I heard people say is simple". This wrong perception is consistent with what Venkat and Graven (2008), suggested that the structuring of Mathematics and Mathematical Literacy is often interpreted as a replacement of the previous Higher Grade (HG)/ Standard Grade (SG) distinction that used to exist in Mathematics during apartheid education system. She argued that this is despite a strong statement from those involved in development of the ML curriculum that ML is not equivalent to Standard Grade mathematics (Brombacher, 2006; Laridon, 2004). In their paper, Graven and Venkat (2006) note that Mathematical Literacy was developed from a historical pressure to seek alternative kinds of mathematically-orientated courses focuses on life- related.

ML2: Maths Lit is for small kinds, because it is so easy.

R: Why are you saying ML is for small kids?

ML2: I heard people say is simple.

It is clear from the above remarks that mathematics teachers have described Mathematical Literacy as being in the form of "scenario", "little story" based subject which indicate the "exploring of a context/ scenario in order to deepen understanding of that context" (agenda 1) (Graven and Venkat 2007, p. 76). Brombacher (2006) notes that FET curriculum is designed in such a way that Mathematics and Mathematical Literacy are different in "kind and purpose" and thus Mathematical Literacy is not subsumed in Mathematics. In as much as one can agree with what Brombacher (2006) is saying, but Mathematics and Mathematical Literacy are not incompatible to each other.

Teaching and learning strategies

When asked about how they are taught Mathematical Literacy, they said:

MLL5: teachers give us case study and scenarios to discuss in group so I find ML teachers more hyperactive than those of math".

MLL1: In Maths lit class, a teacher gives us practical example, for example, we may be asked to calculate percentage of teacher's salary increment and staff like that.

ML2: In Mathematics, a teacher gives us a problem on the board and we solve it step by step.

ML4: In Maths, we use steps to calculate or solve a sum, we use rules and laws in mathematics, and for example, if you don't know the rules and laws of exponents, you cannot solve exponents' sums.

The statements above seems to suggest that there are strategies in the teaching and learning of Mathematics and Mathematical Literacy which are associated to Mathematics and those that are associated to Mathematical Literacy. In Mathematical Literacy data shows that "teachers give case study and scenarios to discuss in group", while in Mathematics there are "steps, rules and laws" to solve a sum.

What mathematical Literacy learners – ML1 and MLL5 - are saying in the above extract as strategies teachers are using in their class, are not peculiar to mathematics class. In fact, mathematics class should be seen as an active class, whereby practical examples are provided for mathematics to make sense.

In Machaba (2014), it was found that teachers associated strategies which require learners to apply mathematical rules to Mathematics and strategies which require learners to reason to Mathematical Literacy, which reasoning is one of the aspects in mathematics which has been emphasized in mathematics education. The data revealed that learners' and teachers' strategies are domain specific. In other words there are teaching strategies that are associated with M and others associated specifically with ML. The separation which has been created between mathematics and mathematical literacy create some problems of associating strategies to particular subjects. Are we saying mathematical literacy because of its domain specific strategies such as encouraging group work, learner centered approach, and teaching with an understanding is better that mathematics which put its emphasis on rules, procedures – step by step approaches?

In this study the teacher who is teaching both Mathematics and Mathematical Literacy when asked about teaching strategies she used in Mathematics and Mathematical Literacy said:

> I often use group work in Mathematical Literacy than in Mathematics class that demand individual approach, so in ML class learners often discuss, debate, analyze and interpret word problems than in math class a which is so straight forward.

When asked about mathematics and mathematical Literacy attributes:

> MLL1 said: "Mathematical Literacy teachers are flexible than Maths teachers while maths teachers are very strict and stereotype, I mean they are not communicative like ML teachers who let us to discus and debate in class".

When asked to explain "ML teachers are flexible":

> MLL1 said: they allow us to discuss in class, ask each other if we don't understand unlike in mathematics class last year where a teacher just teach us even if we did not understand"

The question to ask is what does it mean for a strategy to be for M and a strategy to be for ML strategy? The idea of associating strategies with a certain learning area has an implication to learning and teaching of M and ML. For learning, it suggests that ML and M learners participate in different discourse practices. ML learners are expected to act, think and belief differently from M learners because they are from different communities of practice and therefore their participation into their community of practices would be different. For teaching it raises critical questions linked to the issue of M and ML being considered "separate" subjects.

Does it mean that they are inherently different discourses, and therefore requiring different identities of teachers? Does it mean that for one teacher to work productively with M and ML, the discourses in M and ML should not be inconsistent (not in conflict) with the identit(y) ies of the teacher? When would it be important for these discourses to be consistent? I suggest it would be in the case where the same teacher is asked to teach both M and ML.

I suggest that the differences between the two subjects (M and ML) should not be inconsistent (in conflict), if they were in conflict what does it mean for the teacher who is teaching both? He would have split identities, which can make the task of his teaching very difficult. Again, for the teacher who is teaching both subjects, the two subjects should not have discourses which are in conflict, otherwise it would require ways of behaving which are different from the identity of the teacher.

Language

When asked about the difference between mathematics and mathematical literacy, most learners differentiate the two subjects in terms of language embedded in them. For example, ML3, ML4 and MLL5 said the following:

> ML3: I don't like ML because it is full of sentences and demands too much of reading. In maths there is no too much sentences, it is just straight forward solve for x is solve for x and nothing else.
>
> ML4: ML is full of paragraph, we maths people don't like to read…mmm too much paragraphs no…
>
> MLL5: Last year, I was doing maths nee – I did not understand some of the things the teacher was teaching like this year, for example a teacher could teach us graphs nee ---but this year I can understand cellphone graphs, like I can see that MTN is expensive than VODACOM looking at the graph.
>
> MLL1: I enjoy ML because when I read a scenario a teacher has given us, I can try to understand it, and use maths to solve a question.

It appears from the above extract that ML, due to its nature of too worded in terms of sentences and paragraph (Scenarios), as described by learners, makes Mathematical Literacy to be different from mathematics which is more abstract maths by virtue of it element of solve for x. ML3 and ML4 seem not to understand what mathematics is. They seem not to understand that "solve for x" is another form of mathematical representation which can be represented verbally in terms of sentences and paragraph. In fact in order for learners to understand mathematics, it must be learnt in the form of worded representations before it can represent algebraically – in letters.

The issue of learners spending a lot of time reading, trying to understand problems – paragraphs, sentences and scenarios - in ML is also consistent with what Graven and Venkat (2007) found in their classroom observations with ML teachers.

They observed that the common use of real situations seemed to open up opportunities for communication. Learners commented that group work and discussion were much more common in ML than they had in Mathematics. It was reported that tasks were often covered over a week or two. They "note that the structure and the pace of work in ML were perceived as being more responsive to the understandings of learners" (Graven and Venkat, 2007, p. 75). For mathematics to be taught with an understanding, it has to be verbalized first before it could be shorthanded in the form of letters. In fact, the way MLL1, ML3 and ML4 are describing how mathematical literacy is, that is how mathematics should be. Mathematics should be represented verbally – in terms of paragraph and sentences before it could be represented algebraically in the form of letters for learners to understand, as in the case of MLL1.

When the teacher asked about the difference between Mathematics and Mathematical literacy said:

> The problem with mathematical literacy learners is English, mathematical literacy is more of worded problems in the form of scenario and case studies that need to be analyzed and interpreted so our learners fail ML because they cannot understand English, and so English is a problem to them. But, in Math class is simple and straight forward learners need not to analyses and interpret it is just straight forwards solve for x, Simplify, factories, proof without too much of words.

The above extract seem to suggest that the difference between mathematics and mathematical literacy is based on worded problems and solve for x syntax for example. The big question to ask is mathematics not has worded problems. In the above extract, the teacher seems not to understand that x is a variable not just an alphabet to symbolize mathematics language, but before it is x it was a worded variable. This implies that there is worded problems in mathematics which have been represented in terms of variables – alphabet or letters. So, the difference between mathematics and mathematical literacy cannot be the one is worded and the other can be characterized by alphabet. It was very strange and unfortunate for the teacher as indicated in the extract that "Math class is simple and straight forward learners need not to analyses and interpret". Mathematics should be analyzed and interpreted. It appears from the above extract that the teacher also seems not to understand what x represent in the context of mathematics.

CONCLUSION

The data analysis suggests that there are ways of behaving, acting and doing which are associated with M, for example, application of rules, solving mathematical problems step by step. Also there are also ways of behaving, acting and doing which are associated with mathematical literacy, for example, understanding what is going on, what the concept means. These differences as indicated by both learners and their teacher, they do not necessarily differentiate what mathematics from mathematical literacy.

In fact some of the Mathematical literacy attributes such as group work teaching approach, using of scenarios or context to understand mathematics, as indicated by learners and teachers, are not strange to mathematics. Again, some of the differences, as indicated by learners and their teacher are based on perceptions as compared to reality. For example, it is a perception that mathematical literacy is easy and low standard than mathematics. In this paper, I therefore argue that Mathematics and Mathematical Literacy are inseparable because both have mathematics as their originating frame.

In some sense the data suggests that there are some differences in the discourses of two subjects, which also confirm the categorisation of the two subjects by the Department of Education, which raises the question of what does it, mean to the same teacher who is teaching both M and ML as is the case in some of the schools. The study reveals that although, learners their teacher understanding about mathematics and mathematical literacy is that the two discourses are different in terms of nature, teaching strategies, and language embedded in them, the two discourses are not incompatible to each other which may results on teachers and learners developing different mathematical identities. The study therefore recommend that, taking note of the fact that mathematics and mathematical literacy are compatible to each other, the two subjects can be combined to be one subject.

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LEARNERS' PERFORMANCE AND DIFFICULTIES IN SOLVING QUADRATIC EQUATIONS BY FACTORISATION: A CASE STUDY OF SIX SECONDARY SCHOOLS IN THE LIMPOPO PROVINCE, SOUTH AFRICA

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Grade 11 learners still experience difficulties in solving quadratic equations by factorisation, specifically with equations including both negative signs, and positive and negative signs. The study investigated the performance and difficulties learners encounter when solving quadratic equations by factorisation. The participants were 214 Grade 11 learners purposively sampled from six secondary schools in the Capricorn District of Limpopo Province in South Africa. This district had performed poorly in Grade 12 results over a period of four years. Data were collected through a diagnostic test followed by the focus group interviews held in each of the six schools with participants. This study revealed learners' performance and difficulties encountered in the diagnostic test and the reason why participants experienced difficulties in solving quadratic equations by factorisation. In analysing the data, the following categories were used to gain insights into learners' performance and difficulties: correct, incorrect, incomplete and blank responses. The analysis used percentages and absolute numbers to analyse the performance of learners. Data generated from the focus group interviews were analysed qualitatively to determine the difficulties in solving quadratic equations by factorisation. The findings revealed that, although learners had difficulties in solving quadratic equations by factorisation, they performed better in the topics of difference of two squares, quadratic equation of two terms and that of positive terms only as compared with the ones with both negative terms and positive and only negative terms. Learners performed poorly in word problems involving quadratic equations compared to symbolic equations as most of the learners left blank spaces. Hence it is concluded that learners' difficulties were associated with understanding of a problem, devising a plan, carrying out a plan and looking back. Furthermore, the language of teaching and learning may have contributed towards the poor performance in solving quadratic equations by factorisation and word problems by English Second Language learners.

Keywords: Factorisation, learners' difficulties, learners' performance, mathematics education, quadratic equations, word problems

INTRODUCTION

Problem solving is an important approach in the teaching and learning of mathematics. Studies have been conducted on problem solving in mathematics and much research has attempted to clarify the meaning of this approach to teaching mathematics (e.g., Voskoglou, 2008). According to Schoenfeld (1983), a problem is a situation one does not know how to solve. This study deals with the performance and difficulties in solving quadratic equation tasks through factorisation by Grade 11 learners. The study was undertaken in secondary schools in the Capricorn District of Limpopo Province in South Africa.

According to the new South African Curriculum Policy Statement (Department of Basic Education [DBE], 2012), a major goal is for the learners "to investigate and solve problems in a creative manner" (DBE, 2011, p. 9). Grade 11 learners in this regard are expected to solve problems related to quadratic equations through factorisation as part of the content to be mastered in secondary school mathematics. Various approaches are used by teachers to explain mathematical concepts but learners still experience challenges in solving problems effectively.

Although learners are expected to investigate and solve mathematical problems in a creative manner, they still perform poorly in this subject (National Education Evaluation and Development Unit [NEEDU], 2013). Learner performance in mathematics can be influenced by various factors such as human and material resources, poor teacher qualifications, poor teaching methods and unprofessional teacher conduct (Tachie & Chireshe, 2013). Learner performance in South Africa is poor when compared to other developing countries, which are poorer than South Africa (Armstrong, 2014). Mbugua, Kibet, Muthaa and Nkonke (2012) argue that poor performance in mathematics is due to under-staffing, lack of motivation, inadequate teaching and learning, the negative attitude of teachers and learners and ineffective teaching methods. This is manifested in the lack of learners' problem solving skills in mathematics.

Informed discussions with some teachers led to the identification of difficulties learners face in solving mathematical problems such as Grade 11 quadratic equations. The poor performance of the learners who struggle with mathematical problems highlighted the importance of studying the difficulties associated with solving quadratic equations through factorisation. In subsequent discussions with the teachers, we decided to give the learners a diagnostic test for the researcher to understand the difficulties in solving quadratic equations. This paper describes and reports on the findings regarding the performance and difficulties the Grade 11 mathematics learners encountered; and why learners encountered difficulties when solving quadratic equations through factorisation and word problems involving quadratic equations.

CONCEPTUAL FRAMEWORK

The study is underpinned by the notion of Polya's (1945) problem solving strategies. Problem solving has been transformed into a set of definitive strategies that describe human information processing for the researchers and those strategies prescribed for mathematics teachers regarding what learners should be taught (Kilpatrick, 1987). Polya (1945) identified four stages through which problem solvers pass: (1) understanding or identifying a problem; (2) devising a plan or analysing a problem; (3) carrying out a plan or executing a problem; and, (4) looking back or reflecting on an action taken. These problem solving strategies are learned in teacher education courses and passed on to the learners in the classroom. The four stated steps are described as well as the role they played in this study in the ensuing discussions with participants.

The first step of problem solving is the understanding of a problem, whereby learners should identify a given problem. This first step enabled the researcher to understand if learners were in a position to state and explain the type of quadratic equations posed, such as (1) $x^2-3x=0$, (2) $x^2-4=0$, or (3) $x^2-3x-4=0$. Furthermore, in solving quadratic equations, the following should be identified: the coefficient of x^2, coefficient of x and a constant variable in any given quadratic equation $ax^2+bx+c=0$ where $a=1$ or $a\neq1$, $b\neq0$ and $c\in R$. The equations involving negative signs such as $ax^2-bx+c=0$ or $ax^2+bx-c=0$ or $ax^2-bx-c=0$ or $-ax^2+bx+c=0$ require learners to demonstrate an understanding of placing signs between factors in brackets. The demonstration of understanding of word problems involving quadratic equations should be displayed by learners in solving those problems.

In the second step, that is, devising a plan or analysing a problem in any given problem, learners should be able to choose an appropriate strategy that is learned effectively by solving many related problems (Polya, 1945). The analysis of the problem plays an important role as it enables learners to identify which strategy is appropriate in finding factors of any given equation. The strategy that can be used, for instance, in number 1 is the common factor method to determine the values of x; the second one is to find the difference of two squares $(x-2)(x+2)=0$ to determine the value of x; and the third one comprised an understanding of the use of all the terms to find the factors of the equation. This number 3 equation requires learners' understanding of allocating signs between the brackets $(x-4)(x+1)$ and the equation with $a\neq1$ either positive or negative. In word problems, learners are expected to change words into mathematical equations in order to solve the unknown, such as: let Sara's age be x and that of her father $x+6$ and multiply them to be $x(x+6)=150$.

The third step in the process is the logical step of carrying out a plan. In this step, learners should keep in mind the steps recorded when planning strategies to solve a problem.

Implementation of a chosen strategy or strategies demonstrate to learners if steps taken to solve the problem(s) are appropriate or not, for example, if learners factorised instead of using the common factor method in $x^2-3x=0$, or giving incorrect factors in the equation $x^2-3x-4=0$. The third step requires learners to go back either to step 1 or 2 if they struggle to execute the problem. The learners should determine Sara's age and that of her father and if not satisfied, they should look back at step 1 and 2 to find the solution to the problem.

The last step is looking back or reflection of the plan, which is also important step for learners as they realise the mistakes or errors made in solving problems. In reflecting on the process of problem solving, learners establish which approach worked and what did not work. In this step learners confirm if their solutions are correct or incorrect and also where they went wrong. Solving quadratic equations through factorisation requires a learner to reflect on the factors of the equation(s), such as $(x+1)(x-4) =0$ and to determine if he or she can multiply the brackets back to get the original equation.

LITERATURE REVIEW

The quadratic equation is an important topic, not only in secondary school mathematics but also in historical development (Didis & Erbas, 2015). Saglam and Alaca (2012) concur that the quadratic equation is important in secondary school mathematics because it serves as a bridge between linear equations, functions and polynomials. However, many errors are made by secondary school learners in solving these equations.

Research has explored the errors learners display in solving quadratic equations (Makonye & Nhlanhla, 2014; Zakaria & Maat, 2010). A limited number of studies focus on the techniques learners engage in while solving quadratic equations (Bossé & Nandakumar, 2005). Other studies focus on learners' understanding and difficulties in solving quadratic equations (Kotsopoulos, 2007; Tall, Lima & Healy, 2014; Zakaria & Maat, 2010) while others have focussed on the teaching and learning of quadratic equations (Olteanu & Holmqrist, 2012; Vaiyavutjamai & Clement, 2006).

Bossé and Nandakumar (2005) argue that solving quadratic equation by factorisation is a challenge when leading coefficients or constant variables display many pairs of variables. Poor understanding in the use of procedures in solving linear equations may lead to difficulties in solving quadratic equations (Lima, 2008; Tall et al., 2014). Vaiyavutjamai and Clements (2006) suggest that learners' difficulties in solving quadratic equations are due to lack of instrumental and relational understanding. Zakaria and Maat (2010) postulate that learners make errors in transformation and process skills when solving quadratic equations. Learners experience difficulties in solving quadratic equations associated with arithmetic and algebraic manipulations errors (Didis & Erbas, 2015). Makonye and Nhlanhla (2014) argue that most errors arise from problems with factorisation caused by misinterpretations of the tasks required of learners.

Research has revealed that a reason why learners struggle to solve algebraic word problems is because learners cannot generate the equations representing relationship with the problems (Kieran, 1992). The difficulties in solving algebraic word problems do not lie in a formal algebraic system but in the language of the problems in the comprehension phase (Koedinger & Nathan, 2004). The inability of learners to solve word problems is also attributed to the learners' psychological processes (Briars & Larkin, 1994). Didis and Erbas (2015) found that learners' difficulties in solving word problems are caused by the lack of comprehension of the context and therefore learners cannot formulate the equation to be solved. This study intended to answer the following research questions:

1. How learners performed when solving Grade 11 symbolic quadratic equations and word problems involving quadratic equations?
2. What difficulties do Grade 11 learners encounter in solving symbolic quadratic equations and word problems involving quadratic equations?

METHODOLOGY

The study followed a case study approach using qualitative methods. The participants were 214 learners from six schools. Schools were chosen for their accessibility to the researcher and the poor Grade 12 performance during the past four years. The schools were approached to understand their performance in mathematics and found the schools to have performed below 50% for four consecutive years. Further, the researcher was able to develop a close relationship with the sample, thus trustworthiness of the data was enhanced.

A diagnostic test was administered on learners in the six schools on the same day at the same time to avoid the contamination of the results. The researcher made use of the teachers in the participating schools to administer the test in the five schools while he was administering in the sixth school. Focus group interviews with 30 learners in total (6 learners in each school) were conducted by the researcher in the schools. The criteria used to select learners who participated in the focus group interviews was purposive as were the ones who gave incomplete, incorrect and blank responses. Eight test items were used to test the learners with questions involving difference of two squares and a common factor method equation each, one equation with all terms positive, one with negative and positive, one with all negative signs in-between, one with negative coefficient of x^2 and one with coefficient of $x^2 > 1$, and lastly two word problems. The questions administered were sampled from previous Grade 10 examination papers to enhance the validity and reliability of the test. The purpose was to investigate if Grade 11 learners could solve Grade 10 quadratic equations.

The questions were also sent to two academics in the Department of Mathematics Education at the University of South Africa who have more than ten years' experience in this field.

Changes were effected as suggested by the experts to suit the purpose of the study which was to investigate the learners' performance and difficulties when solving symbolic quadratic equations and word problems involving quadratic equations.

The study followed qualitative approach in which data collected were described (Johnson & Christensen, 2012). The categories were formed to analyse the eight test items as follows: correct response meaning learners got full marks, incorrect response where no learner got any mark, incomplete response where some learners obtained half or less of the total marks and blank space.

The results are analysed across schools and within schools as per test item. Each test item is described and triangulated with the data collected from the focus group interviews with learners.

Participants were interviewed according to Polya's problem solving skills. The following questions were used for the focus group interviews:

i. Do you understand the problem? (understanding or identifying a problem)
ii. Can you explain how you understand and solve the problem? (analyse or devising a plan)
iii. Can you show me the steps you used to solve the problem? (carrying out a plan or executing a plan)
iv. What is your answer and how can you justify your answer? (looking back or reflection)

FINDINGS

The findings are presented according to the following categories as earlier noted: correct response, incorrect response, incomplete response and blank for each test item. The details of each test item based on the four categories are presented in Table 1. The findings are presented in percentages and absolute numbers.

Table 1: Distribution of learners' (N=214) results with percentages and absolute numbers

Test Items	Correct	Incomplete	Incorrect	Blank
Item 1: $x^2 - 3x = 0$	182 (85.1%)	10 (4.7%)	14 (6.5%)	8 (3.7%)
Item 2: $x^2 - 9 = 0$	179 (83.6%)	15 (7.0%)	10 (4.7%)	10 (4.7%)
Item 3: $x^2 + 7x + 6 = 0$	120 (56.1%)	25 (11.7%)	19 (8.9%)	50 (23.4 %)
Item 4: $x^2 - 10x + 25 = 0$	86 (40.2%)	21 (9.8%)	25 (11.7%)	82 (38.3 %)
Item 5: $x^2 + 5x - 6 = 0$	61 (28.5%)	34 (15.9%)	41 (19.2%)	78 (36.5 %)
Item 6: $x^2 - 6x - 7 = 0$	53 (24.8%)	38 (17.8%)	42 (19.6%)	81 (37.9)
Item 7: Peter's father is six times as old as Peter. The product of their ages is 150. What are their respective ages?	18 (8.4%)	43 (20.1%)	26 (12.2%)	127 (59.4 %)
Item 8: The product of two consecutive integers is 72. Find the integers.	13 (6.1%)	29(13.6%)	20 (9.4%)	152 (71.0 %)

Identifying or understanding a problem

The most challenging items to be understood by learners were items 7 and 8 as only a few learners were able to get these test items correct (Polya, 1945. In item 7 only 18 learners (8.4%) were able to respond to the test item and in item 8 only 13 learners (6.1%) gave the correct responses out of the 214 learners. Most learners did not demonstrate the knowledge and skills required to solve word problems involving quadratic equations. Learners who participated in the focus group interviews gave the reason that they did not know how to approach the word problems which they found difficult. For example:

Researcher: Can you identify what the problem is?

Learners: We don't even know how to approach these problems; they are difficult for us

The results revealed that learners performed poorly in item 7 and 8 as a high percentage was found in the blank response. The blank response showed that learners did not understand the questions and hence could not analyse them in order to get the correct methods to solve those test items.

Items 4 to 6 were also challenging to the learners; most were also unable to understand these problems. The test items results showed that in item 4 learners performed better in the correct response with percentage 86 (40.2%) as compared to item 5 and 6 with the percentages 61 (28.5%) and 53 (24.8%) respectively. These test items were symbolic quadratic equations which showed that learners could not solve those equations, which involved positive and negative terms, negative and positive terms as well as negative and negative terms. The results revealed a high number of learners in categories of incomplete, incorrect and blank responses: 21 (9.8%), 25 (11.7) and 82 (38.3%); 34 (15.9%), 41 (19.2%) and 78 (36.5%); and 38 (17.8%), 42 (19.6%) and 81 (37.9%) respectively. Learners in the focus group interviews gave the reason that they encountered challenges in allocating signs in brackets especially when those signs are mixed. For example:

Researcher: Can you explain how you understand and solve the problem?

Learners: We have a challenge when dealing with positive and negative signs in the equations. We don't understand how to explain those signs in order to place them in brackets.

The results showed the understanding of the items 1 to 3 in the correct responses category as 182 (85.1%), 179 (83.6%) and 120 (56.1%). Most learners were able to understand how to use the common factor rule, the difference of two squares and solving quadratic equation with all terms positive. Learners who gave incorrect and incomplete responses factorised item 1 instead of the common factor rule, correctly found the factors of the equation but found it difficult to allocate signs in which most of them had negative signs to all the factors. Most of them gave incorrect factors of quadratic equations.

Devising a plan or analysing a problem

The results revealed that most learners could not analyse the word problems of the test items 7 and 8. The learners who gave the incorrect and incomplete responses as well as those who left blank spaces were interviewed in the focus groups. For example:

Researcher: Can you change this word problem into mathematical symbols?

Learners: It is difficult to change it into mathematical form. We understand the statement but don't know how to change it into mathematics equation.

Most learners' responses in the incorrect and incomplete category did not make sense at all as were not even related to the word problem given to them.

The results of test items 4 to 6 revealed that learners could not analyse those items before responding to them. Most just gave factors without analysing those test items. This problem was confirmed in the focus group interviews where learners indicated that they just gave factors of the symbolic quadratic equations without analysing what those equations expected them to do. For example:

Researcher: Can you explain how did you analyse this problem?

Learners: Sir, we just gave factors of the equation without analysing it as quadratic equations with negative and positive signs are problematic to us.

These learners seemed confused when solving quadratic equations with negative and positive signs, positive and negative signs as well as the ones with all negative signs.

The results for the test items 1 to 3 revealed that most learners were able to analyse those items as they realised that the first one had two terms with unknown variable x and required them to use the common factor rule. Test item 2 comprised of two terms with the first one with variable x and the second one without any unknown variable. Most learners realised that the equation was the difference of two squares, which required them to give those square factors where the last factor in the bracket contained a negative sign and the second last factor in the other bracket contained a positive factor. Test item 3 revealed that most learners used the trial and error method as they just explained the factors of the last term in solving the problem. For example:

Researcher: How did you analyse test item 3?

Learners: We found the factors of x2 and factors of 6 as 3 and 2.

The factors of 6 should have been 6 and 1 as the middle term was 7x. This illustrates their use of trial and error method to solve the test item.

Executing or carrying out the plan

The results for items 7 and 8 revealed that only 18 learners (8.4%) and 13 learners (6.1%) gave correct responses. The blank spaces on learners' responses to these two test items were found to be 127 (59.4%) and 152 (71.0%), which showed that the two word problems were not understood; hence learners could not analyse them in order to solve them. The incorrect and incomplete responses were 43 (20.1%), 26 (12.2%), 29 (13.6%) and 20 (9.4%) in which most learners did not make sense in solving the two word problems. Learners in the focus group interviews gave reason that the two word problems did not make sense to them.

Researcher: Can you show me the steps used in solving the word problem and again why others decided to leave blank spaces?

Learners: We are completely lost; we don't know what to do. These word problems are completely confusing us.

The focus group interviews revealed that learners did not have the knowledge and skills to solve the word problems. It seemed that learners were confused and did not know how to approach this kind of problems. The issue of language of teaching and learning could have impacted in learners' performance of item 7 and 8 as most of them could not understand the word problems.

The test items 4 to 6 results showed that some learners gave correct responses: 86 (40.2%), 61 (28.5) and 53 (24.8%) respectively. These learners solved the test items by giving correct factors with correct signs of the terms in brackets of symbolic quadratic equations for items 4 to 6. The correct solutions or roots of the equations were also found to be correct. The incorrect and incomplete responses were 21 (9.8%), 25 (11.7%), 34 (15.9%), 41 (19.2%), 38 (17.8%) and 42 (19.6%). These results revealed learners' misunderstanding of the symbolic quadratic equations with negative and positive signs, positive and negative signs as well as terms with negative signs as some learners mixed the signs of the last terms in brackets and gave the incorrect factors of the last terms. Some learners decided to leave blank spaces: 82 (38.3%) for test item 4; 78 (36.5%) for test item 5 and 81 (37.9%) for test item 6, which showed that learners were at a loss to solve the three test items. The focus group interviews also showed that learners were confused in solving quadratic equations with mixed signs.

Researcher: How did you solve this problem?

Learners: These equations with different signs are really a problem. The problem is that we cannot give the meaning of those signs in the equations. They confuse us. We don't know what to do in placing the signs in brackets.

Learners seemed to have difficulties in solving quadratic equations with mixed signs and they also confused these operations. Thus, they were unable to solve this type of equations.

The results for test items 1 to 3 indicated that the learners performed well in giving correct responses to these items when compared to test items 4 to 8. The test items results showed 182 (85.1%) learners who correctly executed the equations for test item 1, 179 (83.6%) for test item 2 and 120 (56.1%) for test item 3. Learners showed all the steps in solving the three test items by using the common factor rule in the test item 1, finding the factors of the difference of two squares and finding the factors of the equations which involved all positive terms. Few learners were in the incomplete and incorrect categories in solving test items 1 to 3.

The results revealed that the percentages were 10 (4.7%) and 14 (6.5%) for test item 1, 15 (7.0%) and 10 (4.75) for test item 2, and 25 (11.7%) and 19 (8.9%) for test item 3. The test item 1 revealed learners' misunderstanding of using the factorisation method instead of the common factor rule. In test item 2 learners used the last factors in the first and second brackets as negatives. In test item 3 learners used the last factors of the equation as 3 and 2, which was incorrect to factorise without considering the middle term of the equation. The focus group interviews affirmed that these learners did not display an understanding of the different types of symbolic quadratic equations.

Researcher: Which steps did you follow to solve the problem?

Learners: The problem is when we see x2 we think of factors. We also don't check whether our factors are correct or not.

Reflection or looking back of the action taken

The test items 7 and 8 results revealed learners could not revise what they had done in solving the two test items. For example, learners who correctly identified or understood the problems, analysed or broke them into smaller chunks and successfully executed these test items just left their answers as x=5 or x=-5 for test item 7 and x=8 or x=-9 without concluding that there are no negative ages. Moreover, they did not give the consecutive integers as the question required. The learners who got these test items correct participated in the focus interviews to reflect on their answers.

Researcher: What are the consecutive integers for test item 8?

Learners: The consecutive integers are 8 and -9 as we have found the two values of x.

The learners understood how to approach the problem but could not successfully understand what the answer should look like. Almost all the learners solved the value(s) of the unknown and left the answers after solving the unknown variables. In the test item 7 only 10 (4.7%) of the learners indicated that Peter's age is 5 and his father's age is 25, and gave the consecutive integers as 8 and 9 or as -8 and -9.

The same thing happened in test item 1 to 6 as all the learners who gave the correct responses only gave the values of x without justifying whether those values are the roots of the equations or not. Those learners understood, analysed and correctly solved the test items. Focus group interviews were conducted with the learners who gave the correct responses to those test items in order to understand their justification for their answers. In this discussion the factors of x^2+5x-6=0 as (x-6) (x+1) =0 were found and the values of x as -6 and 1 were given as ultimate solutions.

Researcher: What is your answer and how can you justify it?

Learners: We have found the factors of x^2+5x-6=0 as (x-6) (x+1) =0 and found the values of x as -6 and 1.

It appears that these learners only knew how to solve for x but could not justify why the values are the roots of the equations or not.

CONCLUSION

The quadratic equation is a critical section for secondary school learners to master as required in mathematics in institutions of higher learning. The study revealed that most Grade 11 learners cannot solve Grade 10 quadratic equations including negative and positive signs, positive and negative signs, and negative and negative signs as well as the simple word problems. It was also found that learners could not identify or understand different types of quadratic equations, analyse, execute and reflect on their solutions. Grade 11 learners should be at a position to explain the types of quadratic equations and know what is expected from them. The language of teaching and learning in the schools that participated in this study may be researched on how learners deal with word problems involving quadratic equations as could have contributed to the difficulties English second language learners had in solving quadratic equations and word problems. The rationale for including the issue of language in the recommendation was that most of the learners were found struggling to understand and analyse word problems for item 7 and 8.

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MATHEMATICS LEARNER ERROR ANALYSIS PROTOCOL

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This paper proposes the types of errors that learners may have when solving secondary school mathematics tasks. The errors were deduced from an analysis of learners' scripts as they solved mathematics tasks in an NCS Grade 12 mathematics examination paper. Five hundred (n=500), 2010 Grade 12 examination scripts, from ten examination centres in Gauteng were the source of the data. It was found that learners made varied errors which in most cases were highly dependent on each other, but also noted was that even though they may not answer tasks correctly learners possessed viable mathematics knowledge and skills which were not aptly applied to correctly answer the questions. The study found 22 types of errors. It is hoped that these can help to build teacher knowledge for teaching mathematics through teaching action based on eliciting what learners think about mathematical concepts and procedures.

Keywords: Error analysis, learner, mathematics, protocol

INTRODUCTION

This paper proposes the types of errors that learners may have when solving mathematics tasks basing from learners' responses to differentiation and its applications examination items. Data was obtained from learners' scripts as they solved differential calculus tasks in an NCS Grade 12 mathematics examination paper. Frequent studies show that globally, students often strive for procedural knowledge of mathematics often at the expense of conceptual understanding of mathematics (Porter & Masingila, 2000). South Africa has an inefficient education system in a modern society (Rusznyak, 2008), and so it is necessary to counter that as also, achievement in mathematics remains a challenge across different groups of learners (Reddy, 2006). One problem causing underachievement in mathematics is related to language. This has led some universities to teach some mathematics courses in isiZulu (Van Laren & Goba, 2013). Language is an important cause of learners' errors due to them failing to decode and understand, what the mathematics tasks require of them.

Research question

What errors do high school learners show in mathematics and how can they be explained?

CONSTRUCTIVISM THEORETICAL FRAMEWORK

The errors and misconceptions learners have are best explained through the lens of constructivism.

This is because learners are not expressly taught the errors they have but that they are an expression of their constructive processes as they make sense of new mathematics concepts through the adaptive processes of assimilation and accommodation via their current concepts (Smith, diSessa & Roschelle, 1993). That is why it is important to be empathetic to students' points of view as they learn mathematics so that teachers can profitably help them.

METHODOLOGY AND DATA ANALYSIS

Data was collected through the script analysis of for 500 students on their grade 12 Matric examination scripts for 2010. Data analysis was particularly directed to the calculus questions. The fact that calculus subsumes all most high school mathematics previously studied meant that errors in those concepts and procedures such as in algebra, functions and geometry would inevitably be analysed as well. Thematic analysis through 'grounded theory' and 'constant comparison' (Glaser & Strauss, 1967) within the same script and across scripts was done to explore what types of errors and misconceptions the students made as they strove to answer the questions to obtain a passing mark.

Content analysis was done through scanning learners" responses to mathematics examination tasks. It was quite important to understand what learners wrote to determine how they were thinking in their reactions. I had to infer the strategies they were using to construct their answers.

Analysis of the first item. This item read:

Determine, using the rules of differentiation:

$\frac{dy}{dx}$ if $y = \frac{\sqrt{x}}{2} - \frac{1}{6x^2}$

Show ALL calculations

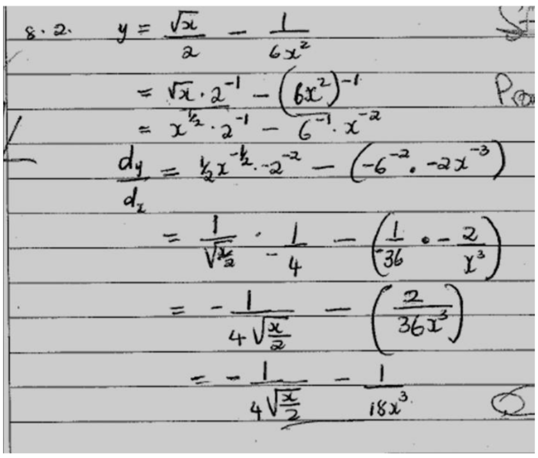

Figure 1: A learner's response to differentiation item

In the first 3 lines the learner shows a high degree of algebra competency, making not even one mistake as he removes denominators in the expression to prepare it for ‘differentiation by rule’.

Procedural extrapolation error:

The learner knows the ‘differentiation by rule’ but is unaware of its limits. If f(x) = x^r where r is a constant real number then its derivative is given by f **'(x) = r x^{r-1}**

Or if y = x^r , then $\frac{dy}{dx}$ = **r x^{r-1}**

This learner goes on to infer that $\frac{d}{d2}(x^{1/2}2^{-1}) = \frac{d}{d2}(x^{1/2}).d/dx(2^{-1})$ = ½ -$2x^{-1/2}$.-2^{-2} by which he regards 2 as a variable. The learner also takes 6^{-2} exactly the same way showing that this error was systematic, the same with the product.

Structural error:

The products show misunderstanding the essential features of a phenomenon. The learner is of the mistaken opinion that ‘the derivative of composition of function; that is product of functions is the same as the product of their derivatives’. Implicitly the learner writes

$\frac{d}{d2}(2^{-1})$ = -2^{-2} this tough a procedure but is essentially a structural error in that the learner equates a variable x to a constant x.

Executive error:

The learner makes a further error by implying that

$$\frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{\sqrt{x/2}}$$

The error can also be seen as an **arbitrary error** where a learner changes the question to suit him/her. I analyse yet another item;

Determine the equation of the tangent to *g* at P (– 3; 11), in the form *y* = … where

$$g(x) = -2x^3 - 3x^2 + 12x + 20 = -(2x-5)(x+2)^2$$

This learner demonstrates good understanding of necessary concepts to answer the question, but that knowledge is not enough for her to completely answer the question. The appropriate knowledge is that; she knows that a tangent is a straight line that must have an equation and most appropriately a gradient. She is also aware that to find a gradient has something to do with the x and y values at the point. The learner also knows how to obtain an equation of a line when one has or can obtain its gradient. So far so good. Notwithstanding, she shows:

A structural error.

In that she believes that the co-ordinates of a single point on the curve, (x_1, y_1) are enough to obtain the gradient of the tangent to the curve at that point.

An executive error.

In writing the gradient as $\frac{11}{-3}$ she shows the error which could have been avoided if the learner conjured up differentiating g(x) and substituting the P value of x in g'(x) in order to obtain the gradient.

Another structural error and application error.

She now believes that the gradient of the tangent and its normal has a product of -1. This reasoning is okay but it is not required here because what is being required is the gradient of the tangent, not the normal. Thus we can say that the learner showed an application error. She has the requisite knowledge which was now misapplied.

Terminology error and decoding error.

I suspect that it may also be a terminology error in that the learner may not have comprehended tangent terminology. In actual fact, the learner gets the equation of the line OP. There is also error of notation because the notation is inappropriate in this case. This could also be a decoding error in that the learner misinterprets what she is supposed to do because of misunderstanding language.

To come up with error types discussed in the following the author analysed other examination items in a similar manner. The reader must always bear in mind that seldom does a learners' mathematical errors occur in seclusion as errors are often highly linked and dependent as shown above. Also I have shown that even if learners may have errors that affect correct solutions they also have very viable and correct concepts which have a bearing on answering the tasks but may have limitations that affect a learner in answering the questions in a way that examiners expect.

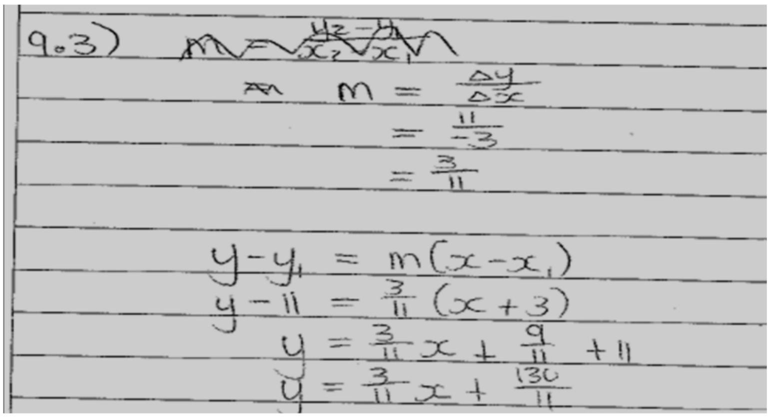

Figure 2: A learners' response to application of differentiation task

(Adapted from: Makonye (2011), PhD Thesis on Learner Errors in Introductory Differentiation Tasks)

EXPLANATION OF ERROR TYPES

The errors that learners showed were neither mutually exclusive nor independent. Instead they were closely interwoven and linked. Thus error analysis became a dynamic process and not a static one. I believe that when other people interpreted the errors in these scripts they could well come with different classifications. I triangulated my error categories with literature to increase validity and reliability. Some of the errors were also discussed with interested people at the Southern African Association of Research in Mathematics, Science and Technology Education (SAARMSTE) conferences in 2010 and 2011 at the Edgewood Campus of the University of KwaZulu-Natal, Durban and the University of North-West, Mafikeng as well as staff research seminars at the School of Education, University of the Witwatersrand. I also discussed the error types at a doctoral seminar at the University of Johannesburg in 2011. Comments and suggestions from all the members of community of practice were incorporated in this protocol.

The types errors and their description are presented below (Makonye, 2011):

1. Systematic error: This type of error occurs due to a strong underlying thinking or schema in a learner's mind that the learner uses to interpret phenomena in that class: to sort or integrate it. Systematic errors are intentional, and therefore are consistent. They are caused by an often invisible misconception that regularly generates the visible error across space and time that others may see. Often, the learner is not aware that the thinking is faulty. Indeed learners defend their thinking against any teaching that might interfere with it. This is because the learners constructed their misconceptions and those misconceptions become part of their psychological selves. Any attempt to temper with the conceptions from the outside world including by a teacher are analogous to a physical threat. That is why it is futile to try to *replace* a learner's misconception. Learners need sympathetic external more knowledgeable others (MKO) that do not criticize them, that show empathy and sympathetically discuss with them the limits of their thinking. MKO need to be diplomatic and encourage learners to gradually revise their conceptions in order to effect the conceptual change required. Constructivists argue that it is unproductive and futile to tell a learner that *your working is wrong*, to confront them with their misconceptions, and *show* them that this is the correct working. Why? Because you will be attacking them, attacking their thoughts, their schemas. That is the reason why systematic errors persist and are immune to instruction targeted to misplace them. I have argued that those misconceptions are part of the learners' semiotic tools. Further there are times when those misconceptions are productive, such as when they produce correct answers. Outsiders would regard systematic errors as due to competency.

2. Unsystematic error: This type of error is not related to incompetency. They could be due to being distracted by some tiredness or anxiety. These errors are non-repeating and non-recurring and learners can correct them by themselves if they check their work. This is a case of 'inside critics failing to fire'. It is unfortunate that most learners do not have the mathematics practice of checking the reasonableness of their answers when they have solved a mathematics problem or completed a calculation, such as solving an equation and verifying their answers maybe by substituting their answers in an original equation. Unsystematic errors may be related to metacognition I will discuss later.

3. Executive error: This error is due to failure to use a mathematical procedure, such as failure to use the long division procedure or multiplying binomial terms. The error is due to lack of proficiency or remembering how a procedure works (Kilpatrick, Swafford, & Findell, 2001). Executive errors relate to failure to use mathematical algorithms. It is beneficial for students to have conceptual understanding of the basis of the algorithm so they can use their own methods to work out a calculation if they have forgotten how to use a prescribed algorithm. Further, the conceptual understanding of the basis of an algorithm helps learners in checking when they use alternative methods to gauge the reasonableness of their answers. Executive errors thus are closely related to structural or conceptual errors discussed next.

4. Structural error (Donaldson, 1963): This error is due to insufficient conceptual understanding of a mathematical process or object. Understanding of mathematical concepts relate to how different topics of mathematics and processes link to each other. Indeed one of the ways for learners to understand mathematical concepts is through use of multiple representations of concepts in verbal, visual, graphical and abstract form. That way a learner realises the common characteristics belying a certain concept and which representation is most suitable is presenting the concept in different situations. A learner who does not understand how mathematical concepts are hierarchical and closely related misses the big ideas that drives the mathematics power used in interpreting and solving diverse problems in our life and at work. While conceptual understanding of mathematics is important, the mathematical processes engulfed by its terminology and procedures makes it not only a potential tool in problem solving but an active tool in solving problems.

5. Application errors: Application errors happen if a learner has grasp of the concepts and algorithms needed to solve a mathematics problem. In this case the learners fail to apply their knowledge to bear upon the problem and generate the required solution to a mathematical problem. Thus a learner may be able to solve equations, but fail to formulate an equation to model a given problem so that they can solve the equation and relate the answers to the problem and so solve it. Thus application errors can be related to decoding and encoding errors I discuss below. We may relate this error to as a modelling error.

6. Meta-cognition error: This error is due to a learner failure to reflect on the sensibility of what they are doing or have done. It is a failure to monitor their own thinking on a mathematical solution. Evidence of this this error comes with unreasonable results that one cannot justify; for example getting a probability of 2, 5. Once a learner realizes the unreasonableness of their error they need to have a relook at their working to identify what was wrong and correct it. Seen in that light, metacognition errors may also be unsystematic errors.

7. Hybridization errors: These errors occur if a learner thinks that some mathematical concepts are related when in fact they are not. They are often due to muddled conceptual understanding. In the same way learners fail to see that despite different representations, the concepts are related. This often occurs in mathematical notation. For example, a learner fails to notice that *f(x)* and *S(r)* are related as they are all functions *which* can be treated in a similar way although the variable x and r are different.

8. Logically invalid inference errors: This occurs when a learner makes a conclusion that is based on faulty reasoning.

9. Interpretation error or decoding errors: In this instance a learner makes an incorrect interpretation of a task because of mis-understanding the language or mathematical terminology. The learner then solves a task which is different to the one asked. In mathematics this is seldom condoned. Indeed in doing mathematics the very first task is to decode, to understand what one is asked of.

10. Arbitrary error: This error occurs if a learner makes any change to a question; such as changing a times sign to a plus sign, or making the numerator a denominator thus converting a question to a form acceptable to him or her. The learner might want the question to be familiar to the one that they know. This change of a question is often seen as declining the mathematical demand of the task and is again often not tolerated in mathematics.

11. Careless errors: These are errors not due to competency but to performance. They are part of unsystematic errors. One has to take careless errors seriously because in a long problem, wrong results from an earlier step due to a careless error can entangle the solution of the problem and effect wrong solutions: compare this to a competent pilot who ‘sleeps’ while landing.

12. Random error: A random error is also unsystematic and a learner does not repeat it. If the learner repeats it then we will be failing to classify it, as it will be a systematic error.

13. Procedural extrapolation error (Hirst, 2003): This results in over-generalising previously learnt procedures in new mathematical situations where that would be wrong. For example, when learners using their arithmetic knowledge on cancellation of fractions fail to identify common factors when simplifying algebraic fractions.

14. Pseudo-linearity errors (Hirst, 2003): This occurs when learners assume linearity in mathematical relationships; such as

$$f(x+h) = f(x) + f(h) \text{ or}$$

$$log(x+y) = logx + logy.$$

These are often due failing to link mathematical concepts and their symbolism, taking mathematical terms as mathematical objects.

15. Equation balancing errors (Hirst, 2003): These are due to previously learnt mathematical processes when solving equations the teacher says 'whatever you do to the left of the equal sign, you do to the right, so as to balance to equation'. Thus learners would assume that adding the same quantity to a numerator and denominator will not affect the value of an algebraic fraction:

$$if\ \frac{a}{b} = \frac{c}{d} \quad \text{then} \quad \frac{a+2}{b+2} = \frac{c+2}{d+2}$$

16. Decoding error: These occur when learners convert word problems in common language to mathematical symbolism. Students fail to do that so answer an unintended question. Sometimes students do not understand mathematical terminology and have decoding errors. They do not understand what is mediated by the signs (Vygotsky, 1986).

For example: *There are three times as many girls as there are boys* is decoded to

$3g = b$; where g stands for the number of girls and b the number of boys. This is wrong because there ought to be more girls than boys.

17. Encoding error: This happens when learners fail to link their mathematical results to the problem given. They cannot relate what they have solved to an original problem.

18. Syntax error: These are related to mathematical terminology. This is an important error because one cannot understand mathematics if they do not understand mathematical symbolism.

$f'(x)$ - first derivative, $S(r)$ – function of r in this case function to find surface area very comparable to f(x). Some students on seeing r compared this to a geometric progression, common ratio r.

19. Fragmentation error: Here learners' ideas of mathematical concepts are disjoint and not connected. Learners cannot see how concepts are related to each other.

20. Misread error: Normally not intended but occur when a learner is distracted. They are part of unsystematic errors. Sometimes these errors are partially accepted in mathematics, where the final answer is the one punished. The method used if acceptable, is often considered if a consistently wrong value due to a misread is used.

21. Theory like error: This error is the opposite of fragmented errors and is major. Students show a mis-understanding on how things work. The treatment of mathematics as a boring mechanical subject is an example of a theory like error.

22. Delayed detachment error: These errors happen when a learner fail to distinguish between concepts occurring at the same time.

DISCUSSION AND CONCLUSION

The two examination items analysed have shown that learners can make a multitude of errors when responding to a single mathematics tasks. These errors are highly inter-related and are not independent of each other. Constructivists (Confrey, 1987; Davis, 1984; Smith et al., 1993) have argued that learners are not directly taught the errors and misconceptions they have. In actual fact teachers implicitly teach them so that they may not have those errors. By teaching learners the most direct route to solving mathematics questions in the behaviorist sense, teachers may be failing to have the whole picture of the phenomenon. It has been observed that learners build their own correct and incorrect mathematics concepts. Misconceptions are the by-product of the mathematics learning process and teachers must welcome errors and misconceptions always so as to build teacher knowledge for teaching mathematics. Teachers must respect learner errors and misconceptions and contrive to make them resources for mathematics teaching (Makonye & Khanyile, 2015). Thus learner errors and misconceptions are learners' step transitions from no knowledge, to partial knowledge and then finally, full knowledge of mathematical concepts and processes. It is hoped that this protocol may help mathematics education practitioners to 'elicit and use evidence of student thinking' (NCTM, 2015; Silver, 2015) as an action principle in improving the teaching of mathematics to all learners so that it is understandable to them.

The recommendation of this research is that teachers must always be on the lookout of learners' errors in their writings or conversations. Often learners have very good reasons for them as they are sensible to them. Teachers must engage the learners on their errors to help them build viable mathematical concepts from these. They must never ridicule learners for their errors as they would lose a golden opportunity to understand learners, and also they could dampen those leaners participation and interest in mathematics.

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MATHEMATICS TEACHER NOTICING: WHAT ROLE CAN TEACHER QUESTIONING PLAY IN SCAFFOLDING STUDENT LEARNING?

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This study uses video-recorded mathematics lessons to investigate how teacher questioning helps teachers to notice the salient features of students' complex mathematical thinking during lessons. Two mathematics teachers participated in the study. This study is grounded in the sociolinguistic framework, and the sociological construct of framing. Data for the study is constituted from the analyses of the transcriptions of the video-recorded mathematics lessons. Teacher questioning, in the form of Socratic questions, and mathematics teacher noticing during classroom lesson interactions are analysed interpretively. Findings show, using specific examples, how Socratic questions can enhance the construct of mathematics teacher noticing during lessons. In addition, this study provides some practical examples of what could happen to classroom mathematics discourses when both mathematics teacher noticing and teacher questioning are used to scaffold student learning activities.

Keywords: Learning, mathematics, questioning, scaffolding, student, teacher, teaching

INTRODUCTION

Mathematics Teacher Noticing

Recent lines of research in mathematics education have shown an increasing interest in exploring the construct of mathematics teacher noticing (MTN) (Jacobs, Lamb, & Philipp, 2010; Russ & Luna, 2013; Sherin, Jacobs, & Philipp, 2011). The construct of MTN allows both teachers and researchers to examine the "who, what, when, where, why, and how of teacher attention during instruction by looking at a whole range of activities teachers do and do not notice in the classroom" (Russ & Luna, 2013, p. 290). Before I discuss the epistemology of MTN, let me start by recognising that the term noticing is frequently used in everyday real-life situations when referring to observations that people make. In the context of this study, I use the terms MTN or simply noticing to refer to the processes that teachers manage in the classrooms which are loaded with "blooming, buzzing confusion of sensory data" (Sherin et al., 2011, p. 5).

Various researchers conceptualise MTN in a variety of ways. For instance, some researchers perceive MTN as paying attention to the aspects of the mathematics learning activities; however, for others, there is a general agreement that it is an active process in which both teachers and students participate (Huang & Li, 2012; Rosaen, Lundeberg, Cooper, Fritzen, & Terpstra, 2008; Walkoe, 2015).

Whilst the researchers conceptualise MTN in myriad ways, researchers also agree that the common thread of all these understandings about noticing is that it is about how individuals internalise complex situations (Jacobs et al., 2010). Central to some of the research on MTN are that teachers analyse classroom interactions through different lenses which are usually informed by their own experiences, educational philosophies, cultural backgrounds, and their work experience contexts (Miller & Zhou, 2007). For the purposes of this study I adopt the lens of MTN which was coined by Van Es and Sherin (2002) which is commonly used and whose three tenets are:

> (a) Identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about [the mathematics] classroom events (p. 573).

The above three tenets are also referred to as the *Learning to Notice Framework*. Insofar as, firstly, *identifying what is important or noteworthy about a classroom situation* of the framework is concerned: often, during teaching, teachers notice, and attend to, specific episodes of students' interactions about subject content, key mathematics concepts, representations, as they arise. Frederiksen (1992) coined the phrase "call-out" to refer to the important aspect of teaching teachers, identified when they were watching professional development videos with teachers. In this study I refer to the call-out as "the ability of teachers to hone in on what is important in a very complex situation" (Van Es & Sherin, 2002, p. 573). For example, during a lesson, a teacher, through the use of teacher questioning, might notice that students' prior knowledge on a difficult concept is weak – teachers could then make a decision on whether to revisit or not the concept, in order to advance instruction on the difficult concept. Secondly, *making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent*: this encompasses the ability of teachers to notice how isolated events of a lesson are connected to the broader objective of the lesson. For example, when teaching the concepts of inverse functions – $f(x) = \frac{1}{x+2}$, isolated events could be finding the domain and range of the function, and the broader objective could be sketching the graph of the functions and naming its characteristics.

Last, *using what one knows about the context to reason about [the mathematics] classroom events*: this relates to how teachers use their specific context to notice how the classroom discourses are unfolding.

These specific contexts include, but are not limited to, subject matter knowledge, pedagogical content knowledge of teachers, and knowledge of their students and how they understand the subject matter, as well as the school location. For example, using the South African educational context, teachers may realise that some of the teaching contexts may not be palatable to certain designated schools, due to their socioeconomic status. The point here is that MTN is tied firmly to the type of context in which a teacher finds himself or herself in. I recognise that, in principle, teaching is a complex process where many events take place at the same time; these include, but are not limited to: students internalising of key mathematics concepts, interactions between students and teachers, students to students interactions, students' questions and their responses, teacher questioning, among many others. Whilst the latter are all taking place, teachers are mandated "to make thoughtful in-the-moment decisions, building on students' ideas and the ongoing lesson" (Walkoe, 2015, p.2). Reforms in education stress the importance of teachers being able to adopt flexible strategies to teaching when noticing of students' thinking processes are highlighted, made visible and can be used to advance instruction (Brown, Stein, & Forman, 1996).

The question we should be focusing on is – how do we develop the ability to notice among teachers during lessons? Strategies to develop MTN are not new (Sherin, 2004; Sherin et al., 2011; Star, Lynch, & Perova, 2011; Van Es, 2011). One such strategy is the use of video analysis which was used by Van Es and Sherin (2002) on elementary school mathematics teachers. The teachers were asked to view a video of a mathematics lesson with the aim of making comments on what they had noticed. These activities were part of the video-based professional development of teachers. The video analysis strategy (Star & Strickland, 2008; Van Es & Sherin, 2002) potentially allowed teachers to remove themselves from the in-moment decision required in a classroom situation, in order to step back to examine the actions of teaching closely.

Given the nature of the complexity of MTN, particularly with novice teachers and teachers in pre-service programmes, teachers find it difficult to notice some of the important mathematics details in a recorded lesson (Choy, 2014; Star et al., 2011; Vondrova & Zalska, 2013). Mason (2002, p. 61) concurs and posits "noticing is an act of attention, and as such is not something you can decide to do all of a sudden. It has to happen to you, through the exercise of internal or external impulse or trigger". Here Mason (2002) gives emphasis to advance preparation and prior experience as some of the important requirements promoting noticing in a mathematics lesson. Highly experienced teachers are more likely to adopt noticing as part of their classroom practice. By advance preparation I refer to prior training which teachers could receive on noticing through professional development programmes (Schoenfeld, 2011). Prior training for teachers to improve on their noticing is not as simple as it appears.

Researchers would have to answer the questions: What should this training about noticing constitute? Should noticing training involve the notion of diagnostic teaching – which I presume is the closest to the construct of noticing? Teachers engaged in diagnostic teaching engage their students, through listening to students, noticing important things such as key conceptual points and misconceptions, giving substantial feedback to challenges, provoking and promoting cognitive conflicts, and actively involving students in mathematics classroom discourses (Bell, 1993). Apart from diagnostic purposes, teachers can actually be trained to improve their noticing and how they reason about the classroom events. The use of videos to professionally develop teachers on the construct of MTN seems to be quite promising (Sherin, 2004; Sherin & Han, 2004). The use of videos allows teachers to study and observe some of the salient classroom features that they would not have been able to see in real time while they are teaching. Hence I claim that video analyses of other teachers' lessons or their own lessons support the learning to notice. This type of learning to notice can be achieved because: firstly, teachers value the idea of analysing their own recorded lessons (Roberts & Wilson, 1998). Secondly, the collective analysis of the videos allows teachers the agency for engaging in in-depth analysis of key features of noticing that are related to teaching and learning. Thirdly, collective analyses of videos allow multiple perspectives on teacher noticing to be explored (Lambert & Ball, 1998). In this study I propose teacher questioning as an important feature that can advance MTN during classroom discourses.

Teacher questioning

Earlier on I alluded to the importance of a context where MTN takes place – in this study, the mathematics classroom is the context where the teacher is confronted with the blooming, buzzing confusion of sensory data emanating from the students' interactions. I refer to these students' interactions as noticed-episodes (NE). The teacher cannot focus on all the NE, hence, s/he selects what NE to focus on with the aim of advancing instruction. Advancing instruction takes different forms, depending on the challenges the students are facing in real time during the lesson. In addition, this action-based response to the teacher is also dependent on the level of professional experience of teachers. The point here is that experienced teachers are more likely to follow up more nuanced NE. For example, a teacher can initiate further probing of the NE through asking questions.

MTN is becoming an area of research interest in mathematics education. The growing body of research on MTN focuses on the teachers' attention to student mathematical thinking, with the aim of creating opportunities for student learning. This study seeks to investigate the construct of MTN through the lens of teacher questioning. In this paper, I explore the link between MTN and teacher questioning. Before I discuss the link between MTN and teacher questioning, I explain – what constitutes teacher questioning?

One of the critical roles of teachers is to guide the development of mathematical discourses during lessons and to ensure that students actively participate and engage in it (Hunter, 2009). Central to the development of mathematics discourses in the lesson is the way teachers orchestrate the discourses through teacher questioning. Teacher questioning can enhance students' mathematical thinking and provide the teachers with the much-needed feedback about students' understanding about mathematical concepts (Roth, 1996; Settlage, 1995; Van Zee & Minstrell, 1997). Although the two constructs are different, they are not mutually exclusive. In this study I approach the construct of MTN using the lens of teacher questioning from two positions; firstly, that MTN is about what key points of learning can teachers conceptually observe, and secondly, that MTN is what is made visible to the teachers about students' conceptual understandings during lessons when the teachers use teacher questioning. To be clear, the aim of this study is to understand how teacher questioning can help teachers to improve on their MTN when such questions are used to probe students' understandings of conceptions – both pre-conceptions and alternative conceptions. Equally important for this study is, for example, when teachers notice that students are struggling with the understanding of a concept, how teachers use Socratic questioning (for instance) to allow the teacher to notice students' mathematical thinking processes. Overall, the study seeks to answer the following research question: How does teacher questioning help teachers to notice the salient features of students' complex mathematical thinking during lessons? Hence, this research builds on the noticing research in mathematics education, by focussing on MTN, viewing it through the lens of teacher questioning. Given that the study presented here is part of a larger project of continuous professional development (CPD), it examines the understandings of the teachers' notions to notice classroom interactions, as well as the use of the video to support the latter. This study also concerns itself (though not as the main focus) with intersecting with other studies which focus on lesson flow, student learning, and students' conceptualisation of key concepts (Field & Latta, 2001; Kennedy, 2005).

THEORETICAL PERSPECTIVES

This study is grounded in the *sociolinguistic* framework as proposed by Carlsen (1991), and the sociological construct of *framing* (Hammer, Elby, Scherr, & Redish, 2005). Before I discuss the sociolinguistic perspective on teacher questioning and MTN of this study, I offer a cautionary preface to the readers that teacher questioning research rarely constitutes sociolinguistics in nature – this study merely uses sociolinguistics as an alternative paradigm to study MTN and, in particular, teacher questioning. By way of defining it, in the context of this study:

> "Sociolinguistics is concerned with the interdependency of language and situation . . . [and] emphasise the role of social context in the interpretation of [mathematics classroom discourses]" (Carlsen, 1991, p. 158).

Teacher questioning during lessons is viewed as a mutually-generated process by both students and teachers – the point here is that in a mathematics classroom situation, whatever students communicate is dependent on the other speakers – teachers included, and these communications of students cannot be assessed in isolation. Alpert (1987) concedes that teachers are not always concerned with the long-term outcomes of teaching; there are times when they focus on the immediate goals of the lessons or the real time interactions in the lesson which lead to stimulating active participation among students. If the latter holds, then the teachers' attentions are to be directed towards MTN. Research on teacher questioning requires an in-depth understanding of three themes: context of the questioning; contents of questions; and teachers' and students' reactions and responses to questions (Carlsen, 1991). From a sociolinguistic perspective, and for this study, during classroom interactions, students' responses to questions are guided by the questions posed by the teachers, and the *context* which was created by the teacher and any other previous questions. In addition, context is also seen as 'setting the scene' for further questioning and between the students and teachers (Cazden, 1988). In sociolinguistic research, *content of questions* refers to the essence of the question – it is about the subject matter of the mathematics discourse taking place in the classroom or as portrayed by the question. *Responses and reactions to questions* speak to the differentiated responses that students and teachers receive, depending on who asked the questions. The detailed analyses of these themes of teaching questioning presented here are not part of this study and are not at all exhaustive.

The sociological construct of *framing* refers to an individual mathematics teacher's tacit understanding of *how do I make sense of what is happening here?* – in terms of teaching and learning (Hammer et al., 2005). [Epistemological] framing relates to the transfer of knowledge that occurs during learning activities where students engage within intellectual communities that connect to the contexts in which students use their knowledge (Engle, 2006). Some researchers have argued that the teachers' practices are driven by the way they frame the learning and teaching mathematics activities – framing also speaks to the dynamics of the individual teacher's knowledge on a moment-to-moment basis during a lesson (Berland & Hammer, 2012; Hutchison & Hammer, 2010).

As in the case of MTN, researchers have found that the way teachers frame mathematics classroom activities influences the attention they give (or do not give) to students' conceptual thinking (Redish, 2004; Russ & Luna, 2013).

THE STUDY

This study is part of a larger project called the Local Evidence-Driven Improvement of Mathematics Teaching and Learning Initiative (LEDIMTALI). The aims of the LEDIMTALI project are to improve the learning and teaching of mathematics in under-resourced schools in the Cape Metro in South Africa.

The initiatives of the project focus on CPD of mathematics teachers. Mathematics lessons of teachers were analysed to investigate how teachers' questioning helped teachers to notice the salient features of students' complex mathematical thinking during lessons. The findings of the study could be used by the research development group (RDG) of the project to inform the nature and composition of the CPD activities for teachers. Recent lines of research show an increase in the use of video to carry out CPD activities of teachers (Putman & Borko, 2000; Van Es & Sherin, 2002; Wang & Hartley, 2003) – this study is different in that the researcher analyses the video-recorded lessons with the aim of observing the teachers' in-moment interactions which are at the intersection of MTN and teacher questioning during lessons.

RESEARCH DESIGN

Data for this research includes the analyses of the transcriptions of video-recorded lessons of two mathematics teachers. Through the analyses of the videos, the researcher is afforded the opportunity to examine all the salient features of the teachers' practice. Despite the growing body of research in noticing, those that focus on MTN and teacher questioning are scarce. For instance, in this study, the researchers are able to notice the multiple perspectives at the intersection of MTN and teacher questioning. These interactions involving teacher-student discourses were transcribed verbatim. From these interactions I identified episodes of each lesson's mathematical discourses on MTN that seemed to be driven by teacher questioning or vice versa. The types of teacher questioning strategies within these episodes were identified and coded to form three categories of a Socratic questions framework – which include pumping, reflective toss, and constructive challenge (Mhakure & Jacobs, 2016; Van Zee & Minstrell, 1997). *Pumping* refers to questions which are used to prompt learners to elaborate on their mathematical thoughts, procedures and/or ideas ((Hogan & Pressley, 1997). *Constructive challenge* includes questions redirecting students after they have answered the teacher's initial question incorrectly. R*eflective toss* refers to questions that are used by teachers to "throw responsibility of thinking back to the student in response to a prior utterance made by the student which may be a question or statement" (Chin, 2007, p. 825). This Socratic questioning framework was then used to analyse the teachers' lessons.

FINDINGS

In this section I analyse the classroom interaction of the teachers, using notions of the MTN and Socratic questions framework that I have described in the previous section. Socratic questioning involves the teacher using a series of questions to probe students' mathematical thinking (Chin, 2006, 2007). The letters used in the transcript mean the following: A and B are the two teachers, T represents the teacher, SF represents female students, and SM represents male students.

Episode 1 – Teacher A:

The lesson under discussion here is on perimeter of shapes – for Grade 8; the question was on finding the perimeter of an isosceles triangle given its base of 5 centimetres and a side of 10 centimetres.

> T: In this question you are given an isosceles triangle – where one side of the isosceles triangle is 10 centimetres and then the base of that triangle is 5 centimetres, what would the perimeter of that triangle be?
>
> SF: 30
>
> T: What is the perimeter? (*A pause*) . . . Work it out, work it out? . . . Is that your perimeter? 30? How did you get your 30?
>
> SF: (*Indistinct – not sure*)
>
> T: How many lengths are there - 2 lengths?
>
> SF: Times 2 breadths.
>
> T: Times 2 breadths? But now that formula, does it fit the triangle?
>
> SM: No.
>
> T: What is the formula for the perimeter of a triangle?
>
> SF: Side one (*pause*).
>
> T: Side one plus side 2 . . .?
>
> SF: Plus side 3.
>
> T: So, did you draw that triangle with the information from the question?
>
> SF: No, sir.
>
> T: Isosceles triangle. I have got 5 as my base. What are my units?
>
> SF: Centimetres.
>
> T: 5 centimetres and the one side, I said, is 10 and then my perimeter would be what?
>
> SM: Side, plus side.
>
> T: Side plus side, plus side, so perimeter equals to?
>
> SM: 25.

In the above excerpt, the teacher starts by asking the students *"What would be the perimeter of that triangle?"* (line 1). One of the students (SF) gives a wrong answer – this prompts the teacher to ask a further pumping question – "How did you get your 30?" (line 3), in order to compel the student to elaborate on her initial answer. The teacher immediately notices further that the students are confusing finding the perimeter of a triangle to that of a rectangle and engages the student with a constructive challenge question: “But now that formula, does it fit the triangle?” (line 7). Instead of telling the student the answer, the teacher throws the responsibility of finding the right formula for finding the perimeter of a triangle back the student – reflective toss questioning: “What is the formula for the perimeter of a triangle?” (line 9).

As soon as the teachers notices that some of his students have problems in conceptualising how to find the perimeter of an isosceles triangle, he scaffolds the activities, leading to the solution of the problem. First, the teacher (line 11) uses the *contributive completion*, where he starts a statement and expects the students to complete it. In addition, the teacher also notices that students might have forgotten about ‘recommended practices’ when solving problems, such as drawing a sketch diagram first before attempting to solve the question (line 13).

Episode 2 – Teacher B: In this episode the teacher is teaching a grade 8 lesson on flow charts – calculations involve finding the output values given the corresponding input values. This question involves carrying out a substitution procedure in a quadratic function using minus 8 (-8) as an input value.

1. T: Practice number 5 reads $y = x^2$. (*A pause while teacher is writing on the board*). Now, do you remember what we said in our last lesson? What is our Input value here?
2. SF: Negative 8.
3. SM: Negative 8.
4. T: Negative 8. So to get the output, what must we do?
5. SM: Times -8 it by itself.
6. T: Now, you go back to your equation there and it becomes $y = (-8)^2$ . . . what is $(-8)^2$?
7. SM: -4
8. SM: Positive 16.
9. T: Is +16 correct?
10. SM: -16.
11. SM: He says, -16 . . . is that correct?
12. T: What does $y = (-8)^2$? What does that mean?
13. SM: 8 x 8, negative 8.
14. T: -8 x -8 and negative x negative, what do you get?
15. SM: Positive.
16. T: 8 x 8?
17. SM: 64. So this becomes, so the output is 64.

In the excerpt above, Teacher B starts by saying to the students: "*Now, do you remember what we said in our last lesson?*" He starts the lesson by asking a *prior knowledge* question. In other words, he asks the questions in order to understand where the students' conceptual understanding of the flow chart is.

This approach is quite interesting as far as the scope of this study is concerned because it stresses the notion that it does not really matter which comes first, MTN or teacher questioning. In the example described in the excerpt, the students identifies correctly the input value of negative 8; however, it is the substitution of this input value into the quadratic function which confuses some students (lines 6, 7 and 8). The latter is as a result of the teacher having asked a *pumping* question (line 6). There are lots of interactions taking place between teacher-students, and student-student (lines 2 and 3, 7 and 8, 10 and 11). Of particular interest is when a student asks the teacher a verification question "is that correct?" (line 11). Instead of giving a direct answer to the student, the teacher decides to ask a *reflective toss* question, thereby compelling the students to think about the conception – *"What does $y = (-8)^2$? What does that mean?* (line 12). One can also argue that at the centre of this interface between MTN and teacher questioning is the notion of scaffolding which is firmly embedded in both the iterative processes of MTN and teacher questioning.

CONCLUSION

The discussion in this study contributes to the growing body of literature on MTN and teacher questioning, particularly on what happens at the intersection of the two. The analyses of the two episodes of teacher A and Teacher B give specific examples of how Socratic questioning – as a form of predominant teacher questioning in this study – can enhance the construct of MTN during mathematics lessons. In addition the study also shows that during classroom mathematics discourses, it does not really matter which of the two constructs appears first – the point here is that MTN can lead to productive teacher questioning, and that the reverse is equally true. Equally important for this study is that it provides some practical examples of what could happen to classroom mathematics discourses where both MTN and teacher questioning are used to scaffold student learning activities. The framework for teacher questioning in this study, albeit brief and not exhaustive, can be used as a valuable guideline by practising teachers whose intention is to improve on their repertoire of questioning and noticing skills.

This study drew its findings from MTN and teacher questioning solely from the analyses of two video-recorded mathematics lessons of two teachers. Given that this study is part of a larger CPD project for teachers who are practising in an education system that uses high-stake examinations, the questions for future research could be: What MTN and teacher questioning skills do teachers in high-stake examinations environments need? This line of research will help develop the practising teachers' craft of knowledge in so far as MTN and teacher questioning is concerned in high-stakes examinations teaching environments.

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STUDENT'S VIEWS ON LEARNING MATHEMATICS IN THE UNIVERSITY OF TECHNOLOGY USING TECHNOLOGY

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Many education institutions in South Africa have introduced e-learning as a tool of learning and professional development, with the hope that such intervention can improve the teaching and learning of mathematics particularly in large classes. A further problem is that here are many students who have matriculated at basic school level but not adequately prepared for higher education. In this paper, we want to identify some issues associated with students' access to and use of e-learning resources among the Mathematics III students. We examine the students' perceptions of their adoption of technology and their understanding of how and what can be done to help students to use technology at their disposal for teaching and learning purposes.

Keywords: Learning, mathematics, student, technology

INTRODUCTION

The aim of teaching engineering mathematics in the faculty of engineering, at a University of Technology, is to provide engineering students with the essential mathematical techniques required in the their field of study. Such techniques are used in decision making in a diversity of engineering related disciplines. The various techniques are studied for developing understanding and comprehension of the mathematical methods necessary in the field of engineering. The techniques are also vital since they establish sound technical proficiency. The understanding of mathematics has been recognised as a challenge to students over time. Müller (1940) proposed that mathematics should not be viewed as a separate entity which has little to do with the rest of the engineering subjects. It should be viewed as one of the integral tools that are used by modern humans in today's engineering and real world applications.

Engineering mathematics students often find it difficult to see beyond the content they are engaged to understand the reason for learning the subject. The major challenge emerges in the application of mathematics techniques and knowledge to the engineering class of their discipline. This becomes even more evident when they cannot apply what they have learnt outside the narrow boundary of their textbooks to their fields of study.

In order to improve student's understanding of engineering mathematics, lecturers could adopt different teaching approaches. Constructivist approaches to learning view the student as someone who already has a certain amount of mathematical knowledge and with a preferred learning style (Salmon *et al*., 1991). New knowledge is acquired by making sense of it in terms of what is already known. Learning can be enhanced by engagement in a series of carefully designed tasks which people engage in within a social environment. Learning in the 21st century necessarily requires lecturers to introduce innovation to their mode of delivery so as to improve the instructional teaching methods.

The widespread influence of technology in every sphere of life, has made e-learning to become one of the essential supporting teaching method that is assumed to blend the traditional context of learning in the universities. Collis & Moonen (2002), defines e-learning as the learning through electronic technology which encompasses a range of technologies. E-learning is considered in the proposed research as supplementary to the traditional teaching methods, which has been the dominant approach in the institution in this study. E-learning technologies allow students to engage on their own in self-directed learning. The learning resources can at any time be repeatedly used. The learning content is easy to update. It is in this context that the current study intends to explore the take up of e-learning practices in the learning of mathematics at a University of Technology by identifying factors that currently limit students' access to and use of technological tools in their learning. An additional purpose is to identify particular areas of difficulty in the Mathematics III course that could be targeted for an intervention utilising online and other technological resources.

LITERATURE

In recent years in South Africa and other countries, there has been large increases in attendance of universities of technology as opposed to traditional universities. This is caused by the lower entrance criteria which makes even those that are partially prepared for university education gain access. Currently, the majority of students furthering their studies in the field of engineering at a university, have to do bridging course through an extended curriculum programme that is aimed at an introductory phase to developmental mathematics (Adelman, 2006) and (Jenkins & Bailey, 2006). The developmental mathematics differs from one university to the next. Baileyn (2009) noted that the students experience a problem when they are supposed to cross from this phase to mainstream appropriate university level. This has actually been the students' primary barrier to completion of their diplomas and degrees in regulation time (Bailey, 2009). The challenges students face go beyond the understanding of mathematical concept and include issues such as major pedagogical and mathematics curriculum changes that have been introduced at the basic school level. A clear distinction needs to be made between basic school and university mathematics education in the area of teaching approaches and methods.

In addressing the challenges students have in mathematics, South African schools will have to introduce subjects that deal with how the students should study and how they can develop motivational stance in fulfilling their academic achievements (Powers, 2008) and *(Calcagno et al*. 2008). There is no doubt that university mathematics departments have to deal with students that have challenged preparation, lack background knowledge and unorthodox attitudes towards mathematics learning. The students find themselves taught by university lecturers who have not received any professional development in terms of the teaching of the content, although they are experts in the content of mathematics. Hence they do not consider background educational or pedagogical issues in mathematics or may even be unaware of such. Some face challenges in embracing change with regards their university teaching practices because of the quality and availability of learning facilities as well as high teaching workloads.

METHODOLOGY

The research was based on an analysis of the examination paper written by Mathematics III students as well as the responses to a questionnaire survey from 150 Mathematics III students in the Departments of Electrical and Mechanical Engineering of the AB University. The questionnaire consisted of section A based on biographical information, section B focusing on challenges in learning Mathematics III, and section C focusing on adoption and use of e-learning in the university. It is estimated that the response represents around 55 percent of Mathematics III students in the university.
The research questions that guided this study were:

1. Which concepts encountered in the Mathematics III course present the most difficulty to the students?
2. What are some factors that currently limit students' access to and use of technological tools in the learning of mathematics?

The data were analysed using the basic descriptive statistics and the relationships within the dataset were captured using the EvaSys and SPSS software. The answers to these questions will provide the basis upon which the intervention (including the use of e-learning platforms and online resources) would be designed.

FINDINGS

The findings are reported under two headings, the first of which is the performance of the students in the Mathematics III course. The next section focuses on the findings about the technology readiness of the students.

Students' performance in Mathematics III

In terms of students' perceptions of the topics in mathematics III that they experienced as difficult, 82 percent indicated that they found Laplace transformation difficult.

Eleven percent found Fourier series and Harmonic analysis and 7 percent found Linear. Differential Equations with constant coefficient also difficult. In terms of the analysis of the actual examination paper for Mathematics III, the averages for each question appears in a clustered column chart 1 below.

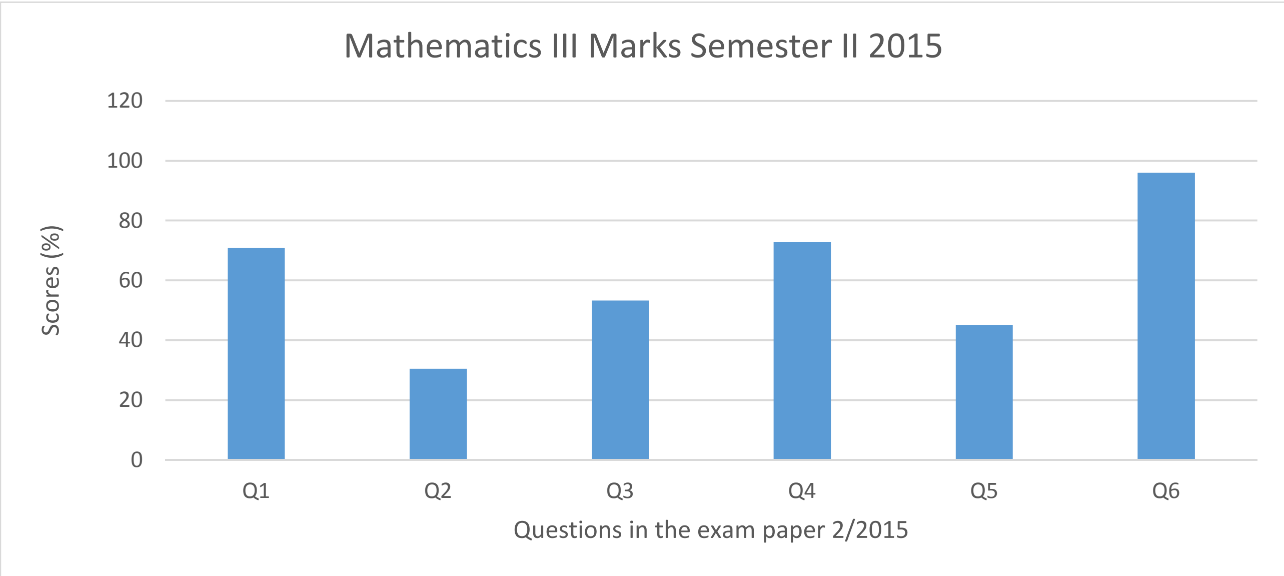

Figure 1: Students result in November 2015 examinations.

The examination question paper covered the following content areas:

Q1 Undetermined Coefficients

Q2 Application of Laplace transformation

Q3 Solution of Differential Equation using Laplace transformation

Q4 Application of undetermined coefficients

Q5 Fourier Series

Q6 Harmonic Analysis

The results in Figure 1 is the graphical representation based on student performance in the examination, shows that the application of Laplace transformation was the section with the lowest average. This explains why such a large percentage reported that they found Laplace transformations difficult. The analysis of Q2 and Q3 in particular indicated that the reason for poor performance of students, was because they did not understand the content of the module sufficiently well. Also students often did not know how to sketch linear graphs, a skill that is taught in Grade 9 at school. They also exhibited problems with partial fractions.

Many students were unable to use the standard table of Laplace transformation and had problems with many more fundamental concepts in addition to their conceptual difficulties with the actual topic.

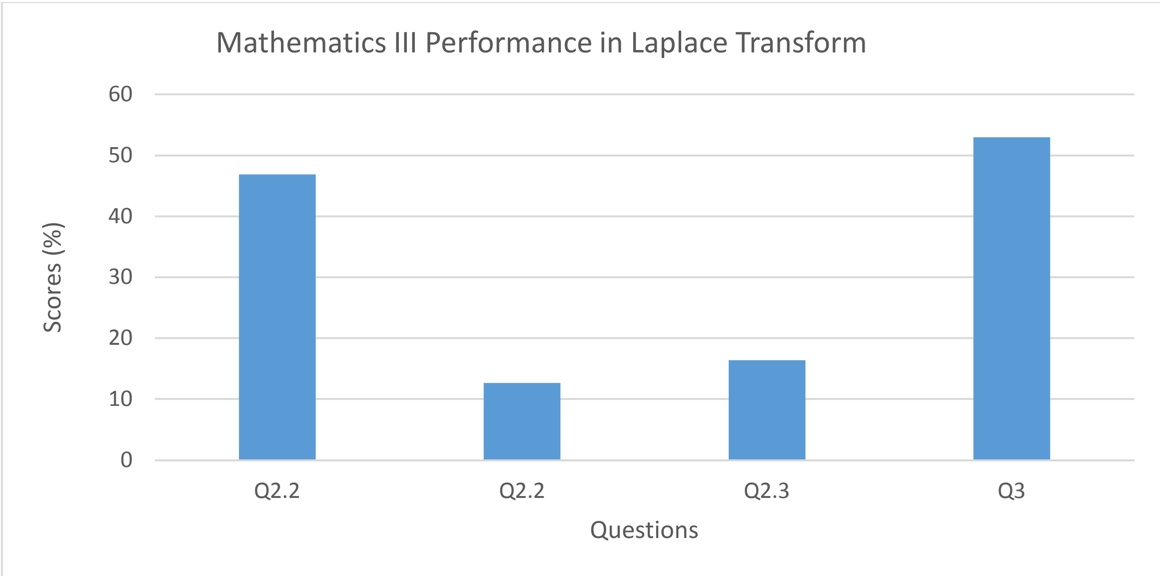

Figure 2: Item analysis of questions on Laplace transformation.

In particular, the questions in Figure 2 were addressing the content knowledge, the knowledge of Heaviside functions, the sketching of the step function and the solution of a differential equation using the principles of Laplace transformation.

This analysis shows that students did not perform well in the Laplace transformation module which was similar to results from previous years. Based on this, the module on Laplace Transformation was identified as a possible focus area that could be targeted for interventions that could improve the learning of the content. This makes it critical that an investigation be conducted on ways in how e-learning can be used to improve the learning of Mathematics III by students. It is assumed that the use of e-learning practices will enhance the learning and problem solving skills in the Laplace transformation module that seems to be very difficult for all mathematics III students. However before this could be implemented, a necessary task was to identify the level of readiness of the students to take on technological tools in their learning. Their readiness was explored via the use of the questionnaires, the results of which are now reported.

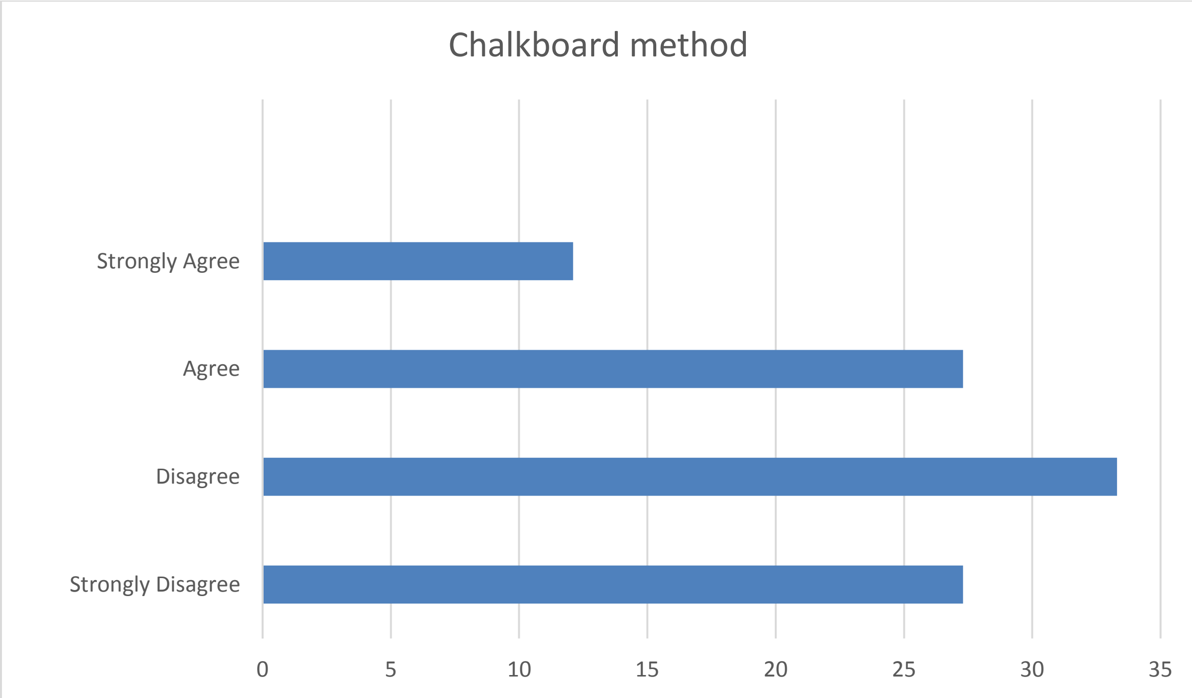

Figure 3: Students views on chalkboard method followed at the university.

It is interesting to observe that 33 percent of Mathematics III student found the chalkboard method followed in the AB University has not improved the standard of learning. This suggests that these students may prefer the use of technology in the teaching of Mathematics III. However, the responses of most students show that in terms of teaching and learning using technology, they were unclear about what is meant by the use of technology in learning.

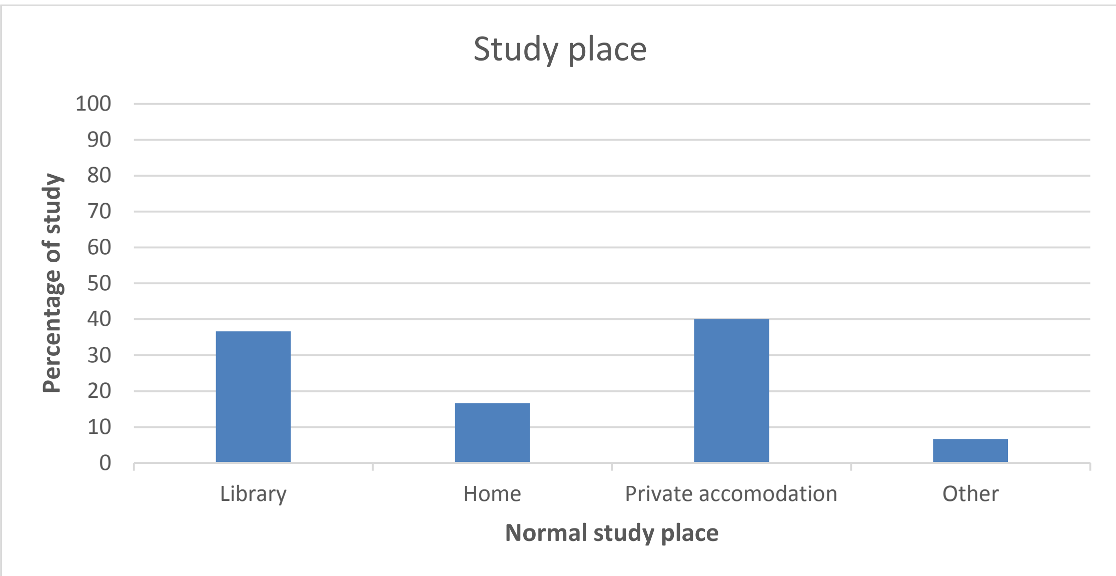

Figure 4: Study place for AB University Mathematics III students.

It was found that only 37 percent of Mathematics III students use the university library to study while 16.7 percent study at home. Although most students have accommodation whilst studying at the university, the majority of them indicated that they do not have internet connectivity in their accommodation which makes it difficult for those that can afford computers to use them for their studies.

The students' opportunities for technology engagement seems to be hampered by their limited access to computers itself. The students expressed that they do not use computers for teaching and learning purposes since only 25 percent indicated that they use them almost every day. They only rely on calculators as the immediate technology tool that they can use for their studies. Although they have cellular phones, they do not use them as a teaching and learning tool due to the internet connectivity that is a challenge at university and home.

From the small 25 percent group who indicated they use the computer, most of them do not use it for improving their learning in mathematics. Only 33 percent of students that use computers do so in order to search for more knowledge in Mathematics III, while only 10 percent use the internet to look up information for Mathematics III. These findings show that students themselves need to make a shift from relying on textbook based information towards using technology in their learning. In addition it was found that student do not have experiences of learning online as a group interactively. Ninety percent of the respondents indicated that there have never used internet to collaborate with a group of Mathematics III students. The internet offers endless learning possibilities and it was surprising that 100 percent of students indicated that they have never used educational software and programmes such as GeoGebra and Khan Academy.

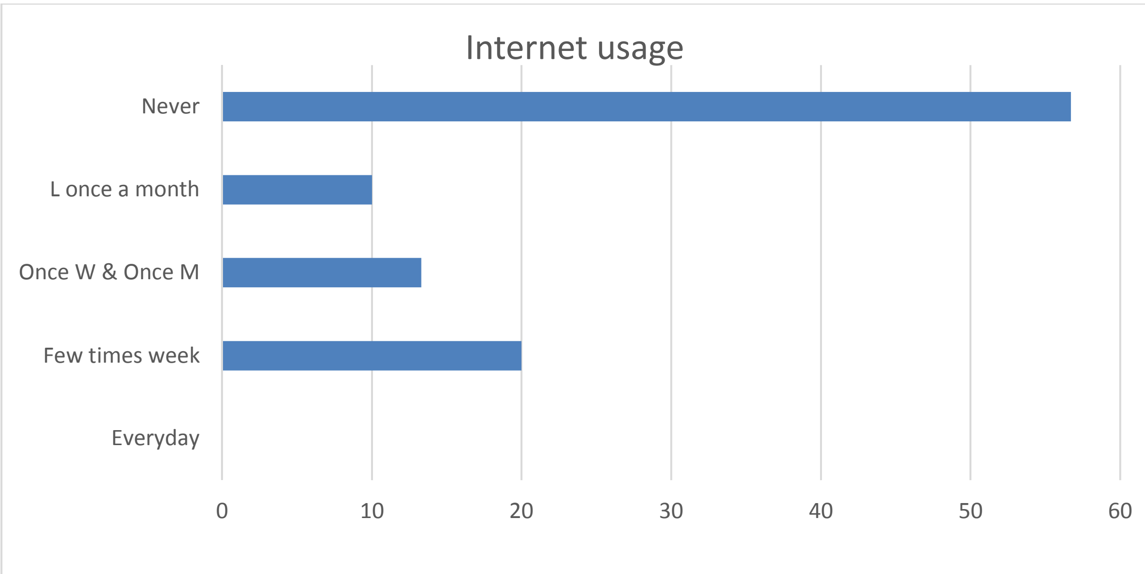

Figure 4: Students views on internet usage at the AB University.

In Figure 4, it is indicated that the majority of students, 58 percent never use internet for learning purposes, while only 10 percent used internet once a month and only 20 percent used it few time a week. Indeed this students are mostly dependent on the lecturers' notes and few available textbooks. The majority of the students have never learnt mathematics through learning mediums such as Facebook, Blackboard due to the response in Figure 4.

This suggests that the students may need an orientation programme to the use of technology where they could be introduced to the various online opportunities for learning mathematics that are freely available. Despite their limited knowledge of the available technology the students still felt that the curriculum needed to include technological links. About 52 percent of students indicated that in Mathematics III, there must be curriculum adaptation for e-learning and about 55 percent requested that e-learning course materials must be developed in the department. However the students were not impressed with the level of Information and Communication Technology (ICT) support provided by the university with 67 percent agreeing that the growth in demand for ICT support for student instructional use of technology is outpacing the university ability to provide this support. Surprisingly, considering that this is the 21st century, most students articulated a preference for face to face contact. Of the respondents, 50 percent of students preferred tutorial classes, and 13 percent chose teaching assistant support as the learner mechanisms that they would prefer. There were however 36.7 percent who said they preferred e-learning tutorials.

DISCUSSION AND CONCLUDING REMARKS

The data from the students indicate that in this institution, there needs to be a change in mentality of how teaching and learning could move from the traditional method to more progressive methods utilising blending and e-learning. To enable this change in mentality, several interventions may be needed. There is a need to advertise the possible resources for teaching and learning that are available within the institution to both lecturers and students. Students were not aware of some of the well-known online resources that are freely available. Hence, students may need to be introduced to these resources for e-learning. More crucially, there is need for the institution to develop computer literacy workshops for the first year students so that when they got to third year level, they would have improved serious challenges on computer use for teaching and learning. This lack of exposure to online resources may be a result of their learning experiences at school level where the use of technology in teaching and learning is very limited in most public schools.

In order to implement the use of technology to learn mathematics III in particular Laplace transformation, will have to consider the Technology Acceptance Model (TAM) developed by (Davis *et a*l. 1989).

The model indicates that ICT' success in an individual, is based on behavioural intention which ultimately determined by individual's positive attitude towards the platform as well as how he perceive its use. The TAM is most relevant in the research study since the success of the investigation is based on self-efficacy and instrumentality. Bandura (1982) developed a concept of self-efficacy, this concept implies that the more students find it easy to work with e-learning the better. Usually what students find it as an interesting tool to use in solving mathematics problems, makes them feel that they are in control over it and ultimately learning takes place immediately. Many researchers in the field of technology such as (Ifinedo 2006), (Wahid 2007), (Park 2009), (Chuttur 2009), (Liu et al. 2010) and (Teo 2011) confirm that this model is found to be useful.

There is also a need to consider Laurillard's conversational model which considers learning as learning mediated through conversations between learners to learners and learners to lecturers. The interaction between lecturers and learners as well as the need of meaningful intrinsic feedback using e-learning is indicated by this framework. The framework allows the learning to take place using wide range of technology as a flexible learning approach in the service of flexibility (Collis & Moonen 2002). It is assumed that it can give students an understanding on how to deal with implementation issues of environment, educational effectiveness, ease of use and engagement. The data suggest the use of two models of technology may be a good basis to introduce students in an e-learning platform that will hopefully improve the quality of learning opportunities that they are exposed to.

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EXPLORING GRADE 12 LEARNERS' OPERATION SENSE IN SEQUENCES AND SERIES

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This quantitative study utilised a content analysis design to explore grade 12 learners' operation senses. A randomly selected sample of 40 grade 12 learners drawn from secondary schools in Sekhukhune participated in the study. A test on number patterns, sequences and series was used to assess how learners incorporate number, symbol and function senses during problem solving. A coded matrix was used as a scoring rubric. The study sought to explore learners' competency on the three operation senses. The data was analysed using SPSS version 23. Hypotheses were formulated to test the generated information. ANOVA for testing differences among the three senses and correlational analysis to measure associations among the three senses and regression analysis were conducted. The findings of the study indicate that learners' have a weak operation sense. The average operation sense for the whole group was 37.8%. The averages for number, symbol and function senses were 40.2%, 38.5% and 39.8% respectively. These figures indicate that learners are less competent in solving routine problems involving sequences. Results indicate that learners have the same competencies in the three senses; there are strong correlations between the senses. The implications of this study are that the three operation senses are closely related and develop simultaneously, hence should be holistically approached in mathematics teaching and learning.

Keywords: Function sense, number sense, operation sense, pattern, symbol sense

INTRODUCTION

Mathematics is a science focusing on numbers and symbols and relationships in a sense. Operation sense refers to good intuition about numbers and their relationships, symbols and their meanings as well as knowledge of functions (Van de Walle, 2003). Learners need to master theses three senses in order to successfully solve problems involving sequences and series and mathematics problems in general. Early mathematical skills involve counting numerals, verbalising numbers in words and understanding cardinality without using number symbols (Fritz-Stratmann, Ehlert and Klüsener, 2014). However, the transition from numbers to Algebra requires learners to have knowledge of symbols and their meanings to complete mathematical tasks.

Understanding sequences and series requires learners to possess knowledge of combining numbers and symbols in mathematical sentences in the form of equations, expressions and functions. Number recognition (number sense), understanding of symbols (symbol sense) and ability to connect them through an algebraic relationship (functional sense) are the three competencies that are essential in handling number sequences and series in high school mathematics (LeFevre, 2010). Knowledge of numbers and symbols as well as meanings of these symbols is essential to complete mathematical tasks (Hiebert, 2013). Mathematical symbols are important because mathematical concepts, processes, tasks and solution processes are represented. Knowledge of functions is also essential in establishing relationships between variables in a sequence such the term and its position in a sequence. The combined effect of these three competencies brings a more powerful and useful competency called operation sense which is being envisaged in this study.

Research on instructional practices related to operation sense reveal that learners are taught only the surface aspects of the procedures involved in the operations and that little attention is paid to the underlying concepts (Zirbel, 2006). According to Faulkner (2009) number, function and symbol senses are regarded as key components to mathematical understanding. One of the aims of mathematics teaching is to enable learners to develop number sense and operational fluency which make them competent and confident with numbers (Department of Basic Education, 2011). The notion of operation sense is derived from learner's ability to construct mental objects (Sfard, 1991). According to Piaget (1964), an operation is the 'essence of knowledge' that is central in developing structural understandings. Operation sense encompasses various kinds of flexible conceptions which can be combined by the learner. Operation sense includes an understanding of the types of questions an operation can answer and the effects an operation has on the numbers involved. Operation sense is the key link between number sense, function sense, and the all-important symbol sense (Kaput, 2001). For example, in the sequence: 5, 8, 11, 14...... .A learner with number sense should be able to recognize the pattern in this sequence, and its relationship to addition and multiplication. Symbol sense should include the ability to express and recognize the sequence in the form Tn=, or in this case Tn =. Function sense should include the ability to recognize the relationship between the general term (Tn) and the general linear function.

The aim of this study is to explore learners' level of operation sense by paying attention to the way they apply number, symbol and function senses when solving routine problems involving number sequences and series. Learners' operational sense is assessed by exploring how they investigate patterns, extend, generalise, and formulate relationships, equations and interpreting solutions. It is hoped that the outcomes of the research may offer insights into other areas of algebraic reasoning where more effective pedagogic approaches need to be developed.

PROBLEM STATEMENT

Learners struggle to utilise algebra as a tool for formulating and solving problems involving sequences and series. Learners' limited understanding of algebra is a barrier to effective conceptual understanding and solving pattern problems. Learners lack skills of conceptualising pattern problems, formulating equations, solving and interpreting the solution based on the problem context. Learners' incompetency when formulating and solving equations from sequences could be explained in terms of their conception of number, function and symbol senses. Thus the difficulties that learners encounter in using algebra to solve routine problems involving sequences and series are centred on: understanding relations, formulating equations or functions using appropriate symbols, solving and interpretation solutions in relation to the problem context. All these competences require a learner to be fluent in the three senses.

RESEARCH QUESTIONS

In an attempt to find answers to the above problem, the study sought to provide answers to the following research problems:

1. To what extent do learners reveal symbol, number and functions senses when solving routine questions involving sequences and series?
2. What kind of operation sense do leaners use when solving routine questions involving sequences and series?
3. What operation senses relationships are emerging from learners' work?

HYPOTHESIS

The following hypothesis will be envisaged in this study:

H_1: There is a significant difference in learners' operation sense means.

PURPOSE

The purpose of this study is to explore grade 12 learners' operational sense in recognising, extending, generalisation, formulating, solving and interpreting the solution to routine problems involving sequences and series. By exploring the responses of learners to items based on the three senses it is hoped that the study will establish the level of learners' operation sense. The outcomes of the research may also offer insights into other areas of pattern and algebraic reasoning where more effective pedagogic approaches need to be developed.

RELATED LITERATURE

According Courtney-Clarke and Wessels (2014) number sense refers to learners' understanding of numbers together with their operations in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations involving numbers. Number sense is important because it is the foundation upon which all higher level mathematics is built (Stott, 2014).

At elementary level number sense includes knowledge of different kinds of numbers, the relationship between different kinds of numbers, and the effects of operating with numbers. The skills and dispositions to make sense of numerical situations in everyday life underpin the ability to understand, critically respond to and use mathematics in different social, cultural and work contexts (Clarke and Wessels, 2014). Learners with number sense have a disposition to make sense of numerical situations, look at problems holistically and use numbers flexibly to do mental calculations, produce reasonable estimates of numerical quantities and use figures to support an argument (Sowder and Schappelle, 1989).

Patterns and algebra is a strand of mathematics in which learners have to extend their thinking beyond what they see to generalise about situations involving unknowns. The introduction of number sequences and series in algebra brings other senses such as symbol and function sense. Sequences and series is one of the algebraic topics that deal with symbolising and generalising mathematical number relationships, structures and operations within these structures. According to Matos and Ponte (2008) understanding sequences and series involves representing, generalizing, and formalizing patterns. The central idea of studying sequences and series revolves around investigating, formalizing patterns and regularities, making generalizations and solving mathematical problems. Researchers concur that knowledge of algebra is a crucial requirement to learners' understanding of sequences series (Wessels, 2009; Aniban et. al, 2014). Symbol, number and function senses have emerged as key components of understanding sequences and series. The combined effect of these three senses forms operation sense. None of these is possible without the other.

Arcavi (1994, p. 32) introduces the notion of symbol sense as a 'desired goal for mathematics education'. Symbol sense incorporates the ability to appreciate the power of symbols, to know when the use of symbols is appropriate and an ability to manipulate and make sense of symbols in a range of contexts. Symbol sense also incorporates the ability to initiate symbols, to know when the use of symbols is appropriate and an ability to manipulate and make sense of symbols in a range of contexts. This research is based on the assumption that a learner with symbol and number sense can identify patterns, make conjectures, formulate equations, solve equations and interpret solutions in the problem context when solving sequences and series routine problems. An important attribute of symbol sense includes the ability to initiate symbolic relationships which express the verbal or graphical information needed to make progress in a problem (Arcavi, 1994). By exploring learners' formulation of equations and algebraic statements to test items the research can establish the extent to which they have symbol sense.

Function sense refers to a learner's level of fluency and ease with functions together with their graphs and interpretations. Algebra is a powerful tool for problem-solving activities, particularly if the problem can be formulated in terms of equations.

Algebra of patterns is about investigating, identifying and formulating, solving and interpreting the solution in terms of the problem context. Formulas (functions) are a powerful means to capture and describe the pattern. However, the formulation of these formulae requires learners to have strong number, symbol and function senses. Sequences are important mathematical objects, perhaps best defined as functions with domain the set of natural numbers or a subset of these (Kissane, 2003). A sequence can be thought of as a function, with the input numbers consisting of the natural numbers, and the output numbers being the terms.

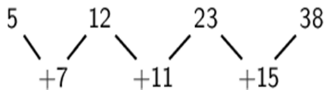
A sequence can be generalised in two ways: a recursive rule, which gives the next number by applying a rule to the number before it and function rule which predict any number by applying a rule or formula to the position of the number. An explicit formula which generalise sequence is a functional rule that relates each term of a sequence to the term number. An explicit formula designates the n^{th} term of the sequence, as an expression of n (where n = the term's location). It defines the sequence as a formula in terms of n. It is expressed either as a subscript notation T_n, or in functional notation, f (n).

For example, when generalising the following sequence a learner demonstrates operation sense by including the following aspects in the working:

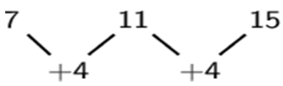
Write down the next two terms and determine an equation for the n term of the sequence:

5; 12; 23; 38; … …

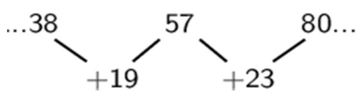
The first difference is calculated by finding the difference between consecutive terms:



A learner with a number sense will be able to study the pattern and extent it by two or more terms by realising the pattern of first differences.



The next two terms will be:



A learner with symbol and function will state the general term for the sequence as:

$T_n = an^2 + bn + c$

To find the values of a, b and c, we look at the first 3 terms in the sequence:

$n = 1$:: T_1= a+b+c= 5

n=2: T_2= 4a+2b+c =12

n=3: T_3= 9a+3b+c=23

Solving this set of simultaneous equations gives a=2, b=1 and c =2

Hence the general term for the sequence is $T_n = 2n^2 + n + 2$ or $f(n) = 2n^2 + n + 2$.

THEORETICAL FRAMEWORK

This study is guided by the Gestalt theory of problem solving, which claims that problem- solving occurs with a flash of insight. Topolinski and Reber (2010) defined insight as an "experience during or subsequent to problem-solving attempts, in which problem-related content comes to mind with sudden ease and provides a feeling of pleasure, the belief that the solution is true, and confidence in this belief" (p. 401-2).Thus according to this definition, insight is a set of metacognitive feelings of ease, pleasure, accuracy, and confidence that can accompany memory retrieval during problem solving. Gestalt psychologists argued that problem solving is a productive process. Productive thinking is solving a problem with insight. This is a quick insightful unplanned response to situations and environmental interaction. By contrast, behaviourists argue that problem-solving is a reproductive process in which learners apply previously learned behaviours to solve a new problem.

Gestalt psychologists emphasises the process of problem solving than the solution process. The theory hypothesises that solutions come from an insight into the problem and occurred when the solver restructures the problem. In particular, in the process of thinking about a problem the solver restructures the representation of the problem, leading to a flash of insight that leads to a solution. According to Metcalfe and Wiebe (1987) insight occurs when a problem solver moves from a state of not knowing how to solve a problem to knowing how to solve a problem. Insight learning occurs suddenly when a learner discovers new relationships within his/ her prior knowledge as a result of reasoning or problem solving processes that re-organizes or restructures that knowledge. During insight, the problem solver gathers tools (numbers, symbols, equations, functions) to solve the problem, devises a plan for representing the solution process. There are several ways in which conceptualisation can happen during insight: insight involves building a schema in which all the parts fit together, insight involves suddenly reorganizing the visual information so that it fits together to solve the problem, insight involves restating a problem's givens or problem goal in a new way that makes the problem easier to solve, insight involves removing mental blocks, and insight involves finding a problem analogy, that is, a similar problem that the problem solver already knows how to solve.

Wenger (1987) introduced the idea of visual salience to describe problem–solving approaches based on the gestalt view of learning. He identified two approaches: pattern salience and local salience. Pattern salience (PS) concerns the recognition of patterns in expressions and equations.

If a pattern is recognized by the learner by means of a gestalt view, it may recall a standard procedure and invite its application. Local salience (LS) concerns the salience of visual attractors such as exponents, square root signs and fractions, that is, the structural form of a mathematical problem. Using this extended definition of visual salience, developing a feeling for when to resist or succumb to both pattern and local visual salience is part of the acquisition of a gestalt view and thus of algebraic expertise.

In short, a gestalt view includes both pattern salience, involving the recognition of visual patterns, and local salience, involving the attraction by local algebraic symbols. In both cases, a gestalt view is needed to decide whether to resist or succumb to the salience. A gestalt view, therefore, includes the learner's strategic decision of what to do next.

RESEARCH APPROACH AND DESIGN

A quantitative research approach was utilised to describe learners' operation sense with sequences and series. Aliaga and Gunderson (2000) describes quantitative research methods explaining phenomena by collecting numerical data that are analysed using mathematically and statistically based methods. A content analysis research design was utilised. It establishes associations between variables (Hopkins, 2008). This type of research describes what exists and may help to uncover new facts and meaning. The purpose of descriptive research is to observe, describe and document aspects of a situation as it naturally occurs (Polit and Hungler, 1999). A quantitative design was preferred in this design because the goal is to describe and to test relationships among the operation senses (Fraenkel and Wallen, 2003). In this study the researcher presented the data in numerical form, and analysed through the use of statistics. This study is concerned with four processes of problem solving: understanding, formulating, solving/ executing and interpreting. The emphasis on the four processes is on how learners apply the three operational senses when solving the problems. Content analysis was also utilised as a research tool to determine the presence or absence of number, symbol and function senses within learners' work. Content analysis is a procedure for the systematic, replicable analysis of text (Rose, Pinks, Canhoto, 2014). In this study content analysis was used to classify of parts of learners' texts through the application of a structured, systematic coding scheme from which conclusions were drawn about their operation sense. The units of learners' texts that were interest during the coding process were the four senses. Descriptive statistics, such as frequency counts, can be used to summarise findings from the sample and appropriate inferential statistics used to test any hypotheses that have been formulated.

Research Instrument

The research instrument for this study consists of a test administered to 40 grade 12 learners who were randomly from a class of 45 selected at a supplementary instruction school.

The sampling technique applied this research was simple random sampling random sampling. Krippendorf (2004) identified probability sampling procedures as applicable to text or content analysis design. Test items require learners to recognise patterns and extend them beyond what they see, generalising patterns, establishing and formulating equations that describes the sequence. A rubric was used to score learners' performance. The scoring key assessed learners' work based on their ability to exhibit numbers sense by assessing their ability to identify and recognise a pattern, extending it by one or two more terms. Symbol sense was assessed by checking the formulation of equations using correct symbols and identifying unknown(s) variable(s) in the equations or inequalities. Function sense was assessed by checking whether the learner can treat the n^{th} term or sum of the sequence as a function. The rubric uses alphabetical codes (N= number sense, F= function sense, and S=Symbol sense). A learner who shows competency with number, symbol and function senses is considered to have operation sense. A learner who lacks any one of the three senses will be considered to lack operation sense (Schifter, 1998).

Participants

Participants for this study consists of 40 grade 12 learners selected randomly from a Mathematics and Science enrichment and supplementary instruction school. The learners were enrolled at different schools scattered in Sekhukhune District of Limpopo province. The researcher taught the topic and compiled a set of question based on the topic "Sequences and Series". A rubric or code matrix was used to assess the learners' work. Permission to engage learners was sought with the Limpopo Department of Education, the circuits, principals and supplementary instruction centre managers. Participants were issued with consent forms written in plain language statements that clearly describes the aim of the research and the nature of involvement of participants. Participants were clearly informed of their rights and any risks associated with participation. At all times the researcher observed the welfare of the participants and respect the dignity and personal privacy of the individuals. Letter codes (L1 – L40) were used to identify participants so that they remain anonymous throughout the study.

Procedure

The research was conducted with a group of 40 grade 12 learners. The researcher engaged the learners in the topic 'Patterns, Sequences and Series' and guided them through the concepts and latter assessed them. The data for the study were obtained from learners' responses to the test administered by the researcher. Content analysis of learners' responses to the test items provided data for the research. A scoring rubric was used to assess learners' number sense, symbols sense and function sense. The performance of each learner for each component of operation sense was expressed as a percentage and the overall operation sense was calculated by combining the competencies from the three senses.

RESULTS AND DISCUSSION

Operation senses competencies

After administering the test, learners' test row scores were analysed. Learner coded L6 obtained 75% on number sense, 75% on function sense, 50% on symbol sense and an overall operation sense of 58%. On the other hand learners L16 and L18 scored 8% in number sense, 4% function sense, 5% symbol sense and an overall operation sense of 6%.

Demographic variables

Table 1: Demographic Variables

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Male	18	45.0	45.0	45.0
	Female	22	55.0	55.0	100.0
	Total	40	100.0	100.0	

Demographic data of the participants is shown in table 1 above. The group consists of 18 (45%) male and 22(55%) female learners. The average number sense level for male learners was 43.3% with a standard deviation of 14.1% while the average number sense level for female learners was 40.2% with a standard deviation of 15.3%. The average symbol sense and function sense levels were almost the same around 40%. The overall average operation sense levels for males and females were 38.9% and 36.5 % respectively. These findings are in sharp contrast with Sabbatini (1997) who observed that males generally outperform females in scientific fields. There are a number of factors that can explain the current trend, such a teacher variable and the general competence of learners. A study by Chisholm and September (2005) revealed that South African female learners perform better than their male counterparts in maths and science. Their corresponding standard deviations were 5.13% and 3.65% respectively suggesting that there was little variation between the two groups and within the groups. The mean operation sense for the whole group was 37.6% with a standard deviation of 13.5%.

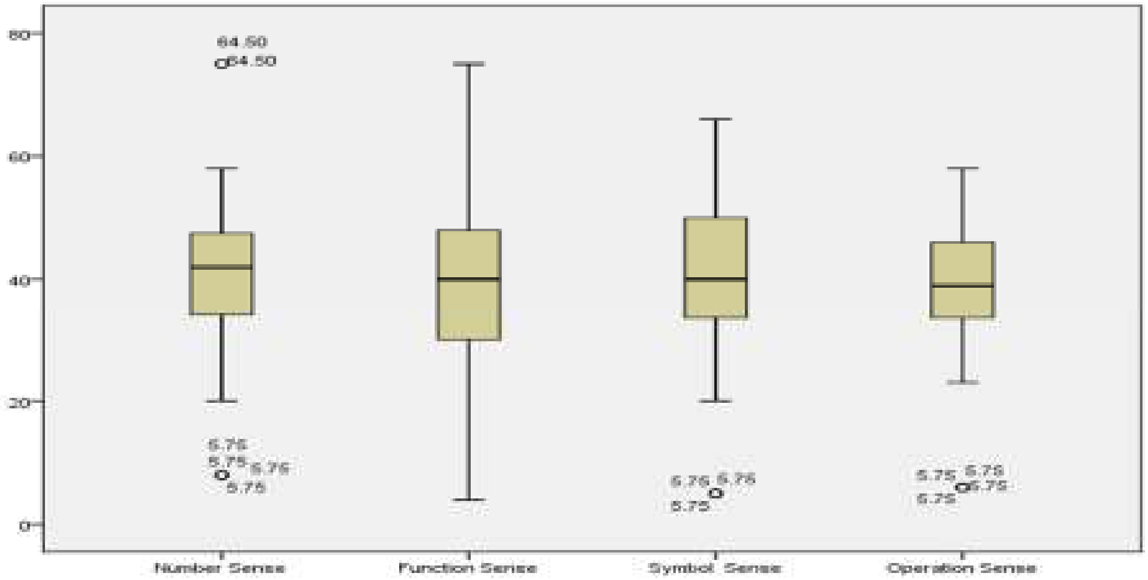


Figure 2: Box Plot

The number sense box plot is comparatively short and symmetrical; this suggests that most of the learners have the same competency with numbers. Learner number six (6) performed extremely well, hence has a strong number as compared to other outliers on the lower end. Learners (L16) and L18 indicate that they have weak number sense. The function sense box plot is comparatively tall suggesting that learners have quite different competences about this aspect. Learner (L6) was also an outstanding outlier who outperformed all the other learners. The function sense box plot was almost symmetrical and had no outliers on either ends. The symbol sense box plot shows a positively skewed suggesting that most learners performed above median of the group. Two extremely low marks were recorded for L16 and L18. The long lower whisker means that learners' difficulties with symbol sense are varied amongst the most negative quartile group, and very similar for the least positive quartile group. The overall operation sense box plot shows positively skewed distribution with values concentrated above the median. L16 and L18 were the notable extremely low performers in the group though overall the group shows a weak operation sense. The overall picture suggested by the plots indicates that learners have almost similar challenges with function and symbol sense learners, but have variable competency in number and operation senses. Function and symbol sense, as noted by Chae (2005) complement each other since symbols provide the tools with which functions can be represented.

Descriptive Statistics

Table 2: Descriptive Statistics

Sense	N	Min	Max	Mean	SD	Q25	Q50	Q75	SK	KT
Number Sense	40	8.0	75.0	40.175	15.34	33	42	47.7	-0.319	0.959
Function Sense	40	4.0	75.0	38.525	16.78	29	40	48	-0.281	0.595
Symbol Sense	40	5.0	66.0	39.775	15.98	32.8	40	50	-0.755	0.248
Operation Sense	40	6.0	58.0	37.575	13.59	33.3	39	46	-1.1023	0.897

Descriptive statistics results are shown in table 2 above. All the participants (N=40) showed instances of the three senses. Number sense recorded a minimum of 8% and a maximum of 75%. The mean number sense was 40.175% with a standard deviation of 15.34% implying a high variability. The quartiles for number sense indicate that 50 % of learners fall between 33% and 47.8%. A negative skewness of -0.319 indicates that the "tail" of the distribution is more stretched on the left side of the mean while a leptokurtic kurtosis value of 0.959 (which is less than 3) indicates that more of the variability is due to a few extreme differences from the mean, rather than a lot of modest differences from the mean. The overall mean operation sense for the whole group was 37.575 % with a standard deviation of 13.59%.The middle 50% of the participants had an operation sense ranging from 39% to 46%. The overall skewness for the three senses was negatively skewed with a value of -1.1023 while a kurtosis value of 0.897 (less than 3) indicates a platykurtic distribution value.

Anova Test

The one-way analysis of variance (ANOVA) was used to determine whether there are any significant differences between the means of three or more independent (unrelated) the senses. A one-way ANOVA compares the means between the groups of interest and determines whether any of those means are significantly different from each other. To test if there are significant differences in learners' operation sense an Analysis of Variance test was conducted to test the following hypothesis:

H_0: There is no difference in learners' operation sense means.

H_1: There is a significant difference in learners' operation sense means.

Table 3: Anova Table

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	7185.108	30	239.504	115.475	.000
Within Groups	18.667	9	2.074		
Total	7203.775	39			

The results of the test in table above show that there was a statistically significant difference between group means as determined by one-way ANOVA (F (9, 30) = 115.475, p = .000< 0.05).Therefore we reject the null hypothesis since $p<0.05$ and conclude that there is a statistically significant difference in learners' operation senses.

Correlations

Table 4: Correlations

		Number Sense	Function Sense	Symbol Sense	Operation Sense
Number Sense	Pearson Correlation	1	.781**	.799**	821**
	Sig. (2-tailed)		.000	.000	.000
Function Sense	Pearson Correlation	.781**	1	.939**	.939**
	Sig. (2-tailed)	.000		.000	.000
Symbol Sense	Pearson Correlation	799**	.939**	1	.908**
	Sig. (2-tailed)	.000	.000		.000
Operation Sense	Pearson Correlation	821**	.939**	.908**	1.
	Sig. (2-tailed)	.000	.000	.000	

**. Correlation is significant at the 0.01 level (2-tailed).

In order to measure the strength and the direction of the relationships operation sense variables the Pearson product moment correlation coefficients were calculated and are shown in table 4 above. All the correlation coefficients were positive indicating that relationships exist among the variables. Symbol sense and function sense have a correlation coefficient of 0.939 (which is greater than 0.8) which is described as strong correlation compared to 0.781 between function sense and number sense. Symbol sense and number sense have a correlation of 0.799 .All the three components of operation sense were highly correlated to operation sense since all the correlation coefficients were greater than 0.8(0.821, 0.939 and 0.908). Function sense had an almost perfect positive linear relationship with operation sense. The relationships are further ascertained by hypothesis tests on whether these relationships exist. The results in table 6 above indicate that the two-tailed probability value of 0.000 was recorded between any two senses. Since the probability values (0.000) are less than the significance level (0.05), the correlations are significant. Thus we conclude that number sense, symbol sense and functions are closely related to operation sense. This relationship is envisaged in the proceeding section.

Regression Analysis

Regression analysis was carried out to generate an equation to describe the statistical relationship between operation sense (dependent variable) and symbol sense, number sense and function sense (independent variables or 'predictors'). The relationship was modelled in a regression equation. The regression model emerging from the data was modelled as:

Operation sense = 2.742+0.182 ns + 0.447fs + 0.425ss + Error Term

P-values were used to assess the significance of each predictor variable. The p-value for each term tests the null hypothesis that the regression coefficient is equal to zero (no effect). All the p-values were low (< 0.05) indicating that we can reject the null hypothesis and conclude that all predictor variables are statistically significant and meaningful to model because changes in the predictor's value are related to changes in the response variable.

Table 7: Regression Coefficients

Coefficients										
Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	R	R^2	Adj R^2	Std. Error
	B	Std. Error	Beta						
(Constant)	2.742	1.509		1.817	.078	.976[a]	.952	.948	3.1065
Number Sense	.161	.052	.182	3.111	.004				
Function Sense	.362	.065	.447	5.605	.000				
Symbol Sense	.362	.055	.425	6.628	.000				
a. Dependent Variable: Operation Sense									

The coefficient of determination (R^2) for the model was 95.2%. This suggest that 95.2% of the variability in a learner's operation sense is explained by number sense, symbol sense and function sense and only 4.8% is explained by other variables.

CONCLUSIONS, RECOMMENDATIONS AND IMPLICATIONS FOR TEACHING

The purpose of this study was to explore grade 12 learners' operational sense by using content analysis to recognise how they extend, generalise, formulate and solving routine problems involving sequences and series. Learners' work revealed the three senses but their attainment level were around 40% which is low. Learners also revealed a higher number sense compared to the other two. However, there were no significant differences in learners' levels symbol and functions senses when solving problems. On the overall, learners' operation sense was low. Learners are not equally competent in the three senses. Symbol and function senses emerged as the main challenges that derail learners' progress when solving problems on sequences and series. Selection of correct symbols and formulae proof to be a drawback for learners and this negatively affects their problem-solving competencies. The study established that symbol sense and function sense are closely related and develop at the same time.

The findings of this study have significant implications for mathematics teaching and learning as well as problem solving. The results suggest that learners' operation sense, which involves the ability to use symbols with understanding, fluidity and flexibility with numbers and formulation of relationships using functions is not yet well developed. The three senses are strong predictors of operation sense and therefore deserve to be at the heart of teaching sequences and series. Operation sense is the key link between number sense, function sense, and symbol sense. The teaching and learning of number patterns, sequences and series requires learners to have strong number, symbol and function senses. Learners need to have a strong number sense in order to recognise patterns. Symbol sense and function sense are essential for making generalisations and conjectures about a given pattern. The combined effect of these three senses constitutes operation sense. The study established that symbol sense and function sense are closely related and develop at the same time. This is so because symbols are components which are used to decode ideas and functions are used to express relationships between ideas, processes and concepts.

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TEACHING MATHEMATICS IN PRIMARY SCHOOLS IN RURAL AREAS NEED NOT BE DEFINED BY THE CONTEXT

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This paper seeks to position teachers at the heart of school improvement by exploring their subjective beliefs on how rural schools can be improved. Teachers remain crucial players in ensuring quality teaching and learning in schools, as it is teachers who interpret and implement the curriculum. This is particularly the case for gateway subjects such as Mathematics. The paper draws from Giddens (1971) structuration theory that transcends from advancing teacher agency and social structures. The paper presents the narratives of four Mathematics teachers in rural areas and seeks to highlight the learner-centred pedagogies that these teachers employed in teaching Mathematics. The data were generated by means of classroom observations and semi-structured conversations. The findings revealed that although the Mathematics teachers in the rural schools felt unsupported by the Department of Basic Education, they did not conform to the stigma attached to rural education where teachers are labelled as being incapable of maintaining quality teaching and learning in teaching Mathematics. Moreover, the findings highlight the need for teacher agency and the fact that social structures in rural areas should not determine teachers' ability to improve their practice.

Keywords: Context, mathematics, rural areas, teacher educator, teaching

INTRODUCTION

In this paper I argue that teaching mathematics to learners in schools in rural areas does not have to be defined by the context within which it is taught. This paper articulates the crises faced by Mathematics teachers in rural areas who are working in dire situations in terms of resources but who have made it through all the odds. It relates to teachers narratives that were generated through semi-structured conversations and classroom observations. Researchers argue that teachers in rural areas embrace the lack of educational and infrastructural resources and believe that it is this deficiency that accounts for their inability to teach effectively (Podmore, Sauvao & Mapa, 2003; Organisation for Economic Co-operation and Development, 2012). These schools are in poverty stricken communities and there is an underachievement in the teaching. This belief hampers education as it encourages teachers to hide behind contextual issues and to forget the significance of education for learners.

This paper will address what teachers in rural areas believe regarding the impact of rurality on Mathematics teaching. The paper uses Giddens' structuration theory to analyse the themes that emerged from the data on teaching of Mathematics in rural areas because this theory articulates that being in the situation cannot always define how one will think or do things. The structuration theory states that the situations of lack of infrastructure are just temporary spaces that people can overcome at any time.

Teachers' qualifications tend to be one of the issues that emerge in discussions on how teachers teach. One such issue significantly impacts on teachers' ability to adequately deliver the curriculum. Research has illuminated the characteristics of teachers who are able to work around adverse situations and who are able to maintain quality Mathematics teaching in low-resourced communities in developing countries, (Christie, 2001; Morrow, 2007).

When referring to schools in rural areas in South Africa, it is generally implied that these schools lack infrastructure; worse, they are usually deemed under-resourced which hinders their ability to teach effectively (Human Sciences Research Council., Nelson Mandela Foundation. and Education Policy Consortium, 2005; Gardener, 2008; Mahlo, 2011). It is noteworthy that these schools are quite far from urban centres where teachers can access teaching and learning materials. At the same time professionally qualified teachers desire to remain in urban areas and this, more often than not, leaves the teaching of learners in rural areas in the hands of under-or unqualified teachers, Green, Adendorff, Mathebula, 2014). The distances between resourced areas and rural schools, as well as the perceived unwillingness of the Provincial Department of Basic Education and Training to provide services to rural areas that are far from the cities, render schools in rural areas, by dint of their geo-politically positioning, deficient. This paper explores rural teachers' delivery of mathematical knowledge by means of teaching, as the latter is an important factor in learners' educational achievement in Mathematics (Hill, Rowen & Loewenberg-Ball, 2005). The schools that were involved in this study are situated in deep rural areas of the KwaZulu-Natal province where limited infrastructure and little development are the bane of the communities' existence.

This study was guided by the following critical research question:

- How do primary school Mathematics teachers rise above the rural area challenges to teach learners?

This paper argues that primary school Mathematics teachers, who lack active support from the Department of Basic Education in rural areas, try their level best to rise above their circumstances and can use what is available within their context in order to assist in teaching leaners understand mathematical concepts.

This paper will contextualise the rural area teachers' experiences of Mathematics teaching without support and strive to make sure that learners are leaning. Therefore, the research aim was to understand how rural teachers make sure that learners learn irrespective of the challenges experienced in the rural areas.

Sampling of the study participants

Purposive sampling was used to generate data from four Mathematic teachers in a rural area. Vos, Strydom, Fouché and Delport (2011, p. 232) refer to purposive sampling as "the selection of participants that [will] best serve the purpose of the study". The four teachers that were selected were deemed best suited to elucidate the phenomenon under study, as they were teaching Mathematics in rural primary schools. Also the sampling of the rural area primary schools was also purposively selected as these four teachers are based in these two rural schools. These two rural primary schools had Mathematics teachers who were teaching in the primary schools.

Ethical considerations

The identities of the participants are kept confidential for ethical considerations (Creswell, 2013); hence they are referred to as teachers A, B, C and D. Before participating in the study, each of the participants signed a consent form, indicating that they had been informed of the aims of the study and that they were cognisant of the voluntary nature of their participation. On the day of the semi-structured interviews, the teachers were informed of their rights as participants in the study. They were also informed that they could withdraw at any time if they felt uncomfortable or threatened in any way. I thus adhered to the ethical requirement of maintaining the anonymity of the participants, as posited by Cohen, Manion and Morrison (2011a). It was also noted on the second day of the semi-structured interviews that the participants were given their transcripts to see and read what perspired during the interviews for their confirmation. All the four participants were happy with what was in the transcripts.

Demographics of the study sample

Teacher A originated from Umlazi BB, but was contracted by the *Fundza Lushaka* bursary initiative to work in a rural area. Two of the teachers were students at the University of KwaZulu-Natal. However, due to the scarcity of teachers in rural schools, they had been offered permanent posts and had subsequently made their homes in the rural area where the schools were situated. The fourth teacher was also desperate for a teaching post and agreed to relocate to a rural area. It is noteworthy that, apart from the many challenges that these teachers faced, they also had to adapt to the much colder climate of the mountainous rural area compared to the sub-tropical conditions they were used to in the coastal city of Durban.

THE CHALLENGES OF TEACHING MATHEMATICS IN SCHOOLS IN RURAL AREAS

The teachers' everyday negative experiences of teaching in a rural area were not important in determining how they managed to teach Mathematics in this context. In this regard, Khoza (2015) specifically defines that the need for learning effectively is implies doing things as required by the environment – this implies that the teacher adapts to the environment rather than being hampered by it. For example, Teacher A narrated how she had to find innovative ways of dealing with the situation of teaching Mathematics. One way she used was to resort to code-switching as a means of guiding her isiZulu speaking learners to understand the mathematical concepts that had to be taught in English. Giddens structuration theory states that the structures do not represent the barriers that these teachers find themselves experiencing but the barriers can allow changes that has to take place in order for effective teaching. Related literature, with specific reference to the NEEDU Report (2012), specifies the requirements for teachers who are expected to teach Mathematics in rural areas. Ball et al. (2008, p.399) explained it as "the mathematical knowledge and skill used in settings other than teaching" this implies that every person may know the differences of those shapes even if they are not teachers. Nkambule (2011, p. 342) asserts that even after 1994 "… rural education and rural development in education has [sic] just remained stagnant." This has resulted in the considerable difference in learner performance in the Annual National Assessment as the lack of resources in rural schools severely challenges the development of teaching and learning. Some of these schools lack electricity, running water and proper classroom space and, as a result, classrooms are overcrowded. There is also a high rate of unqualified Mathematics teachers in rural schools. Gardiner (2008, p. 13) states that rural communities are difficult to reach "…as the physical conditions in schools are inadequate and learner performance in comparison to schools elsewhere (e.g., townships) is at a lower level". Rural education seemingly lack the quality funding for professional teachers and inconsistencies that led to many to conclude that rural education research is poor (Department of Education, 1997). The rural areas are in dire need of qualified Mathematics teachers, but the recruitment and retention of qualified teachers tend to be problematic in disadvantaged areas (Coladarci, 2007). Setati and Adler (2000) assert that there is evidence of teaching Mathematics in rural areas using code-switching as one of the methodologies because the teacher and the learners share the same language; as a result the only resource that can be used is code-switching.

RESEARCH DESIGN AND METHODOLOGY

The research aimed to understand rural teachers' knowledge of the Mathematics discipline and how they understood the influence of this knowledge on teaching their learners in a rural primary school setting.

A qualitative approach was used in order to dig deep into the perspectives and knowledge of the teachers and to best understand their views and experiences. Maree (2009, p. 78) argues that "qualitative research is based on a naturalistic approach that seeks to understand phenomena in real-life situations." To address this approach, semi-structured conversations and classroom observations were chosen in order to generate a deep understanding of the phenomenon under study. The main purpose was to determine whether the teachers were defined by the rural context in which they taught or, conversely, whether their knowledge of the discipline that they had to teach – i.e., Mathematics - defined how and what they taught. As the data source, the teachers were positioned in rural classrooms as their natural space where they were confronted with issues that could be regarded as limiting due the rural context within which they were located. Therefore, a qualitative research method was chosen for an in-depth exploration of their teaching strategies. To achieve this, semi-structured conversations and classroom observations were conducted.

As a narrative research project, the main purpose was to interrogate the phenomenon of Mathematics teaching in rural primary schools. This initiative was prompted by my growing understanding that rural is not necessarily synonymous with deficiency in teaching and learning. This paper therefore reports on the lived realities and experiences of the participants, as proposed by Cohen, Manion and Morrison (2011 b). Both open-ended interviews and semi-structured conversations were conducted with the teachers for the purpose of enhancing the trustworthiness of the findings (Cohen, Manion & Morrison, 2011 b). Following a suggestion by Pinnegar and Daynes (2006), I used a narrative approach as a method of obtaining the data that I required. I define this approach as a specific type of qualitative design where 'narrative' is understood to be the spoken words of four Mathematics teachers who were engaged in semi-structured conversations and who were observed during their teaching practice in their classrooms in rural schools. Although a plethora of literature attests to the inadequate provision of teaching and learning resources in rural schools (insufficient resources, unqualified teachers and lack of support from the Department of Basic Education), the participants in this study painted quite a different picture about their experiences. By combining a narrative and observation approach, I could determine that these teachers tackled their challenging task with zest and made the best of the little they had.

The respondents' narratives

For authenticity of the narratives, the words of the respondents are presented *verbatim*. To produce narrative data, it meant that I had to employ a social approach that was going to allow the teachers to talk about how their teaching of Mathematics is in the rural context.

Teacher-A's story:

> "I was brought into the rural school because of a sponsorship (*Fundza Lushaka*) I got. I am teaching Mathematics to Grade 3. Development in rural areas has been very slow and the recruitment and retention of qualified teachers is problematic as this place is in high poverty. The only way I sometimes survive teaching Mathematics is using code-switching as one of the resources because the learners share the same language as myself; as a result, the only resource that I use is code-switching. In this approach, what I am teaching becomes easier for both learners and myself, though sometimes the Mathematics terms cannot be explained in isiZulu."

The above narrative suggests that this teacher was not born in a rural area; instead, it was her circumstance of being a bursary holder (*Fundza Lushaka*) that brought her to this school. However, this was not the only challenge that this teacher faced. Her reference to abject poverty in her narrative exposed the prevalence of social inequalities which, in turn, translated to learners not receiving learning resources to allow them to understand mathematics. The teacher therefore obviously lacked sophisticated learning materials to support her teaching of mathematical skills, and thus she was forced to use code-switching as a resource in order to make sure that the learners were able to understand what was being taught. It is a teacher's responsibility to make sure that learners understand mathematical concepts. If better resources had been available to facilitate learning, there would have been no need for the teacher to revert to the learners' mother tongue (i.e., isiZulu) to try and teach concepts in English. However, this teacher manipulated the situation in a positive manner by using a structure (i.e., language) that was already available and workable in this environment. Moreover, in order for her to convert a situation that was not conducive to teaching mathematical concepts into a positive learning experience for her learners, this teacher had to delve deeply into her understanding of the environment and the socio-psychological structures that her learners were exposed to in this area in order for her to teach efficiently. She therefore did not capitulate to the barriers within the rural context in which she found herself, but instead allowed the context, where resources were a challenge, to impact her teaching positively. She bravely did not allow the barriers that she encountered in this under-resourced school to deter her from performing her task to the best of her ability.

Teacher-B's story:

> "I do not have a special arrangement for mathematics in my classroom because it is quite small, but I understand that it should be arranged in a way that will instil a love of mathematics in learners. I struggle to have a mathematics corner because there is really no space. There has to be a display of colourful posters and colourful counters for counting. I would love to have a mathematics corner and a reading corner for the learners to understand different things in mathematics. I only take learners outside if the weather is conducive for teaching in order to teach in the manner that I have planned."

Noteworthy in this participant’s narrative is that she really had a problem with classroom space; in fact, she felt that this lack of proper space denied her the opportunity to teach in a manner that would be conducive to learning. Moreover, for her the lack of space also impeded a natural and positive flow of teaching and learning. To expand her classroom outdoors, she had to wait for pleasant weather, as only then would her teaching space be suitable for her planned activities. This narrative does not suggest that the teacher was unable to teach. On the contrary: despite the cramped space she was still willing and able to work with her learners. She also acknowledged that there are other requirements for the learning of mathematics, such as reading and mathematics activity corners where learners can continue with their work in spaces that enrich and enhance learning. The issue of providing sufficient space for teaching and learning is the responsibility of both the Department of Basic Education and the parents in rural areas. As was mentioned earlier, the rural context in which this teacher was based was stricken by poverty and an under-resourced school. Her narrative referred to her awareness of teacher agency, but other structures that need to come into play were lacking. However, this teacher made a concerted effort to teach even though there were severe challenges. This means that the way things were in this community did not stifle her or deter her from doing what she was able to do or knew what she had to do in order to teach mathematical skills. She used her knowledge of the positive impact of an ‘outdoors classroom’ and was prepared to wait for good weather in order to expose her learners to a space that would be conducive to improved learning. The initiative that she took implies that lack of teacher knowledge was not an issue in this instance, as she demonstrated a combination of what Shulman (1987) terms as subject matter knowledge, pedagogical content knowledge, as well as general pedagogical knowledge. Shulman talks about the seven domains of knowledge and proposed that each teacher must possess Shulman’s domains of knowledge in order to be able to teach any subject.

Teacher C’s story:

> “When I came here my mentor teacher told me to focus specifically on what suits the context … but when there are challenges with the resources, there is nothing that teachers can do; hence there is limited support [in this school] in terms of the resources.”

The above narrative suggests that the teachers in this school were advised to conform to the external forces that impacted this rural school and that ‘made it what it was’. The school lacked resources and infrastructure and therefore teachers could not provide what was not available. The respondent’s older mentor suggested that the structures in the rural area should remain as they were. However, according to Giddens’ (1971) structuration theory, it is people who reproduce the structures we find ourselves in. It is therefore the people in a rural school who should construct and restructure the external and internal forces that impact their learners negatively.

If they do, the rural area structures that now seem insufficient for teachers to teach in a way that will benefit their learners in mathematics, will be transformed. However, according to Giddens' (1971) structuration theory, it is people who reproduce the structures we find ourselves in. It is therefore the people in a rural school who should construct and restructure the external and internal forces that impact their learners negatively. If they do, the rural area structures that now seem insufficient for teachers to teach in a way that will benefit their learners in mathematics, will be transformed.

Teacher D's story:

> "I plan my teaching in a way that allows me to start with concrete materials, which are sometimes not available, like counters, an abacus, and anything that learners can use for Mathematics. Then I move to the abstract like … eeeh… using pictures, which are very hard to find. My understanding is that activities which involve the use of concrete materials are done first, the semi-abstract ones will follow, and in this process my activities will be done in a sequence. I try to think of the sequential things that happen in this place and relate my teaching to those things. I struggle to find resources, but sometimes it is not in the town where you will find more examples, but the learners are trying because we use what is [available] in their context, like stones, trees, and animals such as cows going to the dip, one after the other."

What emerges from this narrative is the fact that this teacher used what was available in her rural setting when she worked with her learners. In rural areas, many challenges impede the availability of resources that will work well with what has been planned, but in this instance the teacher did not bemoan her fate, blaming the lack of resources on her rural setting. Instead, she utilised real and concrete resources such as stones and cattle to facilitate counting and understanding of sequence. It was when she wanted to use pictures to demonstrate abstract forms that she encountered challenges, as pictures were not always available. Moreover, this narrative does not suggest that this teacher could not teach or that she lacked pedagogical or content knowledge. The issue is that she was challenged to bring her learners in the rural area to the same level as those of learners in urban and township schools – and she had to do this with limited teaching resources. Her example of innovative teaching strategies therefore strongly suggests that a negative background and limited ways of doing things can be changed when people start to overcome the structures that bind them. In the rural context it means that teachers have the responsibility to reproduce the culture of rural areas in different ways so that effective teaching and learning are facilitated. This narrative is a case in point, as the teacher used what was available in order to teach.

CHALLENGES EXPERIENCED IN TERMS OF TEACHING AND LEARNING MATHEMATICS IN RURAL AREAS

Rural areas are in dire need of qualified Mathematics teachers; however, the recruitment and retention of qualified teachers tend to be problematic in areas of high poverty.

Setati and Adler (2000) found evidence of code-switching as one of the resources teachers use in rural areas when teaching Mathematics, because the teacher and the learners share knowledge of a language that is not the language of instruction. Moreover, Dodeen, Shumrani and Hilal (2012) agree that teachers' low perception of teaching Mathematics is caused by a lack of appropriate qualifications and limited participation in professional development. Charalambous, Philippou and Kyriakides (2007) point out that if teachers are uncertain of their competency to teach Mathematics effectively, it can cloud their skills and sometimes their emotional instability is passed on to the learners. How teachers perceive Mathematics always has an influence on learners' capabilities to learn with interest and confidence. Jackson (2008), for example, states that it is teachers' negative attitudes that affect learners' inability to learn a subject like Mathematics.

These negative attitudes emanate from what they often experience as teachers of Mathematics in rural areas. Lohman (2000) accepts that a lack of learning resources could be a challenge to effective teaching and learning; however, this researcher argues that the rural context oozes high levels of the population's trust in natural resources, although they may not be at the same level as urban areas in terms of access to the internet (Putnam, 2000). Rural schools lack electricity, running water, and proper classroom space; as a result classrooms are overcrowded. There is also a lack of qualified Mathematics teachers. Gardiner (2008:13) concurs, stating that rural communities are difficult to reach "…as the physical conditions in schools are inadequate and learner performance in comparison to schools elsewhere (e.g., townships) is at a lower level." Balfour, Mitchell and Moletsane (2008) assert that there is little development in rural schools, if any.

The teaching and learning of Mathematics in primary schools have been described as being in crisis (Fleisch, 2008). Ball and Cohen (1999) argue that teachers are rarely provided with mentoring opportunities that will assist them in enhancing their teaching skills of mathematics. Instead, they attend short courses or workshops that barely address their challenges in the classroom.

Yet, despite all these challenges in rural areas, there are teachers who, against all odds, break the barriers and manage to assist their learners in the acquisition of sound mathematical skills. Such teachers do not let the rural context define who they are and how they should be teaching. According to Adler (2002), it is "how the teacher uses available resources [that] will support the mathematics pedagogies in the classroom." It is against this backdrop that I investigated the teaching of mathematics in the four classrooms referred to in this paper. The participating teachers faced the issue of a lack of resources head-on and they used context-related resources to make sure that their learners would understand mathematics.

This means that, instead of changing the learning context to their needs, they adapted to the context that was familiar to the learners in order for them to learn better. Schiro (2013) places the teaching and learning environment at the centre of society, which is what these four teachers managed to do. They fought not to be defined by the context in which they were teaching and they made the classroom a place where the learners could be developed. In short, they made a powerful point by not hiding behind the issue of a lack of resources for teaching and learning.

Despite the importance of teachers as an element of educational stratification, very little research has emerged about the kind of teacher who is able to work around adverse situations in order to maintain quality teaching in low-resourced rural communities, with particular reference to the teaching of mathematics. The NEEDU Report (2012) argues that teachers' knowledge of teaching, as well as their competencies, relies on an understanding that underpins the school subject. It is imperative to understand that sound subject knowledge is a necessary condition for effective teaching. If the Mathematics teacher has no knowledge that, for example, building and construction tasks prompt higher order thinking such as comprehension and problem-solving skills, no effort by such a teacher to 'teach' Mathematics will be successful. Moreover, it is an inalienable fact that teacher competence is dependent on sound knowledge and a practical understanding of the principles that underpin the subject being taught.

DATA GENERATION

The two data generation methods, namely semi-structured conversations and open-ended interviews were used in order to increase the authenticity of the data that were generated. I used probing questions in order to increase the authenticity of the data, as suggested by Cohen, Manion and Morrison (2011a). I ensured that I meticulously recorded the conversations and the responses during the interviews for transcription purposes. Particular themes emerged from the data, and these themes were used as subtopics for the data analysis.

Also, it emerged from the open-ended interviews that these teachers had relocated from an urban to the rural area due to employment opportunities. For Teacher A, acceptance of the *Fundza Lushaka* bursary meant that she had to serve in a rural area for three years in order to pay back the sponsorship. The other three teachers urgently needed employment and it was available in the rural area. It was also evident that the two teachers who seemed to be most comfortable with life in this area had found their homes there and had accepted that it was a kind of life they had to get used to.

DATA ANALYSIS

I used inductive analysis as a data analysis technique (Creswell, 2013). This was done in order to create and establish clear relations between the research objectives and the summary of the findings that resulted from the data obtained from four rural teachers' experiences. From the open-ended interviews it was clear that the respondents, who were teaching in a rural setting, had to apply different approaches to teaching as compared to teachers in better resourced areas. They knew that they had to function in schools where resources to teach mathematics were lacking. Moreover, some colleagues told them that there would be little support from the Department of Basic Education despite the fact that the schools were under resourced. Initially, these teachers "went with the flow" as they felt that there was nothing much they could do. This attitude was confirmed by Teacher C who stated, "There is nothing that teachers can do in schools where there are insufficient resources for learners to learn." The implication of this response is that teachers can only focus on what is available and, when they experience challenges in acquiring resources, they capitulate and effective teaching is neglected. However, it was evident that Teacher A wanted to do more for the learners. She reverted to using code-switching as a resource in order to assist the learners to understand mathematics. Teacher D also understood that a classroom setting and appearance need to be conducive to teaching and learning; however, her challenge was that there was not enough space for her to display resources that the learners could refer to. Her answer to this dilemma was to take her learners, weather permitting, to the outdoors where their natural context would be a source of encouragement and enjoyment. Contrary to information revealed during the semi-structured interviews that some teachers in rural areas adopt the attitude that an under-resourced school dictates the manner in which they teach, most of the respondents rose above their barriers and managed to find creative solutions for this particular challenge.

CONCLUSION AND RECOMMENDATIONS

Schools experience a shortage of teachers due to different contextual factors, one being teachers who change their profession due to improved financial gain elsewhere. As a result, the provision of the *Fundza Lushaka* bursary was one initiative to solve the problem of teacher shortages in rural areas. The teachers referred to in this paper felt they had to deal with the challenges of teaching in a rural context by trying to rise above the limitations generated by the lack of support they received from the Department of Basic Education.

Teachers, who are referred to by Khoza (2015) as the agents of change, have a responsibility to change the manner in which Mathematics is taught in rural areas, as they cannot rely on old, ineffective methods any more.

If teachers embrace this mandate, the social structures in rural areas that are embedded in a lack of resources, low levels of infrastructure, and unbreakable power issues can be ignored and therefore overcome. If teachers want to change these limiting situations in their classrooms, nothing can stop them. Therefore, it is suggested that 'rural areas' is merely a term and that teaching in a rural area is not synonymous with a person's inability to teach Mathematics.

There is a relationship between how teachers who do not want to change their situation in rural areas want to be defined, and the low standards of teaching that result in poor mathematical skills in learners. Teaching mathematics effectively involves the ability to process, communicate and interpret numerical information in a variety of contexts (Brown, Askew, Baker, Denvir & Millett, 2010, p. 365). Therefore, the contextual factors that exist in rural schools need to be embraced in order to assist learners to understand mathematical concepts. This is particularly important for teachers who do not have first-hand experience of how contextual factors impact learners in rural areas. Therefore, compared to Teacher A and Teacher D, the response by Teacher C that "there is nothing that the teacher can do more in situations where there are insufficient resources", leaves much to be desired.

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LANGUAGE PRACTICES OF TRILINGUAL UNDERGRADUATE STUDENTS: ENGAGING ONE TASK IN THREE

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This paper explores how and why some trilingual undergraduate students of mathematics use their languages to make sense of mathematics. The findings are drawn from a wider study focusing on language practices of undergraduate students of mathematics in Kenya. The notion of Discourse analysis is used as an analytic tool to describe and explain the language practices of two students. In a country like Kenya where majority students acquire trilingualism through schooling, a practice is advanced where home languages are dominant languages of thinking and understanding tasks, the national language supports communication; while English, the language of learning and teaching, is a language of external communication.

Keywords: language practices, students, trilingual, undergraduate

INTRODUCTION

This paper explores how and why some trilingual undergraduate students use their languages to make sense of mathematics tasks. The concern with the language practices of trilingual students is in keeping with the need for research related to the use of languages other than the Language of Learning and Teaching (LoLT), as suggested during a symposium on “Interactions between Linguistics and Mathematical Education” held in Nairobi in 1974 (UNESCO, 1974). The symposium (UNESCO, 1974) report noted that when mathematical ideas are presented to multilingual students in an additional language, the students may relate these ideas to how they address them in their home languages. Since the symposium, research in mathematics education particularly in Africa has focused on the use of two languages among multilingual learners. Further research among multilingual students can partly be addressed through research into the language practices of trilingual students, by viewing trilingualism as a special case of multilingualism. In fact research (Phakeng, 2013), shows that language practices of trilingual students in mathematics education have not as yet been explored.

The argument I set forth is that trilingual undergraduate students of mathematics who are competent in LoLT switch to their home languages and the national language as resources in making sense of mathematics tasks. I draw on the schooling context in Kenya to consider how and why trilingual students use their three languages and perhaps set forth the process of reclaiming pride in the use of African indigenous languages in mathematical engagement. Relevant extracts from interview transcripts are used.

WHO IS TRILINGUAL?

Research into trilingualism is limited and no one definition has so far been adopted (Hoffmann, 2001). However, researchers in this field (e.g. Hoffman 2001), have identified quantitative and qualitative aspects that are characteristic of trilingualism which they have used to explain it (Hoffmann, 2001). The quantitative aspect of trilingualism shows that trilingualism occurs when a speaker knows three languages. The qualitative aspects are for instance, the existence of different groups of trilingual speakers, depending on both the circumstances and the social context under which they acquire and use the three languages. Further, a trilingual speaker uniquely uses his/her three languages in ways that are determined by his/her communication needs. He/she has the ability to function like a monolingual, a bilingual or a trilingual, depending on the topic, place or interlocutor. Hoffmann explains that to function like a bilingual or trilingual, the speaker requires to code switch between languages. The quantitative and qualitative aspects of trilingualism suggest that while a trilingual person may share some characteristics with a bilingual and multilingual person, a trilingual is not an extension of a bilingual person but a special case of a multilingual person with specific characteristics.

SITUATING TRILINGUALISM IN KENYA'S CONTEXT

The majority of students in Kenya acquire trilingualism at school through the practice of the Language in Education Policy (LiEP). The policy states that during the first three years of their schooling, students are taught through the medium of the home language that is predominant in the school environment. The students may represent a variety of home languages; hence the mathematics classrooms are multilingual. During this period, they are introduced to learning their home languages as well as English and Kiswahili as subjects. Neither Kiswahili nor English are first or home languages for the majority of students. English is the official language while Kiswahili is the national language and co-official language. From the fourth year of primary school, the LoLT is English; Kiswahili continues to be taught as a subject while the home languages are dropped from the academic process.

Research conducted in Kenya has explored the use of two languages, for instance Kiswahili and English or Dholuo and English. This has been particularly from a teaching perspective (Bunyi, 1997; Cleghorn, Merrit & Abagi, 1989). However, what these studies do not tell us is how majority students in Kenya who are exposed to three languages deal with trilingualism. Thus the purpose of the research reported in this paper is to explore how and why trilingual undergraduate students use their languages to make sense of mathematics. The three languages under focus are: the students' home languages, the national language, Kiswahili, and the LoLT, English. Specifically the paper responds to the questions:

1. What are the language practices of trilingual students when engaging with mathematics tasks?
2. Why do they use languages as they do?

These questions focused on the students' language practices through their verbal, written expressions and reflections on their linguistic train of thought, while they engaged with a mathematics task.

LANGUAGE AND DISCOURSE ANALYSIS

The perspective taken in this paper draws widely from Vygotsky's theory that development occurs in and through socially mediated activity and that language plays a key role in this mediation (Vygotsky, 1986). According to Vygotsky's theory, language serves first a regulative, communicative function and later becomes a tool for thinking. Language expresses thoughts through verbal and non-verbalised communication, that is, in spoken and written language, and gestures; while other thoughts remain unexpressed outside the human person (Vygotsky, 1986). It mediates communication within and between humans. Language therefore has the power to transform the way people learn, think and understand.

The works of Gee (2005) on language use show that the primary function of language is to support performance of social activities and identities and, human affiliation within cultures, social groups and institutions. Gee argues that when we use language, we enact certain Discourses (with capital D) in the same or different contexts. Gee refers to Discourse as language plus non-language "stuff" that is:

> ways of combining and integrating language, actions, interactions, ways of thinking, believing, valuing, and using various symbols, tools, and objects to enact a particular sort of socially recognizable identity (pp. 21).

In other words, we use spoken or written language in tandem with non-language "stuff" to perform actions in the world. The actions project us as certain kinds of persons engaged in certain kinds of activities and hence we get associated with groups whose members act as we do (Gee, 2005). The meanings derived in any one Discourse situation are multiple, varied and situated in context of use. Central to Discourse analysis is recognition; if the activity and identity are recognised, then one will have "pulled off" a Discourse of a sort (Gee, 2005, 23). Gee's Discourse analysis was important for analysis of language practices of the trilingual students. The particular Discourses that apply in this paper are mathematics Discourses.

Using a situated and socio-cultural perspective and the notion of Discourses, Moschkovich (2002) observes that mathematical Discourses constitute ways of combining and integrating language with other non-language "stuff", and ways of saying, doing and being in mathematics. The perspective looks at the situational resources students use and ways that mathematics Discourses are relevant to the situation. It helps us to consider the non-language resources that the students are exposed to and can use when they engage with mathematics rather than focus on mathematical vocabulary. Non-language resources include the use of languages in a students' repertoire that are not necessarily the LoLT. Thus mathematical Discourses allow us to more fully describe the variety of resources that students use to communicate mathematically.

Gee's Discourse analysis can be used to analyse language practices within a one language or a multiple language environment. In this paper, the different Discourses involve use of multiple languages in one mathematical task. The language practices of some trilingual undergraduate students were analysed in an effort to understand how and why they used language as they did when they engaged with a mathematics task and the socially situated identities and activities that students enacted.

It is however important to note that Discourses are mediated through natural languages, for example English and Kiswahili. With that regard, and in the absence of literature on language practices involving trilingual students of mathematics, a discussion on language practices involving bilingual and multilingual students sheds light on possible language practices among trilingual students.

LANGUAGE PRACTICES OF BILINGUAL AND MULTILINGUAL MATHEMATICS STUDENTS

Harnessing students' home languages as important resources for learning has been argued as a means to improve multilingual students' participation and performance in mathematics (Setati & Adler, 2000; Setati, Molefe & Langa, 2008). This is especially appropriate when they have limited proficiency in the LoLT, which may prevent them from expressing their mathematics ideas clearly. However, research shows that bilingual students who are competent in the LoLT also draw on their home languages to make sense of mathematics because they face interpretation challenges in mathematics (Clarkson, 2006). Therefore, a common language practice among bi/multilingual students is that of code switching between LoLT and home languages.

Code switching

Code switching is the alternative use of two or more languages in an utterance or conversation in a more or less deliberate way (Baker, 1993; Grosjean, 1982).

The alternation can involve a word, a phrase, a segment of a sentence, a sentence or several sentences. It is a common characteristic of bi/multilingual speech. While the works of Baker and Grosjean portray code switching as a verbal strategy, a corresponding non-verbal strategy of language switching has been proposed and used in research on mathematics education. Language switching refers to the use of two or more languages during solitary and/or mental arithmetic computation (Moschkovich, 2005). This involves switching between two languages when thinking through computations, portraying an internal function. While acknowledging this differentiation, in this paper, I choose to refer to all situations where students switch between languages in verbal conversations and/or in mental computations as code switching.

Linguists explain that code switching happens for a range of purposes. They include, translating content from one language to another and to easily and efficiently express one-self (Baker, 1993; Cohen, 1995; Grosjean, 1982; Kern, 1994). Reasons for which translation has been used in mathematics classrooms include familiarity with the other language(s) and hence the speaker may habitually translate some words (Parvanehnezhad & Clarkson, 2008). Translation also happens as a constant practice with the ultimate goal of transforming information into a more usable representation (Kern, 1994). For instance, Vietnamese mathematics students translated problems from English into Vietnamese while reading and thinking them through (Clarkson, 2006). They later translated them back into English to make the ideas compatible with the classroom language situation. Some bilingual mathematics students also switch between languages in order to express themselves easily and efficiently (Planas, 2011). The switching is not necessarily the result of the students not knowing a word or a phrase in one language; rather, they are taken to facilitate the use of words or phrases in the other language.

The discussion above shows that exploring students' mathematical participation and performance by focusing on code switching between the LoLT and home languages, broadens the view of understanding bi/multilingual students' mathematical ability. However, this view remains narrow since multilingual students have other languages in their repertoire (Gorgorio & Planas, 2001). In fact trilingual students' participation in mathematics Discourse in their third language, which is their national language, has not yet been explored.

METHOD

The wider study, from which this paper draws, was qualitative and involved 15 trilingual undergraduate engineering students taking mathematics in their programs at one public university located in the central part of Kenya.

The university draws its students' population from all over the country and hence the students have varied language backgrounds. Data was collected using questionnaires, clinical and reflective interviews. All the students were academically proficient in the LoLT and in mathematics. Given the opportunity to choose the language for the interviews, all the participants opted to be interviewed in English. The findings presented in this paper are from one of the themes that emerged from two students who engaged in the mathematics task in three languages.

The two students S13 and S14 were in their first year of study taking a degree in Civil Engineering. Their home languages were Kikuyu and Dholuo respectively. Baseline information showed that S13 spoke Kikuyu more than he did Kiswahili or English at home with family members. All three languages were important for his individual engagement with mathematics and he switched between English and Kiswahili when discussing mathematics with his peers and lecturers. S14 commonly used Dholuo with his family; English and Kiswahili were rare languages of communication. When doing mathematics on his own and with his peers, he commonly switched between English and Kiswahili, and he communicated with his lecturers in English.

The task that the students engaged in was;

A hall can accommodate 600 chairs arranged in rows. Each row has the same number of chairs. The chairs are rearranged such that the number of rows is increased by 5 but the number of chairs per row is decreased by 6.

a. Find the original number of rows of chairs in the hall.
b. After the rearrangement, 450 people were seated in the hall leaving the same number of empty chairs in each row. Calculate the number of empty chairs per row.

To guide the analysis, the following questions written in English were asked directly or indirectly of each student: Which other languages did you use as you engaged with the task? Which other languages do you use while engaging with mathematics either in discussion groups or alone? Why did you use the language(s) as you did? All code switching instances were noted.

A recurrent theme in our analysis is that the students switched between English, their home languages and Kiswahili either internally or externally. The home languages were the dominant languages that the students used to think and understand the task and hence make sense of it. They translated part or the whole task to home languages because they were more familiar with the languages and for S13 because code switching was a constant language practice. Furthermore, switching to Kiswahili facilitated ease in their communication.

Thinking through and understanding the task

S13 explained how and why he used his home language, Kikuyu. It was necessary for his thinking and understanding of the task.

S13: For one, when you, the first part when I was reading it, when I was silent, I was trying to get is "what is the question trying to ask?" and I could visualise it in my own language, because / this language is not so…is not "haikuangi ati common kwa kila mtu" [it (English) is not common to everyone]. So at a certain point I could read the question if I have not understood then I try to figure out, "what does it mean?" If I'm given about this information, now I have to digest this information bit by bit in my own language. Then / from there after I have understood, after I have understood now the question, I could now be able in a position to write or answer the question.

S13 read the task in English, but thought through and imagined what was expected of him in his own language, particularly the parts he thought he did not understand. He switched to his own language because English was not his everyday language. After having understood what the task required of him he proceeded to answer it. His own language was his home language, Kikuyu:

R: You mentioned that you usually visualised it in your own language; what is this own language?

S13: Most of the time … after I read a certain piece of question, I have to think it in my language now, my first language…now after thinking in Kikuyu then I could now go to the question, now I have understood in a way that I can now react to the question what it is asking.

R: Does this happen always?

S13: Most of the time; if for example you can give me a question I lack to understand in a way that I can speak it in my own language, most of the time I will not be able to answer that question properly.

He used Kikuyu in the interpretation of the task. The use of Kikuyu was not unique to this task; it was a common practice for S13. He revealed that if he did not understand a question in Kikuyu, he would most likely not answer it correctly. This shows that while he read the tasks in English, he used Kikuyu to think and to seek understanding of the task. He explained how he used Kikuyu in this particular task:

S13: For example, in this part, I could read this one {referring to the first sentence in the question} I know what is talking about in my own language. Then after the first sentence then I continue {with all other sentences} then I can get the real picture of what the question is talking about. For example here, {reads the first sentence}. Now I could ask I could visualise in my mother tongue, "this is 600 / how could they be arranged? Yes they are in a room but how could they be arranged?" After I think it in my own ways now I come back to the questions after I have understood what is in me…

R: Is it something you can write?

S13: I write it in my own language? {R nods and S13 goes on to write the information part of the question in Kikuyu and then reads it aloud}.

Nyumba iganagira itĩ magana matandatu ibangitwo na mĩhari. Omũhari ukoragwa na itĩ ciganaine. Itĩ ni ciabangurirwo na ikibangwo ringĩ, na itĩ iria ciari muhari umwe ikĩnyiha na ithatatu na mihari ikiongerereka na itano. [A house/building can accommodate 600 chairs arranged in rows. Every row has an equal number of chairs. The chairs were rearranged, and the chairs in each row reduced by six and the rows increased by five].

This is how now I understood the question in my own language. Now this is how I understood it in this format, after I understood it in this format, I could now translate this information in now in English, in my own way.

Although S13 had earlier indicated that he used Kikuyu when faced with challenges to his understanding, it seems he sought understanding of the whole task by translating it, sentence by sentence, into Kikuyu. He achieved the desired understanding of the task in Kikuyu then translated what he was writing into English. Up to this point, this trilingual student demonstrated how and why he switched between English and his home language.

For S14, the use of his home language, Dholuo, was as a result of challenges of reinterpreting part of the task in English.

S14: I involved it {Dholuo} at the stages where I was not able to interpret in terms of English.

R: Like which parts?

S14: In part (b) I had to involve, I was a bit confused in terms of these people {450} and the number of seats here. I had to involve Dholuo and Kiswahili so that I interpret that each chair was supposed to accommodate an individual. So depending on the equation that I got in part (b), I had to equate to the number of people so that I could solve it.

R: So did you translate the whole of part (b)?

S14: In mother tongue, yeah.

R: How did it go like? If you can write {S14 writes the translation of part (b) in Dholuo}.

S14: "Ka ji 450 obedo to gi wuoyo kombe, kombe ma odong' onego bed ni ting'o ji 150". [If 450 people are seated and they rearrange the chairs, the remaining chairs should accommodate 150 people] {He then read out the translation}. I set out the equation for the remaining chairs... {Inaudible}.

R: And that assisted in getting the solution?

S14: Yes, because in stage (a) I was now having the equation, how I could put it so that I solve it for the number of empty chairs that was where there was a problem.

S14 had some initial difficulties with (a); he reworked the task to completion and arrived at the expected solution. But it seemed the challenges posed in (a) did not require him to switch languages as he did in (b), where he had difficulties in the interpretation of the task and this caused some confusion. He needed to link the solution arrived at in (a) to the requirements of (b). In order to do so, S14 translated part of (b) into Dholuo. The translation assisted him in linking (a) to (b) and arriving at the solution to (b). From S14's account, it is clear that Dholuo was used as a linguistic resource when he faced interpretation challenges in English. It emerged as the dominant language that shaped his understanding when he was faced with interpretation challenges.

Ease of communication

The students seemed to switch to a third language, Kiswahili, for various reasons. In his verbal explanation during the two interviews, S13 switched to Kiswahili, saying "kwa hivyo, [so that]" "ukuje useme [you say]" and "hapa [here]". The meaning of the words or phrases does not reflect any translation of words in the task and their use did not suggest that S13 derived mathematical meaning from them. Rather, the words seemed to have been taken to facilitate the use of words or phrases in the language. In my view, S13 used the words in order to express himself easily like bilingual students do (see Planas, 2011; Clarkson, 2006). In fact he had earlier on indicated that English is not "common" to him and it could be that he used the next official language as means to communicate with the researcher. While this use of Kiswahili was explicit, S13 explained that he also used it to think. This only happened after he gained understanding in his home language.

I explored how and why he did so:

R: At certain times as you went along you used other languages.

S13: Yeah, mostly I usually use Kiswahili.

R: How does it feature in your work, as you do it? …. What role does it {Kiswahili} play {in responding to the task}?

S13: This one maybe after understanding it in this way {in reference to the Kikuyu translation}, because Kiswahili is the language which is closer to my language, now I come to use it severally...If it's come on mathematics or maybe sciences, I will have to think in those two languages, I read in this one {English}, then in the process I will come now to use this Kiswahili in the meanwhile.

S13 initially seems to be hedgy on his use of Kiswahili but subsequent utterances make it clear that it is not only in mathematics that he switches to Kiswahili.

He claims to use Kiswahili in mathematics only after he has gained understanding in Kikuyu, because Kiswahili is close to Kikuyu[i]. It is not clear whether this occurred in a mixture of Kiswahili and Kikuyu or whether there was a distinction between their functions prior to translating back to English for reporting purposes. However, it is clear that he switches first to Kikuyu to think and understand the task and then if need be to Kiswahili. The closeness suggests that while he was most familiar with Kikuyu, when it came to Kiswahili and English languages, he was more familiar with Kiswahili. This may explain why he used Kiswahili only after gaining understanding in Kikuyu, and finally switched to English so as to report.

S14 explained his interpretation of the task in the LoLT. He however switched from English to Kiswahili to emphasise the claim that the task required the formation of a quadratic equation. The use of Kiswahili was not related to the mathematics but rather to the ease of using Kiswahili words to emphasize his claim. While this switch to Kiswahili was the only notable example of code switching practice in the interpretation, during the reflective interview S14 revealed that he switched to Kiswahili almost throughout the engagement with the task to read some numerals. Prior to the utterances in extract below, S14 explained his use of Dholuo and then continued to explain how and why he used Kiswahili.

R: There was also the mention of Kiswahili, tell me how you used it in this question.

S14: Kiswahili I used it almost throughout. I used it when I was referring to the numbers here like 600, like 6, like 5.

R: Why do you use Kiswahili for the numerals?

S14: I always tend to read them {in Kiswahili}.

R: So even when you are reading this {quadratic} formula here, that 4… {Referring to the constant value 4 in the quadratic formula}

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

S14: But not such cases only at times ndio natumia [it's what I use], and I actually think maybe because I'm living with people… {Inaudible} … so most of the time we speak Kiswahili.

S14 read some numbers in Kiswahili, such as the numbers of chairs, rows and people. These were numbers of countable items and not numbers in equations. While initially he could not explain the language practice, later S14 said he thought he did so because Kiswahili was the dominant language for social communication in his residential area. His environment had socialized him to visualize such numerical figures in the national language.

Therefore, S14 habitually read such numbers in Kiswahili even when the task was presented in English as was the case in the task that was at hand. It seems it was easier and more efficient to think of the numbers in Kiswahili as was the habit. Habitual reading of words in one language and not in the given language is not new in mathematics education. In a study by Parvanehnezh and Clarkson (2008), Persian-English bilinguals switched to read English words, for instance “girl” was read as “doktar” in Persian because of the habitual use of this word in Persian. Therefore, S14 switched to Kiswahili to emphasise his claim and due to the habitual use of the national language and subsequently ease in communication.

Emerging Discourses

At the beginning of this paper, I set forth the argument that trilingual undergraduate students who are competent in LoLT use their home languages and the national language as resources to make sense of a mathematics tasks. The analysis has shown that the Discourses that have emerged are that the two trilingual undergraduate students identified with the use of three languages in their mathematics engagement. The languages used in verbal communication, writing and thinking in relation to their social culturally defined experiences, shaped the identities they enacted and the activities they were engaged in when working on the task. They used their three languages according to their communication needs.

S13 read and wrote the task in English, interpreted it in Kikuyu, and then perhaps used Kiswahili before translating back into English for reporting purposes. S13 not only switched and translated the task content into either of these two languages; rather this was a common practice in other tasks. The use of Kiswahili in his verbal communication is associated with the ease of communicating his understanding of the task. These findings reflect S13’s baseline information that all three languages were important for his individual engagement with mathematics. S14 mainly used English in his verbal explanation and in all his written work. He asserted his claims in Kiswahili; furthermore, he had developed a habit of reading numbers of countable items in Kiswahili. S14 switching to Kiswahili could be seen as a result of his familiarity with certain words and numbers in the Kiswahili language, and not because of a lack of understanding of such words (Moschkovich, 2002). When he needed to reinterpret part of the task; he did so by switching and translating part of the task content to Dholuo in order to form the required understanding. In fact, these findings agree with the baseline information provided by S14, that when engaging with mathematics he switches between English and Kiswahili. The switch to Dholuo was a reaction when S14 faced interpretation challenges. In cases, S13 and S14 switching to home language was an internal function, while switching to Kiswahili was both an internal and external function.

From the discussion above, one of the purposes of code switching was to think through and understand the task. This was done by switching between English and their home languages. They translated whole sentences or the entire task into their respective home languages. This switching was aimed at interpreting the task. The students' code switching practice to their home languages provided access to mathematics and supported the finding of Setati, Molefe, & Langa (2008) that home languages are resources that aid students' understanding of mathematics.

The other purpose of code switching was ease of communicating their understanding but not necessarily to derive the meanings of any particular words. This was particularly seen when switching to Kiswahili. S14 habitually switched to read numbers while S13 switched after having understood the task in Kikuyu.

From the analysis, it is clear that all the languages were not used equally; home languages were dominant languages for conceptual engagement, Kiswahili was used to ease communication and English was positioned as a dominant language for communicating with the researcher during the interviews. The home languages were functioned as languages of thinking and understanding the task, an internal function; while Kiswahili and English were for both thinking and verbal communication, external and internal functions. It is clear that code switching between the languages provided an opportunity for the languages to work together to make mathematics accessible to the students. The findings support the claim that trilingual students can use all three languages in their repertoire as suggested by Hoffmann (2001) and that students who are competent in LoLT also switch to their home languages to engage with mathematics (Clarkson, 2006).

CONCLUSION

The findings presented in this paper contribute to the global debate on language practices among students who are users of more than one language. The study partly attends to the research gap in trilingual context. The findings are particularly relevant to the Kenyan multilingual context, where majority of students are exposed to three languages at school, and other similar contexts like Catalan, India and Malawi. Research in mathematics education in the context of trilingualism is recommended to inform how students use their three languages and hence to support the findings of this study. Perhaps if trilingual students are allowed to use their home languages to explore their understanding of mathematics tasks, we can be in the process of reclaiming our African pride in the context of language diversity.

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THE EFFECT OF CONSTRUCTIVIST-BASED TEACHING METHOD ON LEARNERS' ERRORS IN HIGH SCHOOL ALGEBRA IN MPUMALANGA, SOUTH AFRICA.

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The traditional teaching methods employed in most high school classrooms in South Africa may be responsible for errors learners make in understanding algebraic concepts. This study investigated the comparative effects of a constructivist-based teaching method (CBTM) and traditional teaching methods (TTM) on the exposition and treatment of learners' algebraic errors in Grade 11. TTM referred to any form of classroom activities that characterised teacher's instruction in a control group during intervention. A quasi-experimental approach with a non-equivalent control group design consisting of pre- and post-measures was employed. A convenience sample of 36 learners formed the experimental group and 42 learners formed the control group. The researcher implemented CBTM in one experimental school while the incumbent teacher implemented TTM in a control school located differently from experimental school. An achievement test measured learners' errors in four conceptual areas in algebra. Lesson observations were conducted in the control school to monitor the exposition and treatment of learners' errors. The constructivist learning theory (CLT) framed the study and the results revealed that the CBTM was more effective in reducing learners' errors than TTM.

Keywords: Algebra, Constructivist-based teaching method, Constructivism, Errors, traditional teaching methods

INTRODUCTION

Poor performance in mathematics is a global problem (Baloyi, 2011; Betts, Zau & Rice, 2003; Blanco & Garrotte, 2007; Mji & Makgato, 2006; Prakitipong & Nakamura, 2006). An array of research has been conducted to illuminate as far as possible the factors believed to cause poor performance in mathematics. In some cases the results of these empirical studies have pointed to a strong link between poor performance in mathematics and the tendency of learners to make problem solving errors in algebra (for examples, see, Fajemidagba, 1986; Prakitipong & Nakamura, 2006; Rosnick, 1981). Prakitipong and Nakamura (2006) conducted a study to show that poor performers in mathematics made more problem solving errors than good performers. Luneta and Makonye (2010) confirmed that poor performance in mathematics is associated with errors and misconceptions that learners bring to the classroom. In addition, Luneta and Makonye (2010) noted that algebraic errors present epistemological challenges that have a negative impact in learning calculus.

Erbas (2004) described errors as a way of applying mathematical knowledge incorrectly and also making incorrect conclusions when attempting to represent mathematical expressions and ideas. Blanco and Garrotte (2007) and Erbas (2004) suggested that one of the causes of errors in learning algebra emanates from some obstacles such as lack of closure. Some learners see algebraic expressions as statements that are at times incomplete. Hall (2002) suggested that learners tend to be reluctant to stop before getting to an answer they are comfortable with, which is usually a numerical answer. Prior to the commencement of the current study the first author had observed that one of the most common algebraic errors learners make is writing the expression $3x+2$ as $5x$ and simplifying the expression $7–5y$ as $2y$.

Another side of research has revealed that some of algebraic errors made by learners emanate from mathematics itself or they could be the result of instruction (see, Erbas, 2004; Fleisch, 2008; Hill, Blunk, Charalambous *et al*., 2008). Feza-Piyose (2012) observed that "one of the contributing factors to the poor performance of students in mathematics is quality of instruction received by the majority of South African students" (p. 62). Teachers' instruction may provide another avenue to study learners' errors in mathematics. Although there is a huge body of research on teacher knowledge for teaching mathematics, there is however a paucity of detailed understanding on how teachers use this knowledge to address learners' errors in mathematics classrooms (Ball, Hill & Bass, 2005; Shulman, 1986).

The study of teachers' instruction and learners' errors can be a powerful tool to diagnose learning difficulties, and subsequently help teachers to design adaptive instruction to improve learners' performance.

THE CURRENT STUDY

The study that is reported in this paper is a research initiative to explore teachers' instructions in terms of how they exposed and treated learners' errors in a selected topic in Grade 11 algebra. Mainly the study aimed to investigate the comparative effects of a constructivist-based teaching method (CBTM) and the traditional teaching methods (TTM) on Grade 11 learners' errors in algebra, in terms of exposing the errors and subsequently providing a treatment for the observed errors. The following research questions guided this study:

1. How do the constructivist-based teaching method and the traditional method facilitate the exposition of learners' errors in a Grade 11 algebraic lesson?
2. What is the comparative effect of constructivist-based teaching method and the traditional teaching method on the treatment of learners' errors in Grade 11 algebraic classrooms?

Given the comparative nature of this study the first research question intended to document the relative potential of each teaching method to expose learners' errors in algebra. The second research question was intended to generate statistical measurements to compare the effect of each teaching method on the treatment in relation to learners' errors before and after the intervention.

CONSTRUCTIVIST TEACHING APPROACH

Instruction becomes more effective when it is built on the knowledge and understanding of how students learn. In this regard constructivist teaching approaches come closer to generating pedagogy facilitate the construction of knowledge by learners. Constructivist teaching methods are based on constructivist learning theory, which framed this study. Constructivism explains learning as a continuing development of knowledge involving the ongoing construction of mental interpretation and representations of reality by an individual (Tenenbaum, Naidu, Jegede & Austin, 2001). The tenets of constructivist-based teaching approaches posit that learners should have the autonomy to actively participate during the lesson to construct their own knowledge through group learning, mathematical discourse, and exploratory talk (Hausfather, 2001; Dhlamini & Mogari, 2013). Linked to this theoretical position is the notion of social constructivism based on the seminal work of Vygotsky (1978). According to Ernest (cited in Sriraman & English, 2010), "social constructivism regards individual learners and the realm of the social as indissolubly interconnected" (p. 43). From this understanding knowledge construction is facilitated through interactions with other individuals as well as by their individual processes (Ernest, cited in Sriraman & English, 2010).

Hence, a group approach may be established within the broader theoretical framework of constructivism. Using this theoretical position a group learning approach was largely advocated in the current study. According to Kirschner (2009), within a group setting information processing is characterised by active and conscious sharing. In the current study the CBTM lessons incorporated a group learning approach in an attempt to facilitate the exposition of learners' errors through verbalisation of their mathematical thoughts, which subsequently led to error treatment through guided peer group interactions. In contrast to the CBTM the traditional algebra class (TTM) was considered to be one that was mainly teacher-centred, textbook-driven, transmission-oriented and with practice algebraic problems done by learners individually in a non-group setting. In the TTM environment the teacher takes charge of a lot of the intellectual work in the classroom. The teacher plans the scope and sequence, pre-synthesizes and pre-packages most of the learning (Brooks & Brooks, 1999).

METHOD

In this part of the paper we describe information related to the study methodology, sampling processes, instrumentation and the analysis of data for the study.

Research design

The study employed a non-equivalent control group design consisting of a pre-test and post-test (McMillan & Schumacher, 2010). This design is used in studies that lack random assignment of participants to experimental group and control group. Intact classrooms were used in the study to preserve the normal running of the school. This design is largely used in educational research to prevent disruption of educational activities (for examples, see, Claire & Michael, 2003; Dhlamini & Mogari, 2013; Gaigher, Rogan & Brown, 2006; Turner & Lapan, 2005). In relation to the aim of the study and the related research questions the null hypothesis (H_0) and alternative hypothesis (H_1) were formulated:

H_0: The constructivist-based teaching method is not more effective than traditional teaching methods in reducing learners' errors when Grade 11 algebraic tasks are treated.

H_0: $\mu_{\text{constructivist-based teaching method}} = \mu_{\text{traditional teaching method}}$.

H_1: The constructivist-based teaching method is more effective than traditional teaching methods in reducing learners' errors when Grade 11 algebraic tasks are treated.

H_1: $\mu_{\text{constructivist-based teaching method}} \neq \mu_{\text{traditional teaching method}}$.

Participants of the study

The population of the study consisted of Grade 11 mathematics learners selected from 11 secondary schools located in the White River Circuit of Ehlanzeni[i] District in the Mpumalanga[i] province of South Africa. Two schools were conveniently sampled from this population to participate in the study. The two participating schools were 12km apart to avoid contact and contamination during intervention. From the two schools 36 learners were assigned to the experimental group and 42 to the control group. However, of the 36 learners sampled in the experimental group only 35 participated fully in the study. Also, of the 42 learners in control school 35 participated fully in the study. Full participation meant that learners wrote the pre-test, attended all instructional sessions and eventually wrote the post-test. One teacher participated in the study to preserve conventional conditions in the control group.

Instrumentation

The primary data collection instruments for the study were Algebra Concept Achievement Test (ACAT) and episodes of lesson observations in the control group. The researcher developed the ACAT test covering the following algebraic topics: *variables*, *expressions*, *equations*, and *word-problems*. The ACAT test had a total mark score of 75. The ACAT was developed through months of iterative processes of acquiring existing items from classroom teachers and state-approved textbooks, obtaining feedback from subject specialists and advisers, and conducting repeated content validity assessments.

The ACAT was administered at pre-intervention (that is, as a pre-test before intervention commenced) and post-intervention (as post-test at the end of intervention) stages of the experiment to both control and experimental groups to identify learners' errors. This means that prior to the commencement of the study the ACAT was administered in both groups to assess their initial problem solving status, in particular their tendencies to commit problem solving errors. The post-test served to demarcate the comparative effects, if any, of CBTM and TTM on the exposition and treatment of problem solving errors during intervention.

Content validity, including forms of *face validity*, was established for ACAT. Face validity was established because it was necessary to judge whether the use of ACAT to unearth learners' errors was worth pursuing (Johnson & Christenson, 2012; Rubin & Babbie, 2010). Content validity, which is the degree to which a measure covers the range of meanings included within the concept, was established when mathematics practitioners confirmed that the content of the test adhered to the Grade 11 Algebra curriculum. Both forms of validity were established on the basis of personal judgements. Using the results of a pilot process the Spearman Brown formula was used to measure the "linear relationship between two sets of ranked data" (Charter, 2001, p. 693) in the pre-test and post-test. The value of r=0.82 was computed to confirm that the test was reliable to measure learners' errors in algebra.

Lesson observations were constructed and conducted by the researcher in both groups. Lesson observations in experimental group were continuous and occurred at various points of the CBTM implementation. The focus of lesson observations was the teacher's reaction to the emergence of learners' errors during the algebra lesson. The scope of classroom observations covered observation of the teacher, the learner and the instruction. The following aspects of the lesson development were observed: (1) the format of the lesson; (2) arrangement of teaching and learning setting; (3) teacher's discovery of learners' errors; and, (4) teacher's reaction to observed learners' errors. All classroom observations were carried out through the use of a common the observation schedule. Prior to the commencement of the main study a pilot study was conducted in one school that resembled participating schools in the main study. The pilot study was meant to trial the implementation of the observation process (Johnson & Christenson, 2012; McMillan & Schumacher, 2006). Reliability of the lesson observations was determined through a process of repeated usage of the observation schedule. In addition, comparative checking of consistency in the outcomes was done.

ETHICAL CONSIDERATIONS

Ethical clearance for the study was obtained through our institutional ethics committee and involved schools consented to their participation in the study. In addition, all learners gave informed consent that permitted us to use the collected data in the study.

DATA COLLECTION

The researcher (first author) administered CBTM in the experimental group and the incumbent teacher implemented TTM in control group. The experiment lasted for two weeks, which was in line with the state guidelines on the treatment of topics selected for the study (Department of Basic Education, 2011). The study began with the administration of a pre-test (ACAT) in both groups, which was meant to: (1) determine the initial problem solving status of participants; (2) determine the frequency of algebraic errors participants did at this stage. The learners were assigned with index numbers to conceal their actual identities. They were given codes such as PRE01C, with the "PRE" prefix denoting the pre-test session, and "01" representing learner 1 and the letter "C" referring to the group in which a learner was allocated, which is the control group in this instance.

INSTRUCTION

The experiment started with a pre-test that was administered in both groups. The researcher administered the ACAT in the experimental group while the teacher administered the test in the control group. The test was then followed by classroom lessons on the selected topics of the study. The first author planned and designed instructional activities, which were discussed and given to the teacher prior to the commencement of the study. This arrangement was meant to ensure that similar work would be covered in both groups with only a difference in the mode of presentation. After two weeks a pre-test was administered in both groups to determine learners' post-intervention performance in terms of the success of each instruction to mitigate learners' errors.

In line with constructivism the researcher created group approach learning environments that dynamically altered the conventional roles of teachers and learners, and provided opportunities for collaborative discussions (see, Yackel & Cobb, 1996, cited in Dhlamini & Mogari, 2013). According to Yackel and Cobb (1996, cited in Dhlamini & Mogari, 2013), group learning environments "regulate mathematical argumentation and influence the learning opportunities" (p. 461). The researcher walked from one group to another while learners worked through the algebraic problems in the worksheets. This was done to determine whether the group learning dynamics of listening, writing, answering, questioning, and critically assessing contributions were taking place (Dhlamini & Mogari, 2013). In most instances the researcher intervened during the course of discussions to probe learners to see how they handled each other's misconceptions and errors. The researcher would throw in comments such as: *Please explain further what you mean*. Such interjectors served to expose some of the learners' errors during instruction.

The lesson observations in control school were limited to only three visits in order to allow lessons to run naturally thereby minimising possible disruptions. The visits were scheduled for days that were convenient for the researcher. Instruction in the control group was mainly teacher-dominated. The teacher tended to be all telling and learners inactively sat and received information from the teacher.

DATA ANALYSIS

Descriptive and inferential statistics were used to analyse the quantitative data from ACAT. Data from lesson observations were analysed using typological methods of analysis.

RESULTS

The results of the experiment are discussed in the next sections.

Answering the first research question

The first research question asked: *How do the CBTM and the TTM facilitate the exposition of learners' errors in a Grade 11 algebraic lesson*? In line with the first research question of the study data from the pre- and post-tests were used to document learners' errors in both groups. Table 1 reflect on the frequency of errors at pre- and post-intervention stages in both groups in terms of the four areas of focus for the study, namely, *variables*, *expressions*, *equations*, and *word-problems*.

Table 1: Summary of frequency of learners' errors at pre- and post-stages in both groups

Group	Test	Frequency of errors in algebraic variables	Frequency of errors in algebraic expressions	Frequency of errors in algebraic equations	Frequency of errors in word-problems
Experimental group (n=35)	Pre-test	82	79	68	85
	Post-test	23.0	18.0	15.0	26.0
% of error reduction		71.95	77.22	77.94	69.41
Control group (n=35)	Pre-test	78	81	65	87
	Post-test	45	34	42	56
% of error reduction		42.3	58.0	35.4	36.6

In line with our earlier position on the association between learners' performance and errors they do in mathematics (Prakitipong & Nakamura, 2006) we opted to use this framework to interpret the data in Table 1.

Table 1 shows that both groups performed similar at the pre-test registering almost the same number of errors in each topic of the four algebraic topics. However, Table 1 shows that after intervention there was a significant reduction of error occurrence in experimental group. Table 1 does not reflect on the types of errors observed in each topic. Table 2 provides examples of errors learners did while solving algebraic tasks in the achievement test. We provide examples of some of the learners' errors to illustrate the content of Table 2.

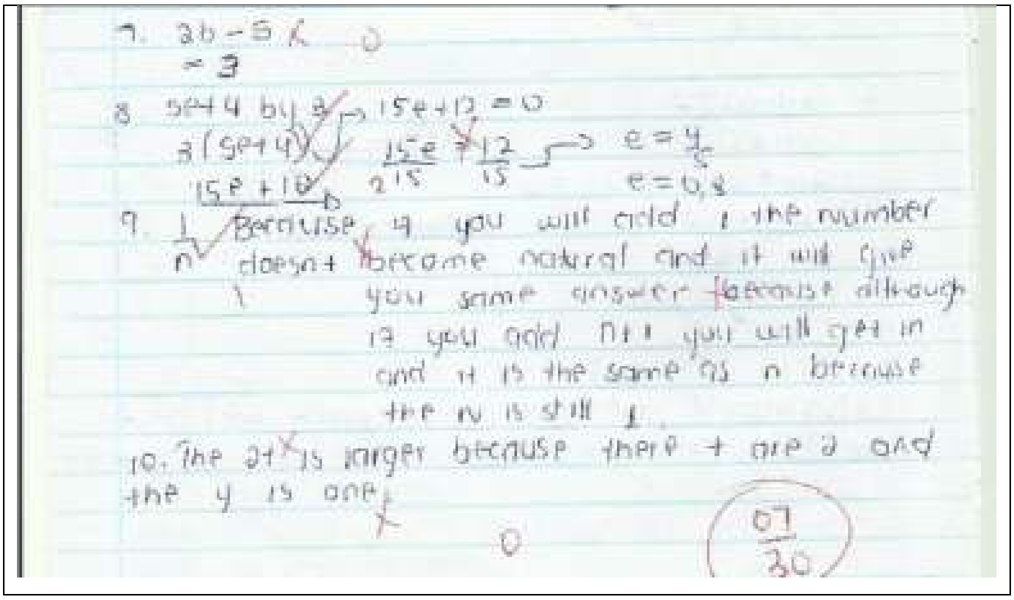
Table 2: Types of errors identified in the four conceptual areas in algebra

Algebraic topic	Errors identified
Algebraic variable	Misinterpretation of product of two variables Wrong assignment of arbitrary values for variables Misjudgement of the magnitude of variables
Algebraic expression	Invalid conversion of expression to equations Reversal errors
Algebraic equation	Inability to identify the type of equation Manipulation and transposition errors
Word-problem	Translation error The use of arithmetic instead of algebraic method

Algebraic variable

Question 10 of ACAT asked: In the equation y = 2t + 3, which is larger y or t. Explain. This test item was meant to give learners an opportunity to reflect on their understanding of the role of a variable in a mathematical equation. See vignette 1 for an example of learner's response.

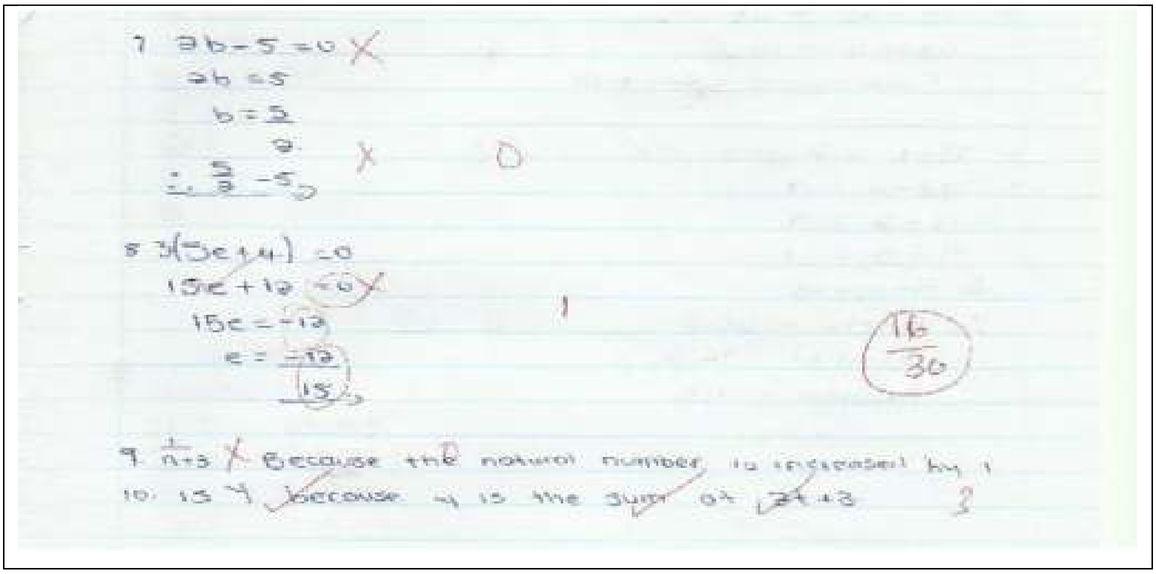
Vignette 1: Example of learners' response to Question 10 of ACAT



Algebraic expression

To illustrate learners' errors in relation to algebraic expressions we refer to items 7 and 8 of ACAT. Items 7 and 8 read as follows: (1) Question 7: Subtract 2b from 5; and, (2) Question 8: Multiply (5e + 4) by 3. Some learners formed invalid equations from the answers in the form of algebraic expressions. These learners proceeded further to solve these equations. We observed two versions of this error. Firstly, when simplifying algebraic expressions learners meaninglessly connected the variables in the problem to form an equation. Secondly, they were reluctant to accept an algebraic expression as the final answer and came up with a solution by solving the invalid algebraic equation they formed. For example, some of the learners answered Questions 7 as 5–2b= 3b and others went further to solve 5–2b as: 3–b and got the answer b=3. For Question 8 some of the learners' solutions were 3(5e+4) = 15e+12= 27e and others as 3(5e + 4) = 5e+12= 0. See vignette 2 for one example of learners' responses.

Vignette 2: An example of learner's response to questions 7 and 8 of ACAT



The data in Table 1 and vignettes 1 and 2 are used in this study as providing answers to the first research question of the study. In addition to this data the results of lesson observations, particularly in the control group, indicate that the TTM provided lesser opportunities to facilitate the exposition of learners' errors during instruction. In this context the teacher tended to dismiss learners' responses that were seemingly incorrect. There were fewer instances of probing to expose and reveal learners' errors. The opportunities for learners to verbalize their problem solving thoughts were less. These findings are corroborated by data in Table 1. The percentages of error reduction in Table 1 serve as an indication of the success of instruction in mitigating learners' errors observed during a pre-test and post-test in both groups. The success rate in error reduction in control group is relatively lower than in experimental group in almost all areas of algebraic focus treated during the intervention.

Answering the second research question

The second research question asked: What is the comparative effect of CBTM and the TTM on the treatment of learners' errors in Grade 11 algebraic classrooms? Parts of the response to the second research question could also be traced in Table 1. Table 1 shows that CBTM is more superior and effective than TTM in terms of the treatment of learners' errors. The CBTM was more learner-engaging, which created instructional opportunities to expose and address learners' errors. The same may not be said about the TTM learning environment. The superiority of CBTM is further verified by the inferential statistical analysis used to compare two instructions in both groups.

The independent samples t-test and the paired samples t-test were used to test the two assumptions of the study to either accept or reject the null hypothesis and the alternative hypothesis.

The researcher assumed that the scores of the learners in the ACAT test would also indicate the extent to which test takers tended to do errors during problem solving. Meaning, a higher score would be an indication that the learner committed lesser errors and vice versa. In addition, the differences in the mean performance at both the pre- and post-stages of the experiment would be a reflection of the effect of each instruction in mitigating learners' errors. The test scores of both groups are presented in Table 2.

Table 2: Descriptive statistics of pre- and post-test scores in both groups

Pre-test	n	$\overline{x}$	SD	Post-test	n	$\overline{x}$	SD
Experimental group	36	25.80	7.31	Experimental group	36	44.63	9.68
Control group	42	24.14	6.94	Control group	42	30.46	6.83

The results in Table 2 suggest that there was a gain score of (44.63-25.8=18.83). The comparable gain of (30.46-24.14=6.32) is observed in the control group. In terms of the aim of the study the observed gains in the experimental group may suggest a significant reduction in learners' errors in algebra. These observations show that meaningful knowledge construction in algebra occurred when CBTM is implemented. In this context meaningful construction of knowledge suggested that learners tended to do fewer errors when subjected to CBTM instruction.

Table 3: Independent t-test for pre-test scores of experimental and control groups

Group	Mean	SD	t	df	p-value
Experiment	25.80	7.31	0.973	68	0.334
Control	24.14	6.94			
Equal variance assumed	(0.693)				

Table 4: Independent t-test of the post-test scores of experiment and control groups

Group	Mean	SD	t	df	p-value
Experiment	44.63	9.68	7.77	68	0.000
Control	30.46	6.83			
Equal variance assumed	(0.592)				

Table 5: The normality test- experiment group

Normality test	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistics	df	Sig.	Statistics	df	Sig.
Post-test-pre-test	0.127	35	0.171	0.969	35	0.424

Table 6: The normality test- control group

Normality test	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistics	df	Sig.	Statistics	df	Sig.
Post-test-pre-test	0.118	35	0.200	0.962	35	0.261

Table 7: Paired samples statistics for the experimental group

Type of test	Mean	n	SD	SEM	Correlation	p-value
Pre-test experiment	25.8	35	7.31	1.24	0.63	0.000
Post-test experiment	44.63	35	9.68	1.64		

Table 8: Paired Sample t-test for pre-test and post-test scores of experiment group

Test	n	Mean	SD	SEM	t	df	p-value
Pre-test-post-test	35	-18.8	7.61	1.29	-14.6	34	0.000

Table 9: Paired Samples Statistics for the Control group

Type of test	Mean	n	SD	SEM	Correlation	p-value
Pre-test control	24.14	35	6.94	1.17	- 0.54	0.376
Post-test control	30.46	35	6.83	1.40		

Table 10: The paired sample t-test for pre-test and post-test scores for control group

Test	n	Mean	SD	SEM	t	df	p-value
Pre-test-post-test	35	-3.23	11.61	1.96	-1.65	34	0.109

Table 3 shows that the Levene's test for equality of variances was $0.334 > 0.05$, indicating that the homogeneity of variance assumption was not violated. In Tables 1to 10 since the difference in the pre-test and post-test scores was normally distributed, the paired sample t-test was applied. In relation to the hypothesis testing the paired samples t-test was used to test the hypothesis of the study. We found that there was statistically significant reduction in learners' algebraic errors in CBTM learning environments from (25.8 ± 7.31 to 44.63 ± 9.68) ($p<0.05$). However, the improvement of learners' tendency not to do errors amounted to (18 ± 7.61). We found that $p<0.05$, suggesting that it was reasonable to reject H_0 of no effect in favour of H_1. These statistical results confirmed that CBTM was more effective in reducing learners' errors.

CONCLUSION

The results of this investigation revealed that the use of CBTM significantly reduced Grade 11 learners' errors in algebra and improved their performance in the selected algebraic topics than TTM. This study showed that most of the errors occurred in the initial stages of the intervention. However, CBTM was more effective in the treatment of learners' errors thus yielding an improved algebraic performance by learners. CBTM may serve as an effective instructional resource to improve learners' performance in mathematics. CBMT is in line with reformed educational initiatives to emphasize classroom instruction that is more learner-engaging to enhance the acquisition of mathematical problem solving skills. We propose that similar studies be conducted in other areas of mathematics to corroborate the results obtained in this study.

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COMPARING THE TESTING FORMAT AND LANGUAGE USED IN THE GRADE 4 MATHEMATICS ANAS AND THE EXEMPLARS GIVEN

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This paper explores the congruence or dissonance between the Grade 4 Mathematics ANA papers and their corresponding exemplars in terms of the testing format and language use to determine the extent to which the exemplars reflect the ANAs. The extent to which the exemplars ease learners' understanding of the ANAs depends on the degree of commonality between the exemplars and the ANAs. Since the inception of the ANAs in 2011, there has been a manifest consistent underperformance in both numeracy and literacy at all levels. The introduction of the exemplars were meant to adequately prepare the learners for assessments. The latest ANA results do not show a marked improvement in learner performance despite the assessments being preceded by supposedly extensive and intensive practice using exemplars. This then begs the question whether the exemplars themselves mirror the ANAs to the extent that attainment in the exemplars equals corresponding attainment in the ANAs. Little, if any, is known about the extent to which there is a deliberate alignment between the content, language and test formats of the exemplars and the ANAs. It is against this background that the present study aimed, through a comparative case study analysis of two documents, namely, the 2013 mathematics ANA and the 2013 mathematics ANA exemplar, to investigate the extent to which the testing format, and language used in these two documents was similar or different. Although the underperformance of South African learners in literacy and numeracy is a source of grave concern across all the grades where the ANAs are administered, it is particularly so at the transition from Grade 3 to Grade 4 which is a make or break stage marked by phenomenal challenges. One of the major challenge which complicates this transition is to some extent linguistic, since at Grade 4 in South Africa the majority of learners begin learning in English, which is an additional language for most. Findings revealed that there were several inconsistencies in the questioning format and language used in the ANAs and in the exemplars. For example, words from the 'mathematical register' in the 2013 ANAs were not present in the exemplars and some items were in the ANAs but not in the exemplars. There were however, questions in the 2013 ANAs that corresponded, albeit too closely, with those in the exemplars. The similarities and differences indicated the extent to which learners were exposed to some of the mathematical language used in the 2013 ANAs.

Keywords: consistency, format, language, mathematics assessments, mathematical register, questioning

INTRODUCTION AND REVIEW OF LITERATURE

The Annual National Assessments (ANAs) formed part of the 2011 Foundation for Learning (FFL) campaign and emerged as a result of increased international monitoring and evaluation of education in 1990 following the Jomtein conference (Howie, 2012). Since 1994 South Africa has adopted assessments in large-scale testing as a means for increasing achievement in education quality. The ANAs are the most recently introduced national assessments; focusing on the two areas of mathematics and literacy which have been identified as critical areas needing intervention both at primary and secondary levels (DBE), 2012). The choice of subjects to be written has been informed by the recognition worldwide of Literacy and Numeracy as the key foundational skills that expose learners to effective learning in all other subjects (DBE, 2012).The ANAs aim to expose teachers to better assessment practices by providing information about what learners know and can do in mathematics and literacy. ANAs also help districts to identify schools in most need of assistance and the department will then provide the necessary assistance. After the assessments are marked and parents are informed about their children's performance, parents can also be in a position to assist their children where necessary (DBE, 2011). ANAs also aimed to identify the weaknesses in literacy and numeracy development (Frengpong, Reddy & Mackay, 2013). The DBE therefore, provides exemplar questions to schools in the course of the year so as to ensure that teachers and learners are exposed to the kind of questions and "different ways of questioning that may best suit different learning styles of learners, without compromising the skill assessed" (DBE, 2012, p 7). From these exemplars teachers could select some items and compose tests for their own classes. (DBE, 2012). Memorandums are also provided to all schools and tests are marked by the relevant teachers (DBE, 2012). After marking the tests, schools report learner achievement in ANA to each parent.

Language policy in South Africa requires that in the first three years of formal learning the Language of Learning and Teaching (LoLT) should be learners' mother tongue (DBE, 2010). Thus in Grade 4 the majority of learners switch to learning in English as the LoLT. Yet 72, 2% of the Grade 4 children entering English-medium classrooms are non-native speakers of English (Robertson & Graven, 2015). Therefore in assessment terms, Grades 1, 2 and 3, learners are mostly tested in their mother-tongue while in Grades 4, 5, 6 and 9, the ANAs are administered in either English or Afrikaans, the two dominant languages of teaching and learning (DBE, 2014), Thus the majority of South African learners learn in a second or third language they are not familiar with (Setati & Barwell, 2008; Setati, Molefe and Langa, 2008) as they use indigenous languages for communication.

This compromises their ability both to comprehend and express mathematical ideas. Therefore, learners have a challenge of being assessed in a language which is not their home language. Heugh (2006) also confirms a zero level of understanding by children being taught in non-mother-tongue languages. The majority of South African learners therefore, have the unenviable task of simultaneously learning the English language and having to access mathematical concepts through a language in which they are not yet proficient. This fact compromises their ability both to comprehend and express mathematical ideas. To make matters worse, from Grade 3, ANA policy states that teachers are not allowed to read or mediate the language (DBE, 2014) and this means learners struggle with language in the ANAs.

The present study explores the items in the 2013 mathematics ANAs and exemplars to see if the exemplars adequately prepare learners for the ANAs. Given that the majority of learners are tested in a language unfamiliar to them, the exemplars therefore have to prepare learners in terms of the questioning or testing format and language used in the ANAs.

RESEARCH QUESTION

Do the 2013 Grade 4 mathematics exemplars adequately prepare learners for the 2013 mathematics ANAs?

THEORETICAL PERSPECTIVE

This study adopts a socio-cultural view of language and learning. Vygotsky’s (1978) influential theoretical work on language and learning, in which language is considered central to learning and learning as a social process embedded in sociocultural settings, informs the study. The study is guided by the view that language is complex and while essential to learning, complexities of language, and differences in language across discourses, can result in difficulties in learning.

METHODOLOGY

This study is part of a broader PhD study which took a case study approach in which the 2013 mathematics ANAs are compared with their exemplars (provided to schools by the DBE a month prior to the writing of the ANAs in September 2013) the reason being to see if the exemplars prepared learners for the ANAs. This paper focuses on the qualitative documentary analysis in which data were brought forth from the national 2013 Grade 4 mathematics ANAs and exemplars. The method used was a content analysis of available information; hence it was a secondary analysis of available data. Content analysis was the best method to use in the analysis of these educational documents, as Cohen, Manion and Morrisson (2007) contend that educational documents can be analyzed using content analysis. The material for conducting the content analysis was readily available and usually the materials can be made available for others to use in case of replication (Cohen, Manion and Morrisson, 2007).

A comparison was made to see whether the number of questions, length of questions, questioning style/format, nature of language demand, mathematical vocabulary used and content area assessed in the two documents corresponded. Therefore, analysis of item by item was done. This would give a sense of which terms and phrases learners may have encountered before they wrote the ANAs and which would potentially be new to them. Analysis of exemplars was important because the exemplars were used by teachers who participated in the broader PhD study to prepare learners for the assessments. Three categories of questions were used to aid the comparison of items in the two documents. These were:

- Questions with minimal instruction or no language e.g. complete or calculate
- Questions with simple instruction (where simple language has been used) e.g. Complete the following table
- Questions with complex instruction (where several mathematical words have been used) e.g. draw the reflection of the arrow on the vertical dotted line.

The similarities and differences in the questioning format were noted, as well as how the differences could influence the familiarity of the learners with the assessment.

The 2013 mathematics ANA consisted of 11 pages and 19 assessment items. Some items consisted of two parts, others three parts (i.e. had sub questions). The mathematics ANA began with some instructions to the learners on page 1, followed by a practice exercise, and then some points learners needed to take note of. The 2013 ANA exemplar, on the other hand, consisted of 12 pages, with guidelines for the use of the exemplar on the first page. In terms of the length, the ANA test and the exemplar were thus almost the same length with the exemplar being a page longer. As for the actual number of assessment items, the exemplar had 30 items, 11 items more than the 19 in the ANA. This on its own points to a key difference between the two where there was a greater concentration of items per page in the exemplar than in the test with an item page ratio of 19:11 and 30:12 for the ANA and exemplar respectively. From comparative analysis it became apparent that the exemplar had less language and fewer diagrams to allow for more items per page than the actual ANA test. The linguistic similarity of these would only be apparent from an analysis of the items in the two documents. This is the focus in the following section.

ANALYSIS, FINDINGS AND DISCUSSION

Almost all (18 out of 19) of the items in the 2013 mathematics ANAs were word problems. Verschaffel, Greer and De Corte (2000) define word problems as any mathematical exercise where significant background information on the problem is presented as text rather than in mathematical notation. The word problems also may vary in the amount of language used in them. I used three categories of instruction to analyse the language demands of items. These were:

- Minimal instruction or no language e.g. complete or calculate.

- Simple instruction (where simple language has been used. For example, complete the following table).
- Complex instruction (where several mathematical words have been used. For example, draw the reflection of the arrow on the vertical dotted line).

In addition to these categories, I compared the mathematical vocabulary used in the corresponding questions. The format of the exemplar resembled the 2013 ANA format and mathematical content assessed. Table 1 summarises the correspondence across the items in terms of similarities in question types and content in the 2013 ANA and exemplar.

Table 1: Comparison of 2013 ANA and 2013 exemplar

2013 ANA			**2013 Exemplar**		
Item	**Nature of language demand and mathematical vocabulary**	**Mathematical content area assessed**	**Item**	**Nature of language demand and mathematical vocabulary**	**Mathematical content assessed**
1	-simple instruction -Value, digit, rounded off, ratio, multiple, pattern, factor,	Learning Outcome (L.O) -number value -whole numbers (rounding off) -ratio -multiples -patterns -factors	1	-simple instruction -value, digit, pattern, number sequence, rounded off, factor, multiple, ratio	L.O -number value -patterns - whole numbers (rounding off) -factors -multiples ratio
2	-minimal instruction(one word) -no mathematical vocabulary	L.O -completing a number sentence (whole numbers)	4	-complex instruction (expanded notation) -expanded notation	L.O -completing a number sentence in expanded notation
3	-simple instruction -patterns	-completing numeric and geometric patterns	18	-simple instruction -patterns	-completing numeric and geometric patterns
4	-simple instruction -patterns	Number patterns(whole numbers and fractions)	No similar item	___________	__________
5	-level 2 word problem	-financial mathematics	9	-simple instruction	-financial mathematics

	-no mathematical vocabulary	(including buying, selling)		-change	(calculate making or giving change)
6	-simple instruction -calculate	-four operations + - × ÷	7	-minimal (one word) instruction -calculate	-four operations + - × ÷
7	-complex worded instruction -number sentence, difference	-number sentences	No similar item	______	_______
8	-complex worded instruction -hands, clock face, quarter	-time (representing time)	25	-complex worded instruction -clock face	-time (naming the time shown)
9	-simple instruction -no mathematical vocabulary	-time	No similar item	_______	_______
10	-complex worded instruction -grid, position	-viewing of objects (locate position on a grid)	No similar item	______	________
11	-complex worded instruction -reflection, vertical, dotted line	Transformations (drawing lines of symmetry)	22	-complex worded instruction -sketch, symmetrical, 2-D shape	-transformations (drawing lines of symmetry)
12	-minimal worded instruction -convert	-length (conversion between cm and m) -time (conversion between minutes and hours)	26	-minimal worded instruction -no mathematical vocabulary	-length (conversion between km and m) -time (conversion between year, weeks and hours)
13	-simple instruction -flow diagram, input, output, rule	-number sentences (relationship or rule presented in a flow diagram)	No similar item	__________	__________
14	-level 3 word problem			__________	__________

	-no mathematical vocabulary	-capacity (solving life problem involving capacity)	No similar item		
15	-simple instruction -fraction wall, fraction strip, calculate, colour in	-fractions (comparing adding, colouring and problem solving of common fractions)	16	-simple instruction -fraction wall	-fractions (comparing common fractions)
16	-simple instruction -Hexagon, pentagon, quadrilateral, triangle, 2-D shapes	2-D shapes (naming)	21	-simple instruction -2-D shapes, trapezium, pentagon, parallelogram, hexagon	-2-D shapes (naming)
17	-simple instruction -faces, triangular prism, rectangles, triangles	3-D shapes (naming)	20	-simple instruction -2-D shape, faces, rectangular prism	-3-D shapes (naming)
18	-Simple instruction -bar graph, tally, difference	-organising, interpreting and analysing data (tally marks, interpreting data)	28	-simple instruction -tally marks, bar graph, frequency	-organising, interpreting and analysing data (tally marks, completing a bar graph)
19	-no instruction -triangles	Shapes(counting)	30	-no instruction -triangles	shapes(counting)

Table 1 shows that there was no one-to-one ordered correspondence in the items, for example, item 2 on 2013 ANA corresponded with item 4 on the exemplar, item 3 corresponded with item 18 in the exemplar and item 19 corresponded with item 30 in the exemplar. The table also shows that 6 of 19 ANA items did not have items in the exemplar that were similar to them, (i.e. items 4, 7, 9, 10, 13 and 14). These six items were a combination of three simple instruction items and three complex or level 3 word problem items. Thus learners encountered these items for the first time in the ANAs and therefore, the items were unfamiliar to them.

The 2013 ANA test item number 1 consisted of six multiple choice questions. The questions had one sentence with three questions having more than eight words[i] and three having less than eight words. Consideration of the length of a sentence on the basis of whether it had more or less than eight words was based on the understanding that an average sentence should have eight to ten words (Korger, 1992). For this research, and supported by broader literature, a sentence with more than eight words was considered too long for Grade 4 learners who used English as an additional language. Some of the 2013 ANA item 1 questions were very similar in wording and content to the 2013 item 1 exemplar questions. For example:

Table 2: Similar questions in item 1 for 2013 ANAs and exemplar

2013 ANA question	Exemplar question
What is the value of the underlined digit in 3 870? A 80 B 8 000 C 800 D 8	What is the value of the underlined digit in 7 999? A 90 B 9 C 900 D 9 000
The number 6 555 rounded off to the nearest 100 is: A 6 550 B 6 650 C 6 500 D 6 600	1.5 6 423 rounded off to the nearest 100 is: A 6 400 B 6 425 C 6 430 D 6 420

Therefore, these questions were most probably familiar to learners since they resemble the ones in the exemplar. Table 3 summarises the nature of similarities and differences between the ANA test and the exemplar.

Table 3: Similarities and differences between corresponding items

ANA test item	Exemplar item	Nature of similarity/ difference
1	**1**	-similar questioning format -exemplar has 6 more questions than the ANA test item
2	**4**	- similarity in completing a number sentence -different way of questioning
3	**18**	-similar way of questioning -different in geometric patterns to complete
4		-none to compare with
5	**9**	-similar in learning outcome -different way of questioning
6	**7**	-similar questioning of the 4 operations -different in number of questions
7		-none to compare with
8	**25**	-both have complex worded instructions -difference in questioning
9		-none to compare with
10		-none to compare with
11	**22**	-Both items require learners to draw a reflection of the shape --difference in the questioning
12	**26**	-similar learning outcome of conversion -different conversions and number of questions based on conversion
13		-none to compare with
14		-none to compare with
15	**16**	-similar in the use of fraction wall -difference in the questioning and number of questions based on the fraction wall
16	**21**	- similar question and way of asking the question

		-different in number of shapes to name.
17	**20**	- similar way used for asking the questions -both items 17 and 20 had two shapes -different in that item 17, learners had to name the first shape and then name the shapes of the faces of the other shape. In item 20, learners were to name the second shape and then say what shapes made the faces of the two given shapes.
18	**28**	-similar in completing tally table -difference in the questioning and no graph to be drawn in item 18
19	**30**	-Exact wording only replacing triangle with square

From the above analysis of the 2013 mathematics ANAs and exemplar, it is apparent that although in some questions the testing format in the ANAs and exemplar were similar, in several cases there was no correspondence with that of the exemplar which the learners used in preparation for the ANAs.

From the analysis, five observations of the similarities and differences were noted:

1. There were items where the wording and questioning format were almost the same. For example, ANA question 1.1 and 1.3 and ANA exemplar questions 1.2 and 1.5, ANA question 16 and ANA exemplar question 21, ANA question 17 and exemplar 20.2.
2. Several items, though not exactly the same had similar language, length and format. For example, ANA item 1 and exemplar item 1; ANA item 3 and exemplar item 18; ANA item 12 and exemplar item 26.
3. Six out of nineteen items were in the ANAs but not in the exemplars. For example, ANA items 4, 7, 9 and 10.
4. Three ANA items not in the exemplars (i.e. items 7, 10 and 14) were of a higher level of language complexity.
5. Several items were similar in the learning outcome assessed but were asked in different ways. For example, ANA item 5 and ANA exemplar item 9 or ANA item 11 and ANA exemplar item 22.

The analysis above enabled me to gauge the extent to which the language of the ANA items, the formatting and the content may have been accessed by learners in the exemplars prior to the ANAs. Although several ANA items' language, length and questioning format were similar to those items that were in the exemplar, this however, does not imply that learners necessarily knew the meaning of questions asked in the ANAs, but does indicate prior exposure to the language and questioning format in the exemplars. The analysis revealed that most questions in the ANAs corresponded with those in the exemplar but also revealed some inconsistencies in the questioning format and the language used in the 2013 ANAs and ANA exemplar. The similarities and differences indicated the extent to which learners were exposed to some of the mathematical language used in the 2013 ANAs.

On the similarities in the vocabulary used, it is assumed that mathematics vocabulary like 'rounded off', 'number pattern', 'factor', 'multiple', 'ratio' and 'calculate' were familiar to learners since they were used in both documents. However, other words from the 'mathematical register' in the 2013 ANAs mathematics, for example, 'the difference between', 'reflection', 'vertical' and 'hands' were not present in the exemplar and thus may not have been familiar to learners. As noted before, this analysis provided some context for learner's prior exposure to the language of some of the ANA questions before they wrote the assessments. However, the broader PhD study revealed that this prior exposure to the language or the question format in the exemplar did not necessarily lead to high levels of success in some questions in the 2013 ANAs (for example, question 12 of conversion) while on the other hand some learners performed relatively higher on items that had strong similarity with exemplar items, for example, items 1 and 16 and 17.

Having seen that consistency of questioning format and language in the ANAs and exemplars leads to learners' better performance in the ANAs, the questions that have to be asked now are: How much consistency between the ANAs and exemplar would be desirable? Doesn't too much consistency lead to the danger of memorization of questions by learners? It is important that when the ANAs and exemplars are set for learners who use English as an additional language in assessments like the ANAs, there have to be a balance of some consistencies in the testing format and language used in both documents so that learners get familiar to the language and with the way of questioning while at the same time it doesn't lead to drilling learners to pass without understanding the mathematics. If the consistency leads to drilling learners to pass without understanding the mathematics, this would make the validity of the ANAs questionable and lessen the reliability of the information of ANAs (van der Berg, Taylor, Gustafsson, Spaull & Armstrong, 2011).

On the other hand, too much inconsistency whereby several items are found in the ANAs but not in the exemplar may disadvantage learners when they encounter a number of mathematical vocabulary and questioning format for the first time in the ANAs. Therefore, inconsistency in vocabulary used and questioning format should be minimal to allow learners to learn new things. Drawing on the Vygotskian perspective of the Zone of Proximal Development (ZPD), learners should be able to demonstrate levels of mathematical competence when linguistic mediation is provided and then gradually develops the ability to do certain tasks without help. Thus Vygotsky (1978) observes that the roles of education and in context of this study, the new material is to give children experiences that are within their ZPDs, and encourage and advance their individual learning. As long as the new information is within the ZPD, children are capable of learning it.

CONCLUSION

The findings revealed that most questions in the ANAs corresponded with those in the exemplars but revealed some inconsistencies in the questioning format and the language used in the 2013 ANAs and ANA exemplar. The similarities and differences indicated the extent to which learners were exposed to some of the mathematical language used in the 2013 ANAs. However several words from the 'mathematical register' in the 2013 ANAs were not present in the exemplars and thus may not have been familiar to learners. Given that Grade 4 is the first year that the majority of learners will be assessed in English as a second language it would seem important that the exemplars familiarise and support learners in developing the language needed for the upcoming assessments.

Consistency between the ANA items and the exemplar items is important as it possibly familiarises learners with what is expected in the ANAs but too much consistency may encourage memorization. For this reason, further investigation into how much consistency would be desirable is required and to also find out how a balance can be achieved such that the consistency is more about familiarity with the way of questioning and not leading to drilling learners to pass without understanding the mathematics.

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THE IMPACT OF TEACHING AND LEARNING CIRCLE GEOMETRY USING GEOGEBRA ON GRADE 11 STUDENT'S PROBLEM SOLVING SKILLS

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Circle geometry is among the new topics introduced in paper two of the Curriculum Assessment Policy statements (CAPS) mathematics curriculum. It's teaching and learning seems to be a challenge to teachers and students respectively. Hence, the need to find better ways of teaching the topic and improve students' problem solving skills led to this study. APOS theory and Van Hiele theory of geometric thought were used as the framework. Following a non-equivalent group quasi-experimental design two secondary schools (experimental and control groups) located in a rural area in Limpopo Province of South Africa were used as sample. The experimental and control groups were taught using GeoGebra and traditional 'teacher-talk-and-chalk' method respectively. The findings suggest that integrating GeoGebra can improve student's problem solving skills in circle geometry.

Keywords: Circle geometry, Geogebra, learning, mathematics, student and teaching

INTRODUCTION

Problem solving in mathematics refers to the ability to perform mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development (Hiebert & Weane, 1993). According to Stacey (2005), problem-solving skills involve a range of processes including analysing, interpreting, reasoning, evaluating and reflecting. Hence, students need deep mathematical knowledge to become efficient in mathematics problem solving. However, over the years, the mathematics pass rates of the students in general and their mathematical problem solving ability at all levels have not been impressive and Limpopo province has been one of the least performing provinces in mathematics (National Senior Certificate Examination Diagnostic Report, 2014). Given the poor state of mathematics education in the country in general and Limpopo province in particular, the inclusion of new topics in the Curriculum Assessment Policy statements (CAPS) was a curriculum change of a big magnitude to teachers and students alike. Circle geometry is one the new topics introduced in CAPS mathematics curriculum. It's teaching and learning poses a challenge to teachers and learners. In the National Curriculum Statement (NCS), phased out in 2013, circle geometry was one of the optional topics meaning that it was only offered by schools and candidates that wished to and they had to write a separate examination paper (paper 3) where the optional topic were tested in addition to the compulsory papers 1 and 2. In the NCS, majority of the students that opted for paper 3 between 2009 and 2013 did not do well in the examination (Department of Education, 2013). Hence, most schools discouraged their students from opting for paper 3. The majority of the mathematics teachers in the Further Education and Training (FET) band (Grades 10 – 12) are pedagogically ill-equipped to effectively teach the new topics and help students to effectively solve problems on those topics as most of the teachers were never taught the topic in their schooling.

Many studies have advocated the integration of computer technology into mathematics teaching and learning to enable students become efficient in solving mathematics problems. Bester and Brand (2013) argue that technology assists students to make meaning of the learning material, and the interactive effects of sound, animation, narration and additional definitions provided by technology (computers) appeal to today's learners, motivating them to concentrate better and to achieve higher average scores. Some studies in South Africa (e.g. Achary, 2011; Ogbonnaya, 2010) have supported the use of computer technology to support students learning of mathematics concepts.

This study explores the impact of teaching and learning circle geometry using GeoGebra® on grade 11 student's problem solving skills. GeoGebra is dynamic mathematics software designed for teaching and learning mathematics in secondary school and college level. The software can easily be used in most mathematical disciplines or topics such as geometry, algebra, statistics and calculus (Hohenwarter & Preiner, 2007).

THEORETICAL FRAMEWORK

This study adopted APOS theory and Van Hiele of geometric thought as the joint theoretical framework. The acronym APOS stands for Action, Process, Object, and Schema. APOS Theory is a theory of how mathematical concepts can be learned based rooted in the work of Jean Piaget, and its fundamental ideas were first introduced in the early 1980s (Dubinsky 1984), and since then, extensive development and application has been carried out by researchers, curriculum developers, and teachers in many countries.

The Van Hiele theory of geometry thin thought king is a framework that describes the development of geometrical reasoning propounded by Pierre Van Hiele and Dina Van-Geldorf. Van Hiele theory describes how students learn geometry through five levels (visualization, analysing, abstraction, deduction and rigor).

METHODOLOGY

This study was a quasi-experimental study, of non-equivalent comparison group design. The experimental group was taught using GeoGebra, while the control group was instructed using the traditional teacher 'talk and chalk' method. The two groups were from two different schools. The control group had 25 participants (10 girls and 15 boys), while the experimental group had 22 participants (9 girls and 13 boys).

Each group was taught by a different teacher. The teachers were holders of university degrees in mathematics and had over twenty years of experience in teaching high school mathematics.

Data were collected by means of pre-test and post-test. The pre-test of 15 multiple choice questions was administered to both the control group and the experimental group. It was based on basic concepts on circles and geometry in general. The pre-test was used to determine if the classes were comparable at the outset by determining the baseline knowledge or preparedness for the learning of circle theorems topic. Data were analysed using descriptive and inferential statistics. The analysis of the pre-test scores of the groups showed that there was no statistically significant difference between the groups.

The post-test was a comprehensive summative 30 questions test based on the principles of Van Hiele's theory on levels of geometrical understanding. The allocation of marks for this test was dependent on the level at which the question belonged according to the Van Hiele's theory of geometrical understanding.

FINDINGS

Level 1: Visual

There was a statistically significant difference in the average marks of the experimental group (Mean mark = 7, Standard deviation = 0) and the control group (Mean = 5.04, Standard deviation = 1.98), $t\ (45) = 4.71$, $p = 0$) at van Hiele level 1. These results suggest that the use of Geogebra in the teaching and learning of circle geometry can result in improvement of students at Van Hiele level 1.

Level 2: Analysis

There was a statistically significant difference in average marks of the experimental group, (Mean = 13.45, Standard deviation = 1.18) and average (mean) mark of the control group (Mean = 9.32, Standard deviation = 3.73), $t\ (45) = 4.98$, $p = 0$). These results suggest that the use of GeoGebra software in the teaching and learning of circle geometry can result in improvement of students, problem solving skills at Van Hiele level 2.

Level 3: Abstract

The independent samples t-test showed that there was a statistically significant difference in average mark of the experimental group (Mean mark = 10.36, Standard deviation = 5.18) and control group (Mean = 8.28, Standard deviation = 4.66), $t\ (45) = 1.45$, $p = 0.15$). These results suggest that the use of GeoGebra software in the teaching and learning of circle geometry can result in improvement of students, problem solving skills at Van Hiele level 3.

Level 4: Deduction

The independent samples t-test ay level 4 shows that there was a statistically significant difference in average mark of the experimental group (Mean mark = 12.23, Standard deviation = 7.55) and control group (Mean = 8.88, Standard deviation = 5.37), $t (45) = 0.15$, $p = 0.15$). These results suggest that the use of GeoGebra software in the teaching and learning of circle geometry can result in improvement of students' problem solving skills at Van Hiele level 4.

Level 5: (Rigor)

The independent samples t-test showed that there was a statistically significant difference in average mark of the experimental group (Mean mark = 5.64, Standard deviation = 3.49) and control group (Mean = 4.28, Standard deviation = 3.32), $t (45) = 1.37$, $p = 0.18$). These results suggest that the use of GeoGebra software in the teaching and learning of circle geometry can result in improvement of students' problem solving skills at Van Hiele level 5.

DISCUSSION

The findings from this study show that GeoGebra enhanced the students' problem solving skills on circle geometry. The results of the analysis of t-test on problem solving skills of students taught using GeoGebra and those taught using conventional method of instruction indicate a statistically significant difference in favour of the students taught with GeoGebra. The students exposed to GeoGebra achieved higher scores compared to the control group students. The findings of this study agree with Okoro & Etukudo (2001), Paul & Babaworo (2006), Egunjobi (2002), Karper, Robinson, & Casado-Kehoe (2005) that students taught with Computer Assisted Instruction (CAI) packages in chemistry, mathematics and Education in general respectively performed better than those taught with normal classroom instruction.

CONCLUSION AND RECOMMENDATIONS

The findings of this study suggests that integration of GeoGebra software in the teaching of circle geometry to grade 11 students will likely lead to improvement in student's problem solving skills. The results of this study are similar to other studies that indicate that the use of technology in classroom instruction enhances student outcomes.

This study recommends that mathematics teachers should be encouraged to use this software and many others in the mathematics classes. Teachers should be introduced to various mathematical software's in order to experience their effects on themselves and their students.

Future studies should be carried out on the effect of integration of GeoGebra software on students' problem solving skills, achievement and motivation for longer periods, using much larger randomized sample sizes, and at different schools with different ethnic composition, and socio-economic status that reflects the entire South African economy.

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LEARNING STYLES OF MATHEMATICS AT AN URBAN UNIVERISTY IN SOUTH AFRICA

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The presentation focusses on learning styles in mathematics classrooms. The study was situated at a university in South Africa.

Keywords: learning style, mathematics, South Africa

There has been much debate about the effects of learning styles. In terms of a balanced teaching and learning approach, an awareness of the value of learning styles is very much of use in mathematics. This study has two main purposes: to identify the prominent learning styles in mathematics classroom; and to provide a view based on a balanced instructional approach to empower as many students as possible with multidisciplinary skills. For this purpose, an explanatory sequential mixed-method was used. In the first quantitative phase, the inventory consisting of 44 items with a five-point Likert scale was used and 302 questionnaires were returned. In the second qualitative phase, interviews with the selected participants were conducted. A content analysis approach was adopted to analyse data and found three relevant categories in terms of suitable instruction. The findings from the quantitative and qualitative phases have been integrated along with existing literature.

Except the dimension of processing information ('Active-Reflective'), there were clear distinctions. Students tended to perceive information through sense ('sensing'), responded strongly to visual forms of information ('visual') and preferred to learn step-by-step ('sequential'). These learning styles were not compatible with lecture method which is very favourable for intuitive and verbal students. Practical examples and problem-first instructional methods with appropriate visual presentations should be considered for sensing and visual students. However, students should build their skills in both their preferred and less preferred modes of learning.

EXPLORING SECONDARY MATHEMATICS TEACHERS' EFFORTS OF MAKING-SENSE OF THE MATHEMATICS THEY TEACH

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This presentation draws on data from a larger study that explored the experiences of secondary BEd (in-service) mathematics teachers, registered at a South African university, regarding their facilitation of mathematics using a problem-solving approach. The focus of this presentation is on the classroom practices of these teachers related to their attempts of making-sense of the mathematics they teach while participating in the BEd programme. This presentation provides ideas on how secondary mathematics teachers can make-sense of the mathematics they teach. The presentation concludes with a discussion of some practical strategies that can be explored to make-sense of mathematics.

Keywords: mathematics, secondary school, teach, teachers

INTRODUCTION

Many learners spend approximately 80% of their time in class listening to teachers without teachers making-sense of the mathematics they teach (Armbruster, 2000). Making-sense involves the teacher making explicit reference to mathematical conventions, symbolisms, definitions, axioms and theorems. According to Armbruster (2000), 63% of teachers in Science, Technology, Engineering, and Mathematics (STEM) fields use "extensive teaching" in most or all classes. Such practice is contrary to a constructivist theory which views knowledge as actively constructed by the learner. From a constructivist perspective, effective learning involves learners constructing mathematical relationships for themselves which enhance making-sense (Twomey-Fosnot & Dolk, 2001). Twomey-Fosnot and Dolk (2001) argue that mathematics teaching should be a making-sense process, with constructivist theory supporting these arguments. Fou-Lai (2000) states that activities such as investigations, intuitive reasoning, problem posing, learning with manipulatives and learning with new technologies are considered valuable for mathematics learning and for teachers to make-sense of mathematics they teach. Unless such mathematical activities are incorporated into mathematics teaching, the realities of making-sense of mathematics cannot be achieved.

The extent to which all learners, including those with learning difficulties, are afforded opportunities to learn in ways that support construction of relationships can be linked to teachers' knowledge and beliefs not only about learners (Yackel & Rasmussen,

2003) but also about pedagogy. Askew, Brown, Rhodes, Johnson and Wiliam (1997) identified the most effective teachers of mathematics as 'connectionists'. Connectionist teachers demonstrate a making-sense approach to mathematics teaching that is rooted in constructivism. Such teachers, do not view learning mathematics as simply being about the assimilation and recall of facts; rather they consciously encourage learners to develop their understanding of the relationships and to establish connections between concepts and processes. This model of teaching takes into account the difference between the mathematical understanding of teachers and that of learners (Bills, 1998) and is characterised by a culture of learning evidence by focused discussion between the learners themselves and between the learners and the teacher.

My presentation draws on data from a larger study that explored the experiences of secondary BEd (in-service) mathematics teachers, registered at a South African university, regarding their facilitation of mathematics using a problem-solving approach. However, in this paper the researcher focuses on the classroom practices of these teachers related to their attempts of making-sense of the mathematics they teach while participating in the BEd programme. Hence, the practices of these teachers to establish their efforts of making-sense of the mathematics they teach became the focus of this paper. This presentation provides ideas on how secondary mathematics teachers can make-sense of the mathematics they teach. The presentation concludes with a discussion of some practical strategies that can be explored to make-sense of mathematics.

THE STUDY

In the study, the researcher followed a multiple-case study design. The researcher purposefully selected four teachers from a group of 12 registered Bed (in-service) teachers in a certain district teaching in four different schools. There were 12 mathematics teachers registered in the BEd (in-service) programme in the district at the time of the study. The interpretive qualitative paradigm underpinned this research study in that it mainly places a high emphasis on the participants to generate an understanding of their experiences, context and ultimately, their reality. Data collection took place during the teachers' third year of study when they have been exposed to techniques of making-sense of the mathematics they teach. The researcher used semi-structured interviews and classroom observations to generate data regarding to how mathematics teachers' make-sense of the mathematics they teach. Content analytic procedures constituted the base of analysis in this study. Data generated from interviews and classroom observations were analysed by identifying categories relating to the research questions and the literature explored.

FINDINGS

The findings of this study indicated that while the four teachers seemed enthusiastic about making-sense techniques and intended to implement such techniques in their classrooms, they continued to teach in predominantly teacher-centred ways. The findings suggest that these teachers still facilitate mathematics lessons using a 'traditional' approach, namely 'telling and showing'. Furthermore, the findings showed that there seems to be a mismatch between what the four selected teachers advocated and what actually happened in their classrooms. The interview findings did not correspond well with the observational findings. The latter findings illustrated that the teachers regularly talked without attempting to give meaning to concepts and that little was done to promote making-sense.

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BREATHING NEW LIFE INTO AN OLD DEBATE ABOUT THE MATHEMATICALLY GIFTED CHILD

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This theoretical paper argues that instead of working harder, we need to work smarter towards closing the performance gap. We have been working harder for over two decades now but we have not achieved the desired results. Therefore there is need for a new mind set in order for the South African education system to achieve its intended goals.

Keywords: Gifted child, mathematics

INTRODUCTION

There are a number of noble ideas which have been raised in this year's conference theme statement. For example the theme suggests the need to reclaim our African pride through mathematics teaching. Mathematics is described as a subject that drives all human development with professions such as Engineering, Medicine, Statistics, Actuarial Science, and Chartered Accountancy all being mathematically driven. This position is further confirmed when one considers that of the 100 identified scarce skills in South Africa, 93 require a decent pass in matric mathematics. An implication drawn for practice is that without a pool of mathematicians, Africa and South Africa in particular, will not be able to compete with the rest of the world in terms of economic growth and scientific research. All these statements suggest that as a nation we know our desired goal but despite this clear vision results from international mathematics competitions as well as our own national assessments indicate that we are not moving anywhere closer to achieving it. Spaull (2013) confirms this sad reality by providing an overview of the quality of education in South Africa since the transition to democracy over two decades back. From his analysis the weight of empirical evidence suggests that there is an on-going crisis in the country's education, and that the current system is failing the majority of our youth. Given this background the theme then suggests that we need to work hard(er).

While this theoretical paper concurs with all these observations; as a point of departure it argues that instead of working harder as the theme suggests, we need to work smarter towards closing the performance gap. We have been working harder for over two decades now but we have not achieved the desired results. Therefore there is need for a new mind set in order for the South African education system to achieve its intended goals.

THEORETICAL FRAMEWORK

Repenning & Sterman (2002) provide a useful framework for thinking about the challenges associated with implementing improvement programs as well as offering some practical suggestions to increase the chances that the next such effort will succeed. While their theory initially emerged from improvement initiatives in a major automaker, the dynamics of improvement that emerged thereof are common over a wide range of organisations that have been studied. As a result their model is quite generic, has been used by many other researchers in a wide range of situations including educational settings (Kennedy, 2010). The starting point for understanding this model is the *Desired Performance* of any organisation – the thermostat setting. Goal setting is one of the basic tools used by organizations to assist in charting a direction and working towards achieving it. Once organisational goals have been set, managers then constantly compare those deisred goals with the organisation's *Actual Performance* to determine a *Performance Gap* if any. When the actual performance deviates from the desired goals, Repenning & Sterman (2001) argued that managers hoping to close the performance gap have only two basic options. Firstly they can respond to a performance shortfall by increasing the pressure on people to work harder - *The Work Harder* loop B1. A second option to close a performance gap is by increasing the pressure on people to improve capability – *The Work Smarter* loop B2. Both are considered as balancing loops that work to keep an organisation at the performance standard - the thermostat setting.

The working smarter loop as a recommendation

In the Repenning & Sterman (RS) model time is allocated between working on tasks (work harder loop) and on investment in improvement (work smarter loop). The model recommends that the more time is invested in improvement activities the more capability is built up in the organization and the more the performance gap is closed. Consistent with the model, the work smarter loop would imply that in order for the South African education to close the persistent performance gap there is need to invest in the improvement of our capability. Helfat (2003) defines this organisation's capability as the combination of human capital (people's skills and knowledge), social capital (relationships between people) and organisational capital (the organisation's processes) and aligning them such that each support the others. Of the three combinations, in education improving the capability of teachers has always taken center stage in many countries including South Africa, with some members of the education community even suggesting that schools be closed for two years and teachers go back for training during that period. However Kennedy (2010) warns us that if all our ideas about teacher quality, are based on the Fundamental Attribution Error, then our efforts to improve teachers will yield only marginal effects on performance. Empirical evidence from many studies remind us that the qualities teachers bring with them to their work are not enough to ensure better teaching practices. For example in a study of secondary schools, Cusick (1983) concluded that the central problem facing those schools was that of "containing" the students who did not want to be there. So when we use value-added measures of student achievement to assess teacher quality, we may still fail to account for all the ways that individual students can influence the achievement of the class as a whole. Cohen (1988) has reminded us that, ultimately, teaching is an attempt to change other human beings (students), and that such enterprises cannot succeed unless the other human beings cooperate. This suggests that it is time to look beyond the teacher to the teaching situation itself and especially the student characteristics. Similarly this paper argues that our capability to improve the South African education system does not only reside in the school teachers. Instead there is a critical component of our capability which the system has neglected for decades – the gifted learners. In a South African study by Oswald & de Villiers (2013) one school principal lamented on this neglect:

> I do feel that the gifted learners should come into their right. They are the future of South Africa. We can all try to do something for the child that struggles, but when we think about our future, the gifted child is the one that needs the attention and it does not happen. This is really sad. We all try to throw our rescue buoys for the child who does not want to work, but the child who can really make a difference for the country, this child is ignored. It is a crying shame (principal 8).

This paper argues that South African researchers and policy makers may also have been ensnared in this paradox of improvement and in the process have overlooked and totally ignored the capability or potential that resides in their gifted students which could help us close the performance gap. Although the justification for nurturing the potential within academically talented youth has been based on our observations in talented sports persons, Lewis Terman's Genetic Studies followed by the Study of Mathematically Precocious Youth – SMPY (Lubinski, Benbow & Kell, 2014) have tracked mathematically gifted youth over decades and are arguably among the most famous longitudinal studies in psychology to date. These studies provide empirical evidence that mathematical precocity early in life predicts later creative contributions and leadership in critical occupational roles. These studies confirm that early manifestations of exceptional mathematical talent did lead to outstanding creative accomplishments and professional leadership. Results from these studies have shown that mathematically talented males and females became the critical human capital needed for driving modern day, conceptual economies.

In South Africa, responding to what they view as a sense of complacency about investment in future innovations, both the NPC (2011) and the Department of Science & Technology task team recommended that opportunities for excellence be provided for the nation's most talented students. According to the NPC's recommendations many of the new graduates between now and 2030 must be in the critical skills categories, such as engineering, actuarial science, medicine, financial management, and chartered accountancy and so the downward trend in the number of learners who pass matric with mathematics must be reversed. This paper argues that one of the definite ways that can contribute to the achievement of these goals in the long term is to ensure that the education system recognises, identifies and deliberately structures a curriculum for the most potential students – the gifted. The single most important investment any country can make is in its people." In the 21st century economy the potential contribution of the gifted and talented to the global economy is becoming increasingly important, which is why policy makers and the leaders of business and finance express a growing interest in gifted education in its various formats. In Europe as well as the United States of America policy makers are urged to meet the needs of their intellectually precocious youth because they represent "extra-ordinary human capital for society at large" (Bleske, Rechek, Lubinski & Benbow, 2004, p. 223). From this perspective, the gifted have been described as "the world's ultimate capital asset" (Toynbee, 1967), and also that they guarantee a constant reservoir of individuals who will lead, both research and development, thus continuing to propel recruitment of the community, the State, and humanity at large toward a knowledge-based economy (Sever, 2011). In many African countries economic and scientific stagnation is the order of the day because we have never bothered to invest in our intellectually precious youth. A *Work Smarter Loop* would require us to invest in these individuals who will yield high returns for all humanity through their exceptional skills thereby closing this persistent gap between desired performance and actual performance.

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